VEKTORANALYS
HT 2014

ED1110 Vektoranalys 4.5 hp

INTRODUCTION

1. TEACHERS

Lorenzo Frassinetti	*e-mail: lorenzo.frassinetti@ee.kth.se*	
Richard Fridström	*e-mail: richard.fridstrom@ee.kth.se*	*group A*
Simon Tholerus	*e-mail: simonth@kth.se*	*group B*
Pablo Vallejos Olivares	*e-mail: pablova@kth.se*	*group C*

Check on the participant list which group you belong to!

2. THE KURS-PM

Copies are here on the desk.
Or can be downloaded from the web site.

3. THE BOOK

Ramgard, *Vektoranalys*, 3:e upplagan, ca 140 kr.
Can be found at Kårbokhandeln.

4. THE COURSE WEBSITE

https://www.kth.se/social/course/ED1110/

From the website it is possible to download:
- the kurs-PM
- the slides of each lesson (*but make your own notes! Not everything will be on the slides*)
- some solved exercises

1. KURSNÄMD

- You should elect three class representatives. One for each group. Please, decide by Friday!
- They will communicate your opinion to me in order *(hopefully)* to improve the way of teaching.
- We will meet them twice (during the second week and the fourth week of the course)
- During these meetings we will have a (free) lunch at the Syster-o-bror restaurant.

2. STRUCTURE OF THE COURSE

each week is divide into three parts:

-1- föreläsningar (Lorenzo Frassinetti)

-2- övningar (Richard Fridström, Simon Tholerus, Pablo Vallejos)

-3- räknestugor (Richard Fridström, Simon Tholerus, Pablo Vallejos)

Check on the participant list which group you belong to for the "övningar" and the "räknestugor"!

Which class for övningar and räknestugor?

1- Check on the participant list which group you belong to

Förnamn	Efternamn	Personnummer	Epost	Program	Underskrift	Grupp
				CELTE		A
				CELTE		A
				CELTE		A
				CMIEL		A
				CELTE		A
				CELTE		A
				CELTE		A
				CELTE		A

2- Check on the course schedule the class location:

Wed 3 sep 10:00-12:00 Övning

HT 2014 CELTE

EXERCISE

Location L41 L42 L43

First class: Group A
Second class: Group B
Thirs class: Group C

The location is different every time! Remember to check!

Föreläsningar

Each lesson (3hours) presents two main topics.
Each topic is presented starting from an introductory problem.

Föreläsning 1 sid 3-28
Kapitel 1-3. GRADIENTEN.
Hemuppgift: Om potentialen, avsnitten 1.1-1.3 och enkel räkneuppgift.

Föreläsning 2 sid 29-50
Kapitel 4-5. LINJEINTEGRALER, YTINTEGRALER.
Hemuppgift: Om arbetsintegraler, sats 4.3 samt exempel 5.4.

Föreläsning 3 sid 51-82
Kapitel 6-7. GAUSS SATS, STOKES SATS.
Hemuppgift: Om Stokes sats, exempel 6.5 samt tal 58.

Föreläsning 4 sid 99-122
Kapitel 10. KOORDINATTRANSFORMATIONEN.
Hemuppgift: Om rotationen, exempel 10.5 samt tal 90.

Föreläsning 5 sid 83-98
Kapitel 8-9. NABLA OPERATORER, KARTESISKA TENSOR, INTEGRALSATSER.
Hemuppgift: Om nablaoperatorn, indexräkning samt tal 72.

Föreläsning 6 sid 123-150
Kapitel 11-12. NÅGRA VIKTIGA VEKTORFÄLT, LAPLACE OCH POISSON EKVATIONER.
Vi blickar också tillbaka på kursen för att få ett helhetsintryck.
Hemuppgift: Om Greens satser, virveltråden samt exempel 12.3.

Övningar

A. The aim is to:
  (1) give practical applications of vector analysis
  (2) train you to go from problem formulation to the final solution

B. "Övningar" (2h) is divided in two parts:
  (1) The teacher will solve some problems related to the topics discussed during the föreläsningar.

  First a simple exercise.
  Second, a more complicated exercise.
  Third, an exercise taken from older exams (only if this is possible).

  (2) In the last 20minuts (+5min for correction):

  you will be divided in groups and you will try to solve a new problem.
  The solution will be evaluated and will give points to pass the exam.
  In the last five minutes the teacher will show the solution
  and you will correct your own exercise.

Räknestugor

“Räknestugor” (2h) is divided in two parts:

1. FIRST PART
   The teacher will present some problems and then you will try to solve them.
   The teacher will be available for any question and clarification.

2. SECOND PART, last 20minuts (+5min for correction)
   You will solve individually a problem.
   The solution will give points for the exam.
   In the last five minutes the teacher will show the solution
   and you will correct your own exercise.

CONTINUAL EXAMINATION

If you are "active" during the whole course, you might be able to reach "E" even without the final exam.

"Active": (a) homework

(b) participation to class teaching.

You will get points for:

(a) home assignments *(given on the first lesson of the week)*

The solution must be given to the teacher on the first lesson of the following week.
Each student must hand in its own assignment. Home assignment solved in group are NOT accepted.
Each assignment can give up to 1 point (4 exercises, 0.25 points per exercise) per week.

(b) problems solved in groups during "övningar"

You will have 20min. Three students per group (groups are made by the teacher!)
Marks are only Pass or Fail.
The problem is "passed" if there are no logic errors in the solution
Group problems give maximum 2 points in total (if you have at least 4 passed problems).
1 point if you have from 2 to 3 passed problems.
You can use your notes and the book.

(c) problems individually solved during "räknestugor".

You will have 20min.
Marks are only Pass or Fail.
Individual problems give maximum 2 points in total (if you have at least 4 well solved problems).
1 point if you have from 2 to 3 well solved problems.
You can use your notes and the book.

IMPORTANT: students are allowed to participate in group problems only if they have attended the whole 2h lesson.

9 points over 10 are necessary to pass the course with mark **E**.

FINAL EXAMINATION

- If you do not reach enough points during the course (9 points over 10) or

- if you want a grade higher than E,

you have to participate to the **FINAL EXAMINATION** (3h +1h for correction).

- 6 exercises (3 points each):
  - Theory : 2 exercises ($\Rightarrow$ 6 points maximum)
  - Calculation : 4 exercises ($\Rightarrow$ 12 points maximum)

18 points maximum

Grade:	F	FX	E	D	C	B	A
Minimum points:	-	7	9	10.5	12	14	16

FX: You can ask to have "complement exercises".
Remember that is the student that needs to contact the teacher to ask for the complement.
You will have 6 weeks to solve these exercises. But the maximum grade is E

Or you can repeat the exam in january.
Points from continual examination will be valid only till **Jan. 2015**

FINAL EXAMINATION

How to add the points obtained during the course
(home, group and individual exercises) to the final examination?

Theoretical part	$\Rightarrow$ **<u>9 points</u> maximum**
- theory *(2 exercises in the final examination)*	$\Rightarrow$ 6 points maximum
- home exercises (the 6 homeworks)	$\Rightarrow$ 6 points maximum

So, if you have 6 points from homeworks and 6 points from theory
you do not get 12points but only 9 points!

Calculation part	$\Rightarrow$ **<u>12 points</u> maximum**
- calculation *(4 ex. in the final examination)*	$\Rightarrow$ 12 points maximum
- group exercises	$\Rightarrow$ 2 points maximum
- individual exercises	$\Rightarrow$ 2 points maximum

This is because to get high grades you need both to:
- know theory and
- be able to do calculations

TENTAMEN

Week 43 2014

- **Fri 24 oct 09:00-13:00** Tentamen

  EXAMINATION
  Location L21, L22, L41, L42

Week 2 2015

- **Thu 8 jan 09:00-13:00** Omtenta

  OMTENTA
  Location D34

3h for the exam + 1h for correction

You will mark your own examination!!
After the examination you have to remove **everything** from your bench.
I will distribute a red pen and the solution.
With the red pen you have to correct your own exam!

WHY VECTOR ANALYSIS?

VECTOR ALGEBRA

sum,
subtraction,
scalar product,
cross product

} of a vector

VECTOR ANALYSIS

derivation
and
integration

} of combinations of scalars and vectors

Essential tool in many areas of engineering and physics

WHY VECTOR ANALYSIS?

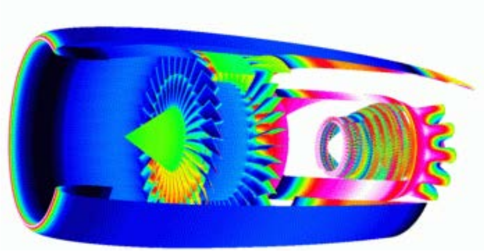
HYDRODYNAMIC: Navier-Stokes equations

(meteorology, aero-space engineering, turbulence…)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \overline{v} = 0$$

$$\nabla \cdot \overline{v} = 0$$

$$\rho\left(\frac{\partial \overline{v}}{\partial t} + \overline{v} \cdot \nabla \overline{v}\right) = -\nabla p + \mu \nabla^2 \overline{v} + \overline{f}$$



WHY VECTOR ANALYSIS?

THEORETICAL ELECTRONICS: Maxwell equations

(electronic engineering, telecommunications…)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{J}$$

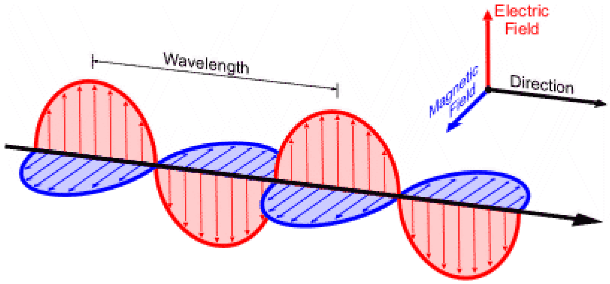
$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Electromagnetic waves:

(electronic engineering, telecommunications…)

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial \vec{j}}{\partial t}$$

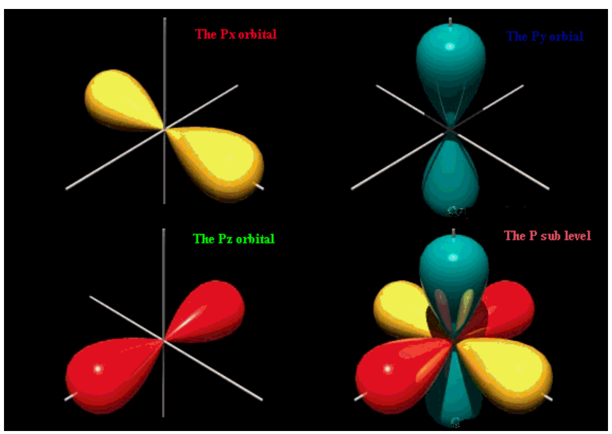


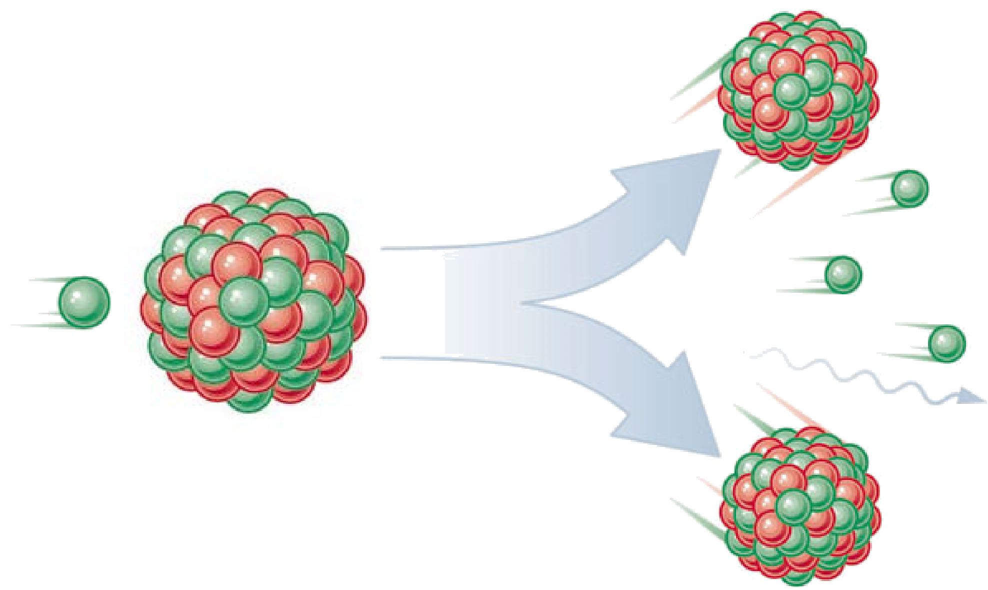
WHY VECTOR ANALYSIS?

MODERN PHYSICS: Schrödinger equation

(atom description, nuclear physics…)

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$





WHY VECTOR ANALYSIS?

PLASMA PHYSICS: Magnetohydrodynamic equations

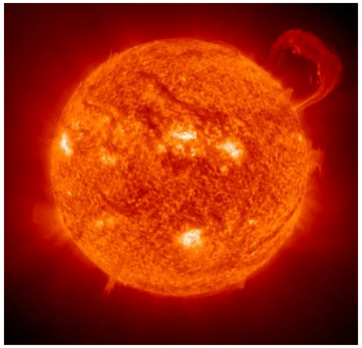
(dynamics of a electrically charged fluid "plasma": nuclear fusion, space physics…)

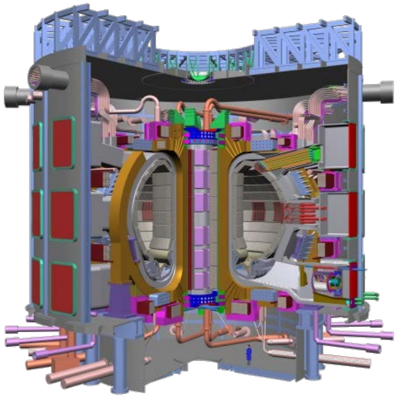
$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}\right] = \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} - \nabla p,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B},$$

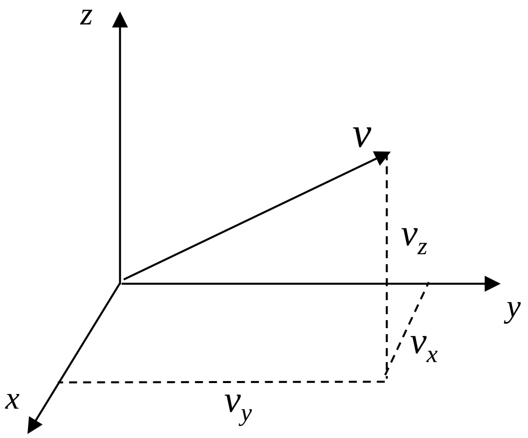
$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0,$$

$$\frac{\partial p}{\partial t} + (\boldsymbol{v}\cdot\nabla)\,p = -\gamma p\nabla\cdot\boldsymbol{v},$$





VECTOR ALGEBRA
(A QUICK OVERVIEW)



$$\vec{v}=(v_x, v_y, v_z)$$

$$\mathbf{v}=(v_x, v_y, v_z)$$

$$\overline{v}=(v_x, v_y, v_z)$$

equivalent notations to identify a vector

always highlight that a variable is a vector!

Absolute value of a vector $\left|\overline{V}\right| = \sqrt{V_x^2 + V_y^2 + V_z^2}$

Sum $\overline{a} + \overline{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$

Subtraction $\overline{a} - \overline{b} = \overline{a} + \left(-\overline{b}\right) = (a_x - b_x, a_y - b_y, a_z - b_z)$

Multiplication with a scalar c $c\overline{a} = (ca_x, ca_y, ca_z)$

Scalar product

$$\overline{a} \cdot \overline{b} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$$

$$\overline{a} \cdot \overline{b} = |a||b| \cos\alpha$$

$$\overline{a} \cdot \overline{b} = 0 \quad \Leftrightarrow \quad \overline{a} \perp \overline{b}$$

Basis vectors
in a Cartesian coordinate system:

$$\hat{e}_x = (1, 0, 0)$$

$$\hat{e}_y = (0, 1, 0)$$

$$\hat{e}_z = (0, 0, 1)$$

$$\Rightarrow \overline{a} = a_x \hat{e}_x + a_y \hat{e}_y + a_z \hat{e}_z$$

Note that they have absolute value =1

VECTOR ALGEBRA
(A QUICK OVERVIEW)

Cross product

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\left|\bar{a} \times \bar{b}\right| = \left|\bar{a}\right|\left|\bar{b}\right| \sin\alpha$$

$\bar{a} \times \bar{b}$ is a vector perpendicular to both $\bar{a}$ and $\bar{b}$

$$\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$$

$$\hat{e}_x \times \hat{e}_y = \hat{e}_z$$
$$\hat{e}_y \times \hat{e}_z = \hat{e}_x$$
$$\hat{e}_z \times \hat{e}_x = \hat{e}_y$$

VECTOR ALGEBRA
(A QUICK OVERVIEW)

A <u>basis</u> is a set of linearly independent vectors, whose linear combination is used to represent any other vectors.

Each coordinate system has its own basis.

When you operates with two or more vectors, it is essential that you know in which coordinate system they are expressed.

Example:

is it correct?

$$\left.\begin{array}{ll} \bar{a} = (3,1,2) & \text{in cartesian coordinates} \\ \bar{b} = (2,0,0) & \text{in spherical coordinates} \end{array}\right\} \Rightarrow \quad \bar{a} + \bar{b} = (3,1,2) + (2,0,0) = (5,1,2)$$

$$\bar{a} = (3,1,2) = 3\hat{e}_x + \hat{e}_y + 2\hat{e}_z$$

$$\bar{b} = (2,0,0) = 2\hat{e}_r$$

To sum the two vectors, first you need to convert one of them in order to have both vectors in the same system!

(more details in the 4th week)

VECTOR OR SCALAR?

(1)	$a\overline{v}$	vector
(2)	$\overline{v}\,\overline{a}$	wrong expression
(3)	$\overline{k}\cdot\overline{n}$	scalar
(4)	$c\times\overline{n}$	wrong expression
(5)	$b\cdot\overline{a}$	wrong expression
(6)	$\overline{a}\times\overline{n}$	vector

Please, use:

vector → blue star
scalar → yellow triangle
wrong expression → red square

Home assignments and final exam: -0.1 points <u>each time </u>you will write a wrong expression

VECTOR OR SCALAR?

(1)	$\left(\left(c\overline{v}\right)\times\overline{b}\right)\cdot\overline{a}$	scalar
(2)	$\left(\overline{v}\times\overline{a}\right)\overline{a}c$	wrong expression
(3)	$\overline{b}\cdot\left(\overline{v}\times\overline{a}\right)c$	scalar
(4)	$\left(\left(\left(\overline{c}\times\overline{n}\right)\times\overline{a}\right)\cdot\overline{b}\right)\overline{r}$	vector
(5)	$\left(\overline{r}\cdot\overline{a}\right)\times\overline{n}\times\overline{v}$	wrong expression
(6)	$\left(\left(\overline{a}\times\overline{n}\right)\cdot\overline{r}\right)\overline{v}\times\overline{n}\,d$	vector

Please, use:

- **vector → blue star**
- **scalar → yellow triangle**
- **wrong expression → red square**

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