AUTOMATIC CONTROL

KTH

Nonlinear Control, EL2620

Exam 14.00–19.00   January 18, 2014

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text) or equivalent basic control book if approved by the examiner beforehand. Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2014-02-07 at STEX, Studerandeexpeditionen, Osquldasv. 10.

Responsible: Elling W. Jacobsen 070 372 22 44

Good Luck!

1. Consider the nonlinear dynamical system

$$\begin{aligned}\dot{x}_1 &= -\frac{2x_1}{1+x_1^2} + x_2 \\ \dot{x}_2 &= -x_1 + ax_2 + u\end{aligned}$$

(a) Assume $a = 1$ and $u = 0$. Determine the equilibrium point(s) of the system and the local stability of these. Also classify the local phase plane behavior around the equilibrium points. (4p)

(b) Determine a linear control law $u = c(x)$ which makes the origin locally asymptotically stable when $a = 1$. (2p)

(c) Consider now the case with $a = 0$ and $u = 0$ and show that the origin is globally asymptotically stable. (4p)

2. (a) Consider the 2nd order system

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1(1 - x_1^2 - x_2^2)\\ \dot{x}_2 &= -x_1 + x_2(1 - x_1^2 - x_2^2)\end{aligned}$$

We want to analyze whether the system has a stable limit cycle.

(i) Show that the set

$$E = \{(x_1, x_2) | -2 \leq x_1 \leq 2, -2 \leq x_2 \leq 2\}$$

is an invariant region for the system. (3p)

(ii) Now use the result in (i) to prove that the system must have a stable limit cycle within the region defined by the set E. (3p)

(b) A mechanical system is described by the two state model

$$\begin{aligned}\dot{x}_1 &= -\frac{x_2}{1 + x_2^2} + u\\ \dot{x}_2 &= -\frac{x_1}{1 + x_1^2}\end{aligned}$$

The equilbrium at the origin with no control ($u = 0$) is unstable. Design a state feedback control law $u = g(x)$ such that the origin becomes globally asymptotically stable. Show that the controller indeed is globally stabilizing. (4p)

3. (a) We shall consider linearizing control of the system

$$\begin{aligned}\dot{x}_1 &= \sin(x_1) + x_2^2 + u \\ \dot{x}_2 &= -x_1 - x_2^2\end{aligned}$$

   (i) Determine a state feedback and state transformation that yields a linear state space description of the system. (3p)

   (ii) Consider now the output $y(t) = x_1(t)$. Determine a state feedback $u = c(x, v)$ which renders the system linear from $v(t)$ to $y(t)$. Will there be problems with unstable zero dynamics in this case? (3p)

(b) The system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - 2x_2 + u\end{aligned}$$

is controlled with the switching control

$$u = -\mathtt{sign}(x_1 + x_2)$$

Show that this corresponds to a sliding mode controller, determine the sliding set and the equivalent control on the set. (4p)

4. Consider the feedback loop in Figure 1. The system $G(s)$ is given by Equation (1).

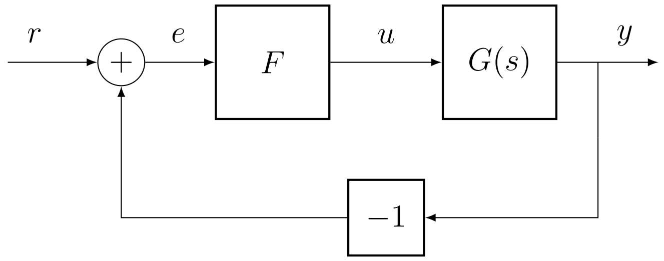

Figure 1: Feedback loop of problem 4.

$$G(s) = \frac{1}{\left(s+\frac{1}{2}\right)^2} \tag{1}$$

The controller F is a linear controller with a saturation H and a deadband D, see Figure 2. The slope of the controller in the linear region is K.

(a) Consider first the case with no saturation and no deadband, i.e., $H = \infty$ and $D = 0$. For which positive values of K is the feedback loop in Figure 1 BIBO stable?

(2p)

(b) For which positive values of K does the small gain theorem guarantee BIBO stability of the feedback loop in Figure 1?

(3p)

(c) For which positive values of K does the passivity theorem guarantee BIBO stability of the feedback loop in Figure 1?

(1p)

(d) For which positive values of K does the circle criterion guarantee BIBO stability of the feedback loop in Figure 1?

(4p)

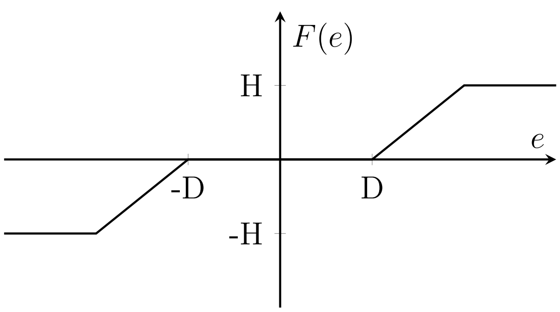

Figure 2: The controller F with a deadband D and saturation H.

5. You are planning to have a party in 24 hours. The current temperature in the dance hall is only 10^oC and it should be 22^oC when the party starts. Since you are concerned with the environmental impact of power production, you want to minimize the energy spent on heating the dance hall. The instantaneous energy input is u and you aim to minimize $\int_0^{t_f} u^2 dt$ while achieving the desired temperature at $t_f = 24h$. The relation between the energy input u and the room temperature T is given by the model

$$\dot{T} = -\delta(T(t) - 10) + u(t)$$

where δ is the effective heat transfer coefficent for heat loss through the walls.

(a) Formulate the heating problem described above as an optimal control problem. (2p)

(b) Assume first that there is no heat loss, i.e., $\delta = 0$. What is the optimal control policy in this case? *Hint: you can find the solution to this problem through simple reasoning.* (2p)

(c) Consider now the case with heat loss coefficient $\delta = 0.02$. What is the optimal control policy in this case? (4p)

(d) Consider again $\delta = 0.02$ and consider now that there is a contraint on the heat input u such that $0 < u < 0.1$. Is the optimal control problem feasible in this case? (2p)