Practice problems 2014

1)

Consider the following two systems. In system A there is one server, the service time is exponentially distributed with parameter $\min(k\mu, m\mu)$, where $k$ is the number of customers in the system and $m$ is a constant. In system B there are m servers and the service intensity of each server is $\mu$. In both cases there is an infinite queue and the arrival process is Poisson, the mean time between arrivals is 1s. $1/\mu$=4s, m=5. Answer the following questions for both systems! (Do not need to calculate state probabilities now, enough if you give parametric solution.)

a) Give the Kendall notation and draw the state diagram of the system.
b) What is the probability of that an arriving customer finds two customers waiting?
c) What is the mean number of customers in the queue?
d) What is the mean time spent in the system?
e) What is the mean time spent in the server?

2)

Consider a local telephone switch with two lines and 1 waiting position. Call arrivals originate from two groups of subscribers, A and B, with five subscribers in each group. Subscribers behave as follows: after finishing a call they are idle for an exponentially distributed time, with a mean of 30min. Then they try to make a new call. If the servers and the queue position are occupied they start a new idle period. Call durations are exponentially distributed with mean of 6 min.

a) Give the Kendall notation and draw the state transition diagram.
b) Calculate the time blocking and call blocking probabilities.
c) Calculate the expected waiting time considering the calls that are waiting for service.
d) Consider now the same system without the waiting position. Assume that subscribers of group B are allowed to make a call only when both lines are free. Draw the state transmission diagram for this case.

3)

The distribution of the length of the telephone calls at the Telia helpdesk can be estimated with the combination of exponential distributions (in minutes) as:

$$b(x) = 0.2 \cdot 2e^{-2x} + 0.6 \cdot 0.5e^{-0.5x} + 0.2 \cdot 0.1e^{-0.1x}\ .$$

Calls arrive according to a Poisson process, one call per minute in average. Assume that one clerk answers the calls, calls arriving when the clerk is busy are blocked.

a) Model the system with a Markov-chain and give the Kendall notation.
b) Based on the model, give the fraction of time when the clerk is busy answering calls and the probability that a call is blocked.
c) Calculate the offered load, the effective load and the number of calls served in 10 hours.
d) Now consider the case when calls arriving when the clerk is busy are put in a waiting queue. To simplify the analysis, you decide to estimate the call length distribution with an Exponential distribution of the same mean as E[X]. Do you overestimate or underestimate the waiting times this way?

4)

Jobs arrive to a server according to a Poisson process with intensity 2 jobs per time unit. The Laplace transform of the service time distribution is given by $S^*(s)=1/(s+2)+2/(s+4)$. Jobs that arrive when the server is busy are dropped.

a) Give the Kendall notation and draw the state transition diagram for the system.
b) Calculate the utilization of the server.

Assume now that the jobs that arrive when the server is busy are placed in a queue.

c) What is the expected waiting time of the jobs?
d) If 10% of the jobs are given non-pre-emptive priority, what is the expected waiting time of an arbitrary job?
e) Assume that the server has to perform a maintenance procedure every time it finishes the service of a high priority job. It takes 1 time unit on average to perform the maintenance, the duration is exponentially distributed. What is the average waiting time of an arbitrary job in this case?