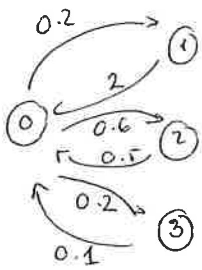
3

μ_1 μ_2 μ_3

$$b(x) = 0.2\cdot\underbrace{2e^{-2x}}_{Exp(2)} + 0.6\ \underbrace{0.5\,e^{-0.5x}}_{Exp(0.5)} + 0.2\cdot\underbrace{0.1\,e^{-0.1x}}_{Exp(0.1)}$$

$\lambda = 1$

a) $M/H_3/1/1$



b) P(clerk is busy) = P(call is blocked) (PASTA)

$= 1 - p_0$

$p_0\, 0.2 = p_1 \cdot 2$ $\qquad p_1 = 0.1\,p_0$

$p_0 \cdot 0.6 = p_2 \cdot 0.5$ $\qquad p_2 = \frac{6}{5} p_0 = 1.2\,p_0$

$p_0 \cdot 0.2 = p_3 \cdot 0.1$ $\qquad p_3 = 2\,p_0$

$p_0 [\underbrace{1 + 0.1 + 1.2 + 2}_{4.3}] = 1$ $\qquad p_0 = \frac{1}{4.3} = \frac{10}{43}$ $\qquad p_2 = \frac{12}{43}$

$p_1 = \frac{1}{43}$ $\qquad p_3 = \frac{20}{43}$

$P(\text{busy}) = P(\text{call blocked}) = 1 - p_0 = \frac{33}{43} \approx 0.767$

c) Offered load $= \lambda \bar{x} = 3.3$ $\qquad \bar{x} = 0.2 \cdot \frac{1}{2} + 0.6 \cdot 2 + 0.2 \cdot 10$

$= 0.1 + 1.2 + 2 = 3.3$

Effective load $= \lambda p_0 \cdot \bar{x} = 3.3 \cdot \frac{10}{43} = 0.76$

Number of calls served in 600 min $= \lambda_{eff} \cdot 600 = \lambda p_0 \cdot 600 = 139{,}5$

d) $M/H_3/1$, but we model with $M/M/1$ $\qquad C_x^2 = \frac{Var X}{E(x)^2}$

$$W = \frac{\lambda \bar{x}^2 [1 + C_x^2]}{2(1-\rho)}$$

$C_x^2 = 1$ for M $\qquad Var(x) = 0.2(\frac{1}{2})^2 + 0.6(\frac{1}{0.5})^2 + 0.2(\frac{1}{0.1})^2 = 22.45$

$C_x^2 > 1$ for H_3 $\qquad Var(x') = \bar{x}^2 = 10.9$

$\Rightarrow$ The exponential model underestimates the mean waiting time.

(41)

1.

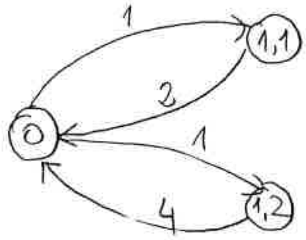
$\lambda = 2$

$$S^*(s) = \frac{1}{s+2} + \frac{2}{s+4} = 0.5 \cdot \frac{2}{s+2} + 0.5 \cdot \frac{4}{s+4} \Rightarrow$$

Service time is hyperexponential:

- $Exp(\mu_1 = 2)$ with probability 0.5
- $Exp(\mu_2 = 4)$ with probability 0.5

a) $M/H_2/1/1$



b)

$$P_{1,1} = \frac{1}{2} P_0 \quad , \quad P_{1,2} = \frac{1}{4} P_0$$

$$P_0 = \frac{1}{1 + \frac{1}{2} + \frac{1}{4}} = \frac{4}{7} \quad , \quad P_{1,1} = \frac{2}{7} \quad , \quad P_{1,2} = \frac{1}{7}$$

$$P_{block} = P_{1,1} + P_{1,2} = 1 - P_0 = \frac{3}{7}$$

$$\text{Utilization} = \rho \cdot (1 - P_{block}) = \lambda \cdot \bar{x} (1 - P_{block}) =$$

$$= \underbrace{2 \cdot \left(\frac{1}{2} \cdot 0.5 + \frac{1}{4} \cdot 0.5\right)}_{\lambda \cdot \bar{x} = \frac{3}{4}} \left(1 - \frac{3}{7}\right) = 0.4286$$

c) $M/H_2/1$

$$\bar{W} = \frac{\lambda E[S^2]}{2(1-\rho)}$$

$$E[S^2] = S^{*''}(s) \Big|_{s=0} = \frac{2}{(s+2)^3} + \frac{4}{(s+4)^3} \Big|_{s=0} = \frac{5}{16}$$

(4.2)

$$\bar{W} = \frac{2 \cdot \frac{5}{16}}{2\left(1-\frac{3}{4}\right)} = \frac{5}{4}$$

d) Same as in c)

e)

$M \sim \mathcal{E}xp(1)$: maintenance time

$$S_{hp} = S + M \quad \Rightarrow \quad \rho_{hp} = \lambda_{hp} \bar{x}_{hp} = 0.1 \lambda \cdot (\bar{x}+1) = 0.275$$

$$S_{ep} = S \quad \Rightarrow \quad \rho_{ep} = \lambda_{ep} \bar{x} = 0.9 \lambda \cdot \bar{x} = 0.675$$

$$E[S_{hp}^2] = E[(S+M)^2] = E[S^2] + 2E[S]E[M] + E[M^2] = \frac{49}{16}$$

$$E[S_{ep}^2] = E[S^2] = \frac{5}{16}$$

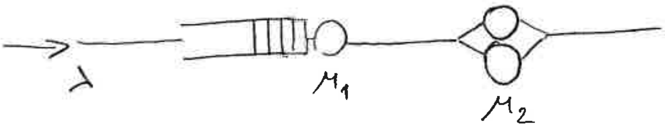
$$\bar{R} = \frac{1}{2}\left(\lambda_{hp} \overline{S_{hp}^2} + \lambda_{ep} \overline{S_{ep}^2}\right) = \frac{1}{2}\left(0.2 \cdot \frac{49}{16} + 1.8 \frac{5}{16}\right) = 0.5875$$

$$\bar{W}_{hp} = \frac{\bar{R}}{1-\rho_{hp}} = \frac{0.5875}{1-0.275} = 0.81$$

$$\bar{W}_{ep} = \frac{\bar{R}}{(1-\rho_{hp})(1-\rho_{hp}-\rho_{ep})} = \frac{0.5875}{(1-0.275)(1-0.275-0.675)} = 16.21$$

$$\bar{W} = 0.1 \cdot \bar{W}_{hp} + 0.9 \bar{W}_{ep} = 14.67$$

a)



$\lambda = 1, \mu_1 = 2, \mu_2 = 1$

$$\rho_1 = \frac{\lambda}{\mu_1} = 0.5$$

$$\rho_2 = \frac{\lambda}{\mu_2} = 1$$

$$\overline{T} = \overline{T}_1 \cdot P_B + (\overline{T}_1 + \overline{T}_2)(1 - P_B)$$

M/M/2/2: $$P_B = \frac{\rho_2^2/2}{1 + \rho_2 + \frac{\rho_2^2}{2}} = \frac{1/2}{1 + 1 + 1/2} = 0.2$$

$$\overline{T}_2 = \frac{\rho_2(1 - P_B)}{\lambda} = 0.8 \text{ s}$$

M/M/1: $$\overline{T}_1 = \frac{1}{\mu_1 - \lambda} = 1 \text{ s}$$

$$\overline{T} = 1 \cdot 0.2 + (1 + 0.8) \cdot 0.8 = 1.64$$

b)

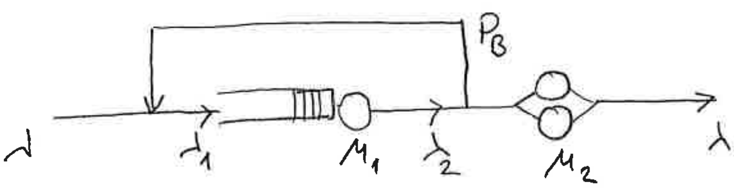
$$P(2) = P_1(2)P_2(0) + P_1(1)P_2(1) + P_1(0)P_2(2) =$$

$$= (1-\rho_1)\rho_1^2 \cdot \frac{1}{1 + \rho_2 + \rho_2^2/2} + (1-\rho_1)\rho_1 \frac{\rho_2}{1 + \rho_2 + \rho_2^2/2} + (1-\rho_1) \frac{\rho_2^2/2}{1 + \rho_2 + \rho_2^2/2} =$$

$$= 0.5 \cdot 0.5^2 \frac{1}{1 + 1 + 0.5} + 0.5 \cdot 0.5 \cdot \frac{1}{1 + 1 + 0.5} + 0.5 \cdot \frac{0.5}{1 + 1 + 0.5} =$$

$$= 0.25$$

c)



$\lambda_1 = \lambda_2$ $\lambda_1 = \lambda + P_B \lambda_2 \Rightarrow \lambda_2 = \frac{\lambda}{1 - P_B}$

$$\overset{1}{\lambda} = (1-P_B)\,\overset{\rho_2\mu_2}{\lambda_2} \qquad P_B = \frac{\rho_2^2/2}{1+\rho_2+\rho_2^2/2}$$

$$1-P_B = \frac{2+2\rho_2}{2+2\rho_2+\rho_2^2}$$

$$1 = \frac{2+2\rho_2}{2+2\rho_2+\rho_2^2}\rho_2\mu_2$$

$$\rho_2^2(2\mu_2-1)+\rho_2\overset{0}{(\cancel{2\mu_2}-2)}-2=0$$

$$\rho_2^2=2 \Rightarrow \boxed{\rho_2=1.41} \Rightarrow \boxed{\lambda_1=\lambda_2=1.41}$$

$$\Rightarrow \boxed{\rho_1=\frac{\lambda_1}{\mu_1}\approx 0.7}$$

d)

$$\overline{T}=\frac{\overline{N}}{\lambda}=\frac{\overline{N_1}+\overline{N_2}}{\lambda}=\frac{1}{\lambda}\left(\frac{\rho_1}{1-\rho_1}+\rho_2(1-P_B)\right)=$$

$$=1\left(\frac{0.7}{1-0.7}+1.41\cdot\frac{2+2\cdot1.41}{1+1.41+2/2}\right)=4.32$$