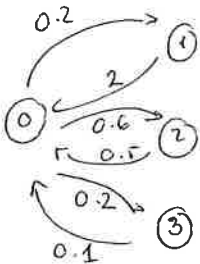


3

$$b(x) = 0.2 \cdot \underbrace{2e^{-2x}}_{\text{Exp}(2)} + 0.6 \cdot \underbrace{0.5e^{-0.5x}}_{\text{Exp}(0.5)} + 0.2 \cdot \underbrace{0.1e^{-0.1x}}_{\text{Exp}(0.1)}$$

$$\lambda = 1$$

a) M/M₃/1/1



b) $P(\text{date is busy}) = P(\text{call is blocked})$ [PASTA]
 $= 1 - p_0$

$$\begin{aligned} p_0 \cdot 0.2 &= p_1 \cdot 2 & p_1 &= 0.1 p_0 \\ p_0 \cdot 0.6 &= p_2 \cdot 0.5 & p_2 &= \frac{6}{5} p_0 = 1.2 p_0 \\ p_0 \cdot 0.2 &= p_3 \cdot 0.1 & p_3 &= 2 p_0 \end{aligned}$$

$$p_0 [1 + 0.1 + 1.2 + 2] = 1 \quad p_0 = \frac{10}{4.3} = \frac{10}{43} \quad p_2 = \frac{12}{43}$$

$$p_1 = \frac{1}{43} \quad p_3 = \frac{20}{43}$$

$$P(\text{busy}) = P(\text{call blocked}) = 1 - p_0 = \frac{33}{43} \approx 0.767$$

c) Offered load = $\lambda \bar{x} = 3.3$ $\bar{x} = 0.2 \cdot \frac{1}{2} + 0.6 \cdot 2 + 0.2 \cdot 10$
 $= 0.1 + 1.2 + 2 = 3.3$
 Effective load = $\lambda p_0 \cdot \bar{x} = 3.3 \cdot \frac{10}{43} = 0.76$

Number of calls served in 600 min = $\lambda_{\text{eff}} \cdot 600 = \lambda p_0 \cdot 600 = 139.5$

d) M/M₃/1, but we model with M/M/1

$$W = \frac{\lambda \bar{x}^2 [1 + C_x^2]}{2(1-\rho)}$$

$$C_x^2 = 4 \text{ for } M \quad \text{Var}(x) = 0.2 \left(\frac{1}{2}\right)^2 + 0.6 \left(\frac{1}{0.5}\right)^2 + 0.2 \left(\frac{1}{0.1}\right)^2 = 22.47$$

$$C_x^2 > 1 \text{ for } M_3 \quad \text{Var}(x) = \bar{x}^2 = 10.9$$

⇒ The exponential model underestimates the mean waiting time.

(41)

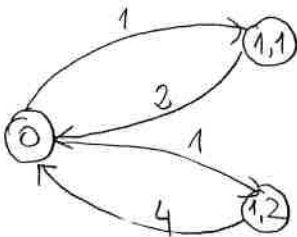
$$\lambda = 2$$

$$S^*(s) = \frac{1}{s+2} + \frac{2}{s+4} = 0.5 \cdot \frac{2}{s+2} + 0.5 \cdot \frac{4}{s+4} \Rightarrow$$

Service time is hyperexponential:

- Exp($\mu_1=2$) with probability 0.5
- Exp($\mu_2=4$) with probability 0.5

a) M/H₂/1/1



b)

$$P_{1,1} = \frac{1}{2} P_0, \quad P_{1,2} = \frac{1}{4} P_0$$

$$P_0 = \frac{1}{1 + \frac{1}{2} + \frac{1}{4}} = \frac{4}{7}, \quad P_{1,1} = \frac{2}{7}, \quad P_{1,2} = \frac{1}{7}$$

$$P_{\text{block}} = P_{1,1} + P_{1,2} = 1 - P_0 = \frac{3}{7}$$

$$\text{Utilization} = \rho \cdot (1 - P_{\text{block}}) = \lambda \cdot \bar{x} (1 - P_{\text{block}}) =$$

$$= 2 \cdot \left(\frac{1}{2} \cdot 0.5 + \frac{1}{4} \cdot 0.5 \right) \left(1 - \frac{3}{7} \right) = 0.4286$$

$\lambda \cdot \bar{x} = \frac{3}{4}$

c)

M/H₂/1

$$\bar{W} = \frac{\lambda E[S^2]}{2(1-\rho)}$$

$$E[S^2] = S^{*''}(s) \Big|_{s=0} = \frac{2}{(s+2)^3} + \frac{4}{(s+4)^3} \Big|_{s=0} = \frac{5}{16}$$

(4.2)

$$\bar{W} = \frac{2 \cdot \frac{5}{16}}{2(1 - \frac{3}{4})} = \frac{5}{4}$$

d) Same as in c)

e) $M \sim \text{Exp}(1)$: maintenance time

$$S_{hp} = S + M \Rightarrow \rho_{hp} = \lambda_{hp} \bar{x}_{hp} = 0.1 \lambda \cdot (\bar{x} + 1) = 0.275$$

$$S_{ep} = S \Rightarrow \rho_{ep} = \lambda_{ep} \bar{x} = 0.9 \lambda \cdot \bar{x} = 0.675$$

$$E[S_{hp}^2] = E[(S+M)^2] = E[S^2] + 2E[S]E[M] + E[M^2] = \frac{49}{16}$$

$$E[S_{ep}^2] = E[S^2] = \frac{5}{16}$$

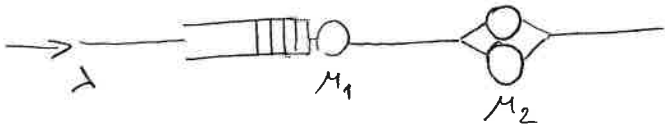
$$\bar{R} = \frac{1}{2} (\lambda_{hp} \bar{S}_{hp}^2 + \lambda_{ep} \bar{S}_{ep}^2) = \frac{1}{2} \left(0.2 \cdot \frac{49}{16} + 1.8 \cdot \frac{5}{16} \right) = 0.5875$$

$$\bar{W}_{hp} = \frac{\bar{R}}{1 - \rho_{hp}} = \frac{0.5875}{1 - 0.275} = 0.81$$

$$\bar{W}_{ep} = \frac{\bar{R}}{(1 - \rho_{hp})(1 - \rho_{hp} - \rho_{ep})} = \frac{0.5875}{(1 - 0.275)(1 - 0.275 - 0.675)} = 16.21$$

$$\bar{W} = 0.1 \cdot \bar{W}_{hp} + 0.9 \cdot \bar{W}_{ep} = 14.67$$

a)



$$\lambda = 1, \mu_1 = 2, \mu_2 = 1$$

$$\bar{T} = \bar{T}_1 \cdot P_B + (\bar{T}_1 + \bar{T}_2)(1 - P_B)$$

$$\beta_1 = \frac{\lambda}{\mu_1} = 0.5$$

$$\beta_2 = \frac{\lambda}{\mu_2} = 1$$

$$M/M/2/2: P_B = \frac{\beta_2^2/2}{1 + \beta_2 + \frac{\beta_2^2}{2}} = \frac{1/2}{1 + 1 + 1/2} = 0.2$$

$$\bar{T}_2 = \frac{\beta_2(1 - P_B)}{\lambda} = 0.8 \text{ s}$$

$$M/M/1: \bar{T}_1 = \frac{1}{\mu_1 - \lambda} = 1 \text{ s}$$

$$\bar{T} = 1 \cdot 0.2 + (1 + 0.8) \cdot 0.8 = 1.64 \text{ s}$$

b)

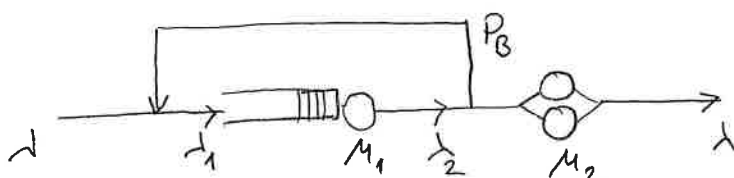
$$P(2) = P_1(2)P_2(0) + P_1(1)P_2(1) + P_1(0)P_2(2) =$$

$$= (1 - \beta_1)\beta_1^2 \cdot \frac{1}{1 + \beta_2 + \frac{\beta_2^2}{2}} + (1 - \beta_1)\beta_1 \frac{\beta_2}{1 + \beta_2 + \frac{\beta_2^2}{2}} + (1 - \beta_1) \cdot \frac{\beta_2^2/2}{1 + \beta_2 + \frac{\beta_2^2}{2}} =$$

$$= 0.5 \cdot 0.5^2 \frac{1}{1 + 1 + 0.5} + 0.5 \cdot 0.5 \cdot \frac{1}{1 + 1 + 0.5} + 0.5 \cdot \frac{0.5}{1 + 1 + 0.5} =$$

$$= 0.25$$

c)



$$\lambda_1 = \lambda_2$$

$$\lambda_1 = \lambda + P_B \lambda_2 \Rightarrow \lambda_2 = \frac{\lambda}{1 - P_B}$$

$$1 \leftarrow \lambda = (1 - P_B) \lambda_2 \xrightarrow{\beta_2 \mu_2}$$

$$P_B = \frac{\beta_2^2}{1 + \beta_2 + \beta_2^2/2}$$

$$1 - P_B = \frac{2 + 2\beta_2}{2 + 2\beta_2 + \beta_2^2}$$

$$1 = \frac{2 + 2\beta_2}{2 + 2\beta_2 + \beta_2^2} \beta_2 \mu_2$$

$$\beta_2^2 (2\mu_2 - 1) + \beta_2 (2\mu_2 - 2) - 2 = 0$$

$$\beta_2^2 = 2 \Rightarrow \boxed{\beta_2 = 1.41} \Rightarrow \boxed{\lambda_1 = \lambda_2 = 1.41}$$

$$\Rightarrow \boxed{\beta_1 = \frac{\lambda_1}{\mu_1} \approx 0.7}$$

d)

$$\bar{T} = \frac{\bar{N}}{\lambda} = \frac{\bar{N}_1 + \bar{N}_2}{\lambda} = \frac{1}{\lambda} \left(\frac{\beta_1}{1 - \beta_1} + \beta_2 (1 - P_B) \right) =$$

$$= 1 \cdot \left(\frac{0.7}{1 - 0.7} + 1.41 \cdot \frac{2 + 2 \cdot 1.41}{1 + 1.41 + 2/2} \right) = 4.32$$