

# Last lecture (7)

- Particle motion in magnetosphere
- Aurora

# Today's lecture (8)

- Aurora on other planets
- How to measure currents in space
- Magnetospheric dynamics



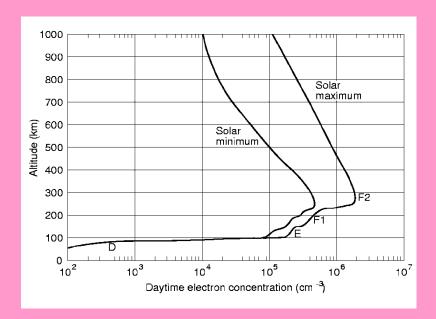
# Today

Activity	Date	Time	Room	Subject	<u>Litterature</u>
L1	2/9	10-12	Q33	Course description, Introduction, The Sun 1, Plasma physics 1	<b>CGF</b> Ch 1, 5, (p 110-113)
L2	3/9	15-17	Q31	The Sun 2, Plasma physics 2	<b>CGF</b> Ch 5 (p 114-121), 6.3
L3	9/9	10-12	Q33	Solar wind, The ionosphere and atmosphere 1, Plasma physics 3	CGF Ch 6.1, 2.1- 2.6, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	11/9	10-12	Q34	Mini-group work 1	
L4	16/9	15-17	Q33	The ionosphere 2, Plasma physics 4	<b>CGF</b> Ch 3.4, 3.7, 3.8
L5	18/9	15-17	Q21	The Earth's magnetosphere 1, Plasma physics 5	<b>CGF</b> 4.1-4.3, <b>LL</b> Ch I, II, IV.A
T2	23/9	10-12	Q34	Mini-group work 2	
L6	25/9	10-12	M33	The Earth's magnetosphere 2, Other magnetospheres	<b>CGF</b> Ch 4.6-4.9, <b>LL</b> Ch V.
L7	30/9	14-16	L51	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
T3	3/10	10-12	V22	Mini-group work 3	
L8	7/10	10-12	V22	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
T4	9/10	15-17	Q31	Mini-group work 4	
L9	11/10	10-12	M33	Interstellar and intergalactic plasma, Cosmic radiation, Swedish and international space physics research.	<b>CGF</b> Ch 7-9
T5	15/10	10-12	L51	Mini-group work 5	
L10	16/10	13-15	Q36	Guest lecture: Swedish astronaut Christer Fuglesang	
T6	17/10	15-17	Q31	Round-up	
Written examination	30/10	14-19	B21-24		



## Mini-groupwork 3

$$\frac{\partial n_e}{\partial t} = q - \alpha n_e^2$$



$$\frac{dn_e(t)}{dt} = 0 \implies$$

$$\alpha = \frac{q}{n_e^2}$$

$$q = 1.7 \cdot 10^4 \text{ cm}^{-3} \text{s}^{-1} = 1.7 \cdot 10^{10} \text{ m}^{-3} \text{s}^{-1}$$

$$n_{e}(150 \text{ km}) = 2 \cdot 10^{5} \text{ cm}^{-3} = 2 \cdot 10^{11} \text{ m}^{-3}$$

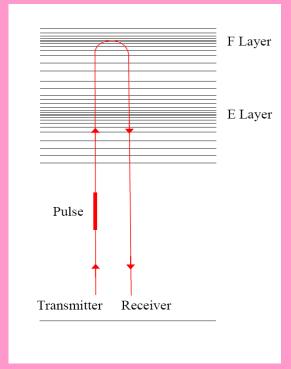
Thus

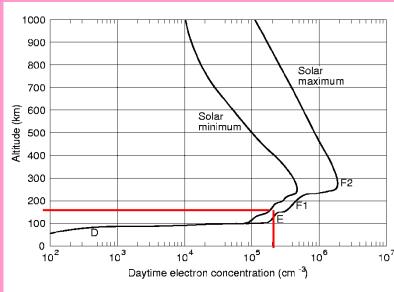
$$\alpha = 4.2 \cdot 10^{-13} \text{ m}^3 \text{s}^{-1}$$



## Mini-groupwork 3

**b**)





$$f_p = \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \approx 9\sqrt{n_e}$$

$$f_p = 5 \cdot 10^6 = 9\sqrt{n_e}$$

$$\Rightarrow$$

$$n_e = \left(\frac{5 \cdot 10^6}{9}\right)^2 = 3 \cdot 10^{11} m^{-3}$$

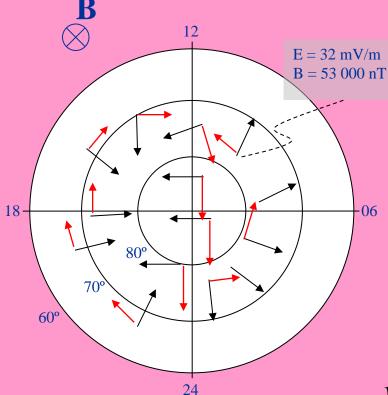
$$h = 150 \ km$$

$$t = \frac{2h}{c} = \frac{300 \cdot 10^3}{3 \cdot 10^8} = 10^{-3} s$$



## Mini-groupwork 3

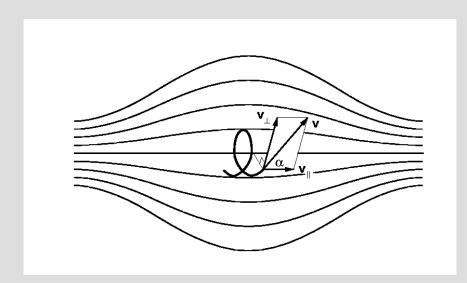




$$v_d = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{E}{B} = \frac{32 \cdot 10^{-3}}{53000 \cdot 10^{-9}} = 604 \text{ ms}^{-1}$$



### **Magnetic mirror**



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

mv<sup>2</sup>/2 constant (energy conservation)

$$\frac{\sin^2 \alpha}{B} = konst$$

particle turns when  $\alpha = 90^{\circ}$ 



$$B_{turn} = B / \sin^2 \alpha$$

If maximal B-field is  $B_{max}$  a particle with pitch angle  $\alpha$  can only be turned around if

$$B_{turn} = B / \sin^2 \alpha \le B_{max}$$



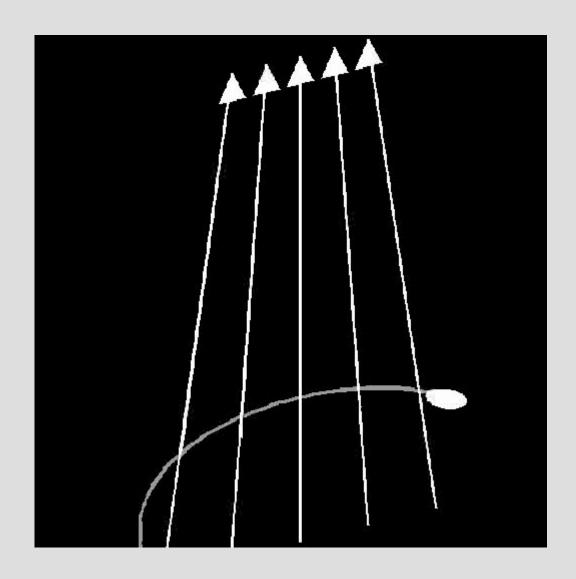
$$\alpha > \alpha_{lc} = \arcsin \sqrt{B/B_{\text{max}}}$$

Particles in loss cone:

$$\alpha < \alpha_{lc}$$



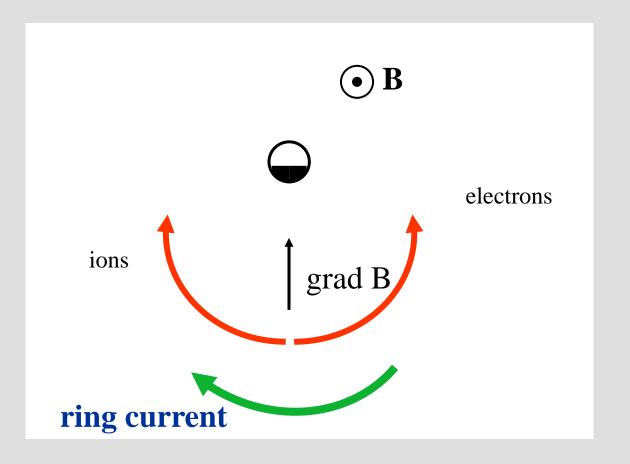
## **Magnetic mirror**





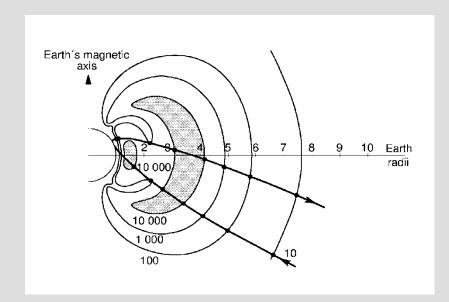
# Ring current and particle motion

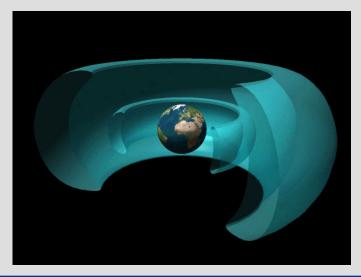
$$\mathbf{u} = -\frac{\mu \nabla B \times \mathbf{B}}{qB^2}$$





#### Radiation belts





#### I. Van Allen belts

- Discovered in the 50s ,
   Explorer 1
- Inner belt contains protons with energies of ~30 MeV
- Outer belt (Explorer IV, Pioneer III): electrons, W>1.5 MeV



#### **CRAND** (Cosmic Ray Albedo Neutron Decay

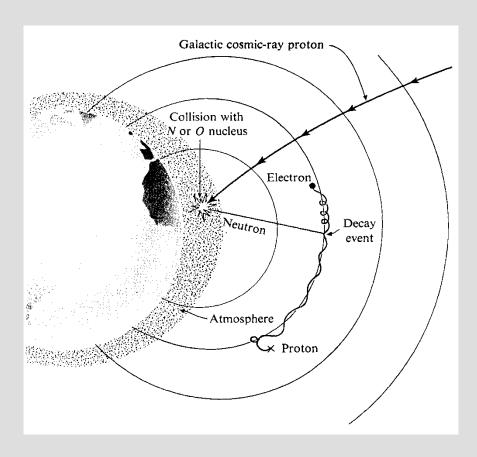


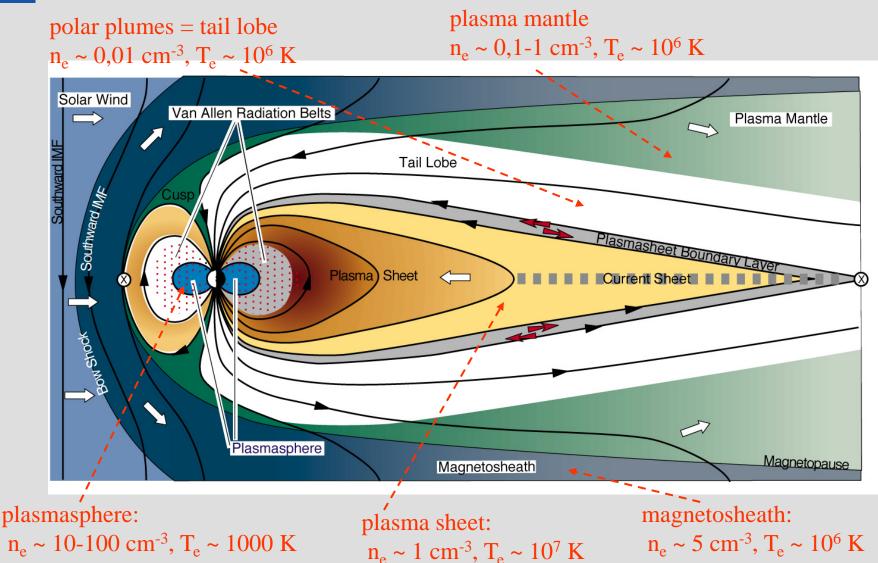
Figure 8. An illustration of the CRAND process for populating the inner radiation belts [Hess, 1968].

Collisions between cosmic ray particles and the Earth create new particles. Among these are neutrons, that are not affected by the magnetic field. They decay, soom eof them when they happen to be in the radiation belts. The resulting protons and electrons are trapped in the radiation belts.

This contribution to the radiation belts are called the *neutron albedo*.



## Magnetospheric structure





# Planetary magnetospheres

	Radius Earth radii	Spin period (days)	Equatorial field strength (μT)	Magnetic axis direction relative to spin axis	Polarity relative to Earth's	Typical magneto- pause distance (planetary radii)
Mercury	0.38	58.6	0.35	10 <sup>0</sup>	Same	1.1
Venus	0.95	243	< 0.03	-	-	1.1
Earth	1.0	1	31	11.5 <sup>0</sup>	Same	10
Mars	0.53	1.02	0.065		Opposite	?
Jupiter	11.18	0.41	410	10 <sup>0</sup>	Opposite	60-100
Saturn	9.42	0.44	40	<10.	Opposite	20-25
Uranus	3.84	0.72	23	60°	Opposite	18-25
Neptune	3.93	0.74	20-150 <sup>*)</sup>	47 <sup>0</sup>	Opposite	( 26 <sup>**)</sup>

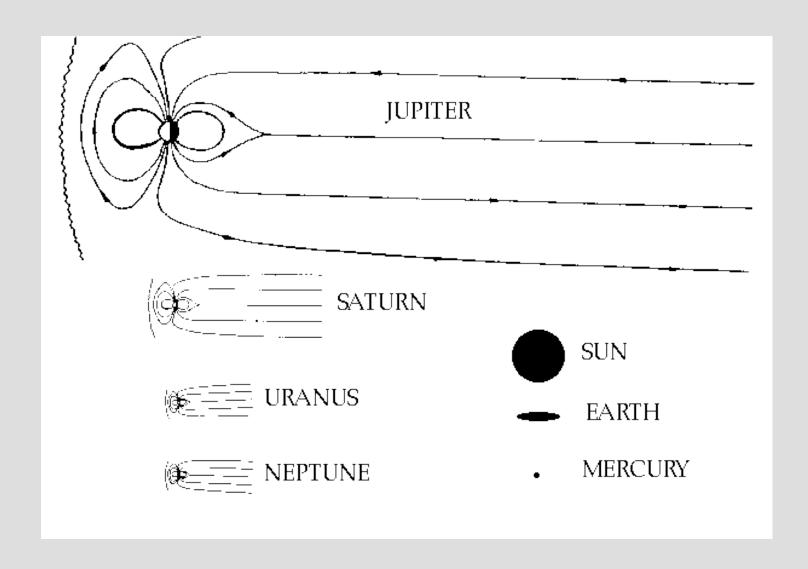
<sup>\*)</sup> The magnetic field differs greatly from a dipole field. The numbers represent maximum and minimum strength at the planetary surface

Very weak magnetic fields

<sup>\*\*)</sup> Based on single passage



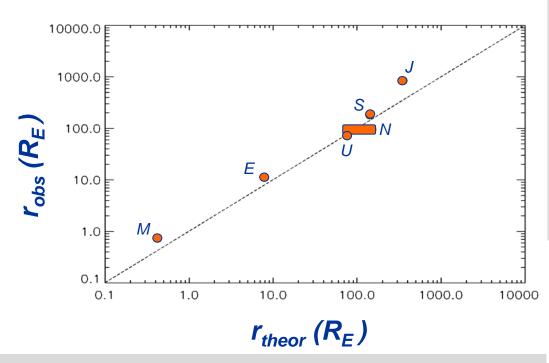
### Relative size of the magnetospheres





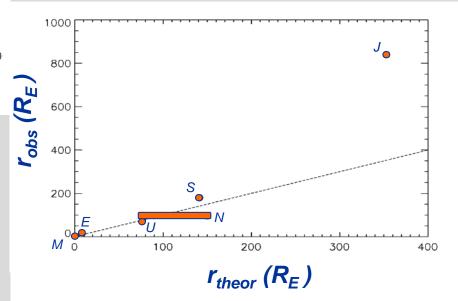
## Comparative magnetospheres

#### Observed vs. theoretical standoff-distance



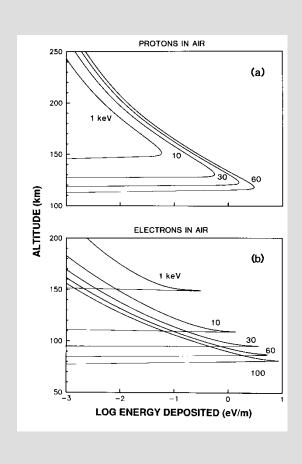
 $r_{theor} = \left(\frac{\mu_0 a}{4\pi}\right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2\right)^{-1/6}$ 

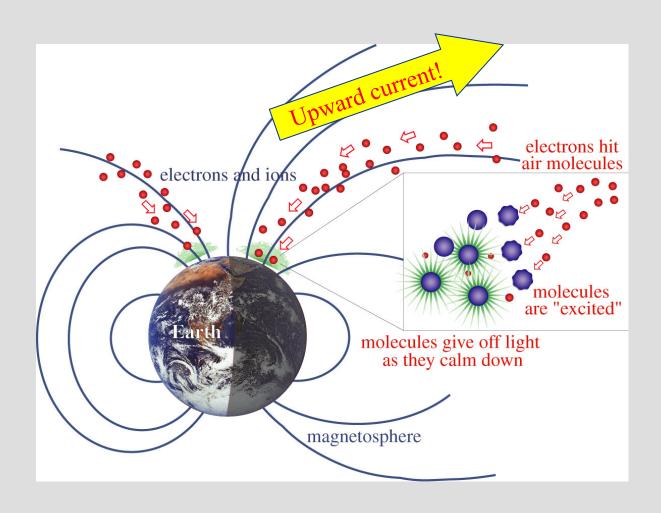
- Model reasonably valid over three orders of magnitude
- Size of Jupiter's (and maybe Saturn's) magnetosphere underestimated





## **Collisions - emissions**

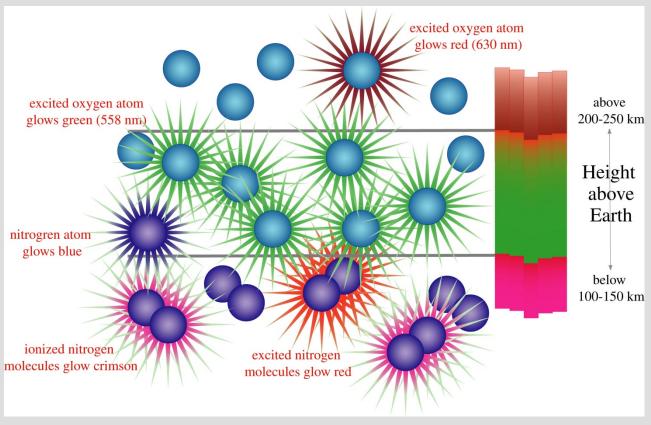






## **Emissions**







# Larger scales

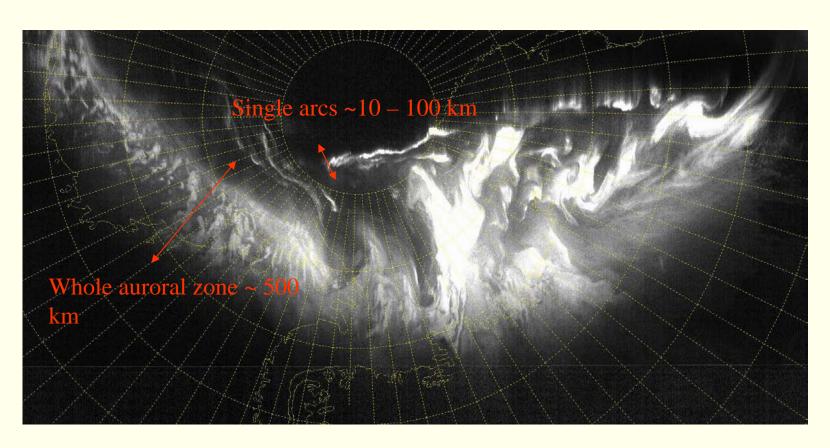
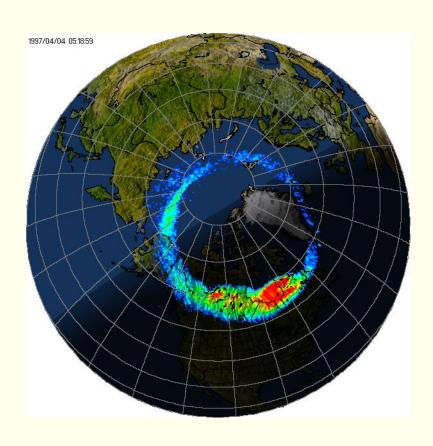


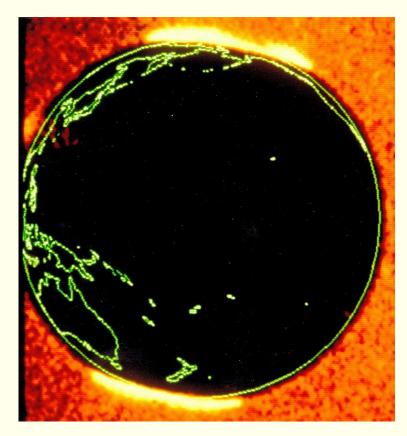
Foto från DMSP-satelliten



## **Auroral ovals**



Polar



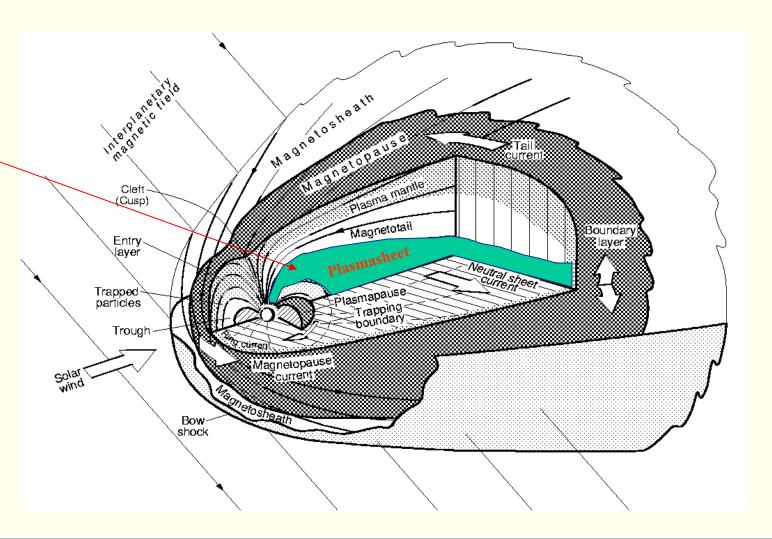
**Dynamics Explorer** 



# The auroral oval is the projection of the plasmasheet onto the atmosphere

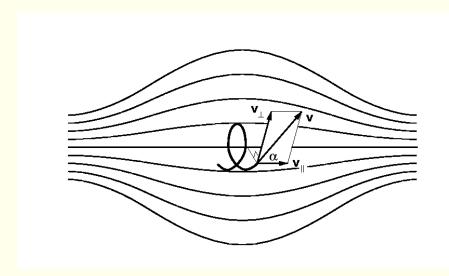
#### **Mystery!**

The particles in the plasmasheet do not have high enough energy to create aurora visible to the eye.





### **Magnetic mirror**



The magnetic moment  $\mu$  is an *adiabatic invariant*.

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$$\frac{\sin^2 \alpha}{B} = konst$$

particle turns when  $\alpha = 90^{\circ}$ 



$$B_{turn} = B / \sin^2 \alpha$$

If maximal B-field is  $B_{max}$  a particle with pitch angle  $\alpha$  can only be turned around if

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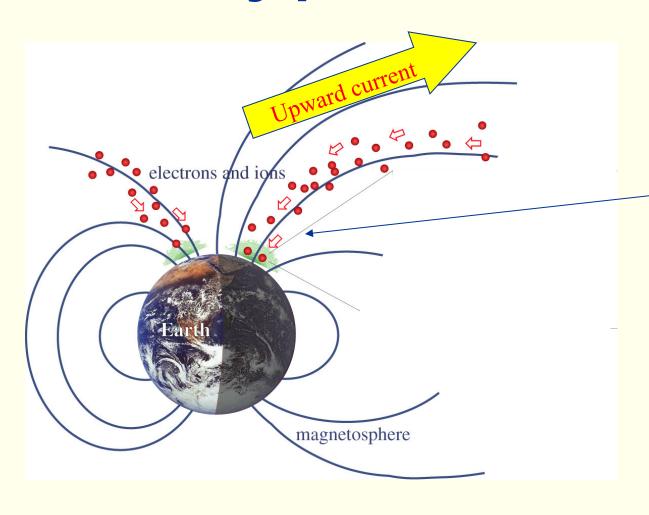
$$\alpha > \alpha_{fl} = \arcsin \sqrt{B/B_{\text{max}}}$$

Particles in *loss cone*:

$$\alpha < \alpha_{fl}$$



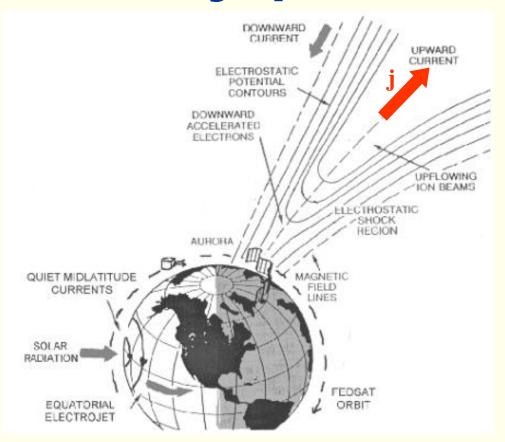
# Why particle acceleration?



- The magnetosphere often seems to act as a current generator.
- The lower down you are
   on the field line, the more particles have been reflected by the magnetic mirror.
- At low altitudes there are not enough electrons to carry the current.



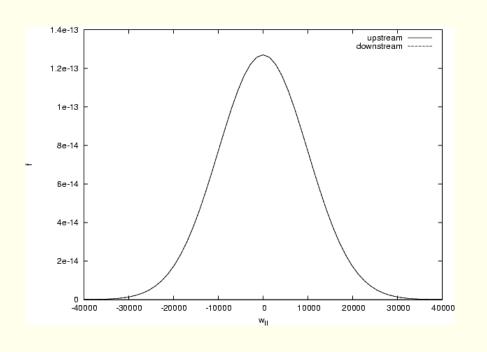
# Why particle acceleration?

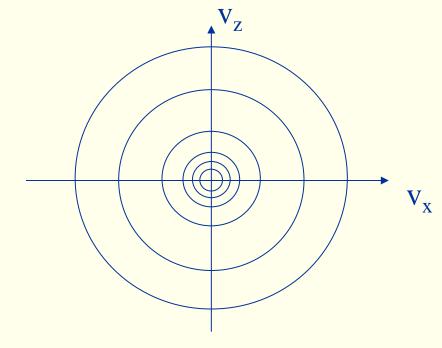


- Electrons are accelerated downwards by upward Efield.
- This increases the pitch-angle of the electrons, and more electrons can reach the ionosphere, where the current can be closed.



## Distribution function



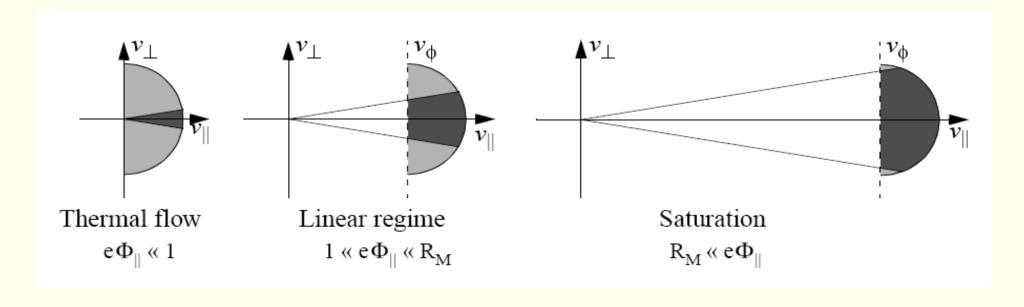


# Example: Maxwellian distribution

$$f = \frac{n}{\sqrt{(2\pi RT)^3}} \exp\left(-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2kT}\right)$$



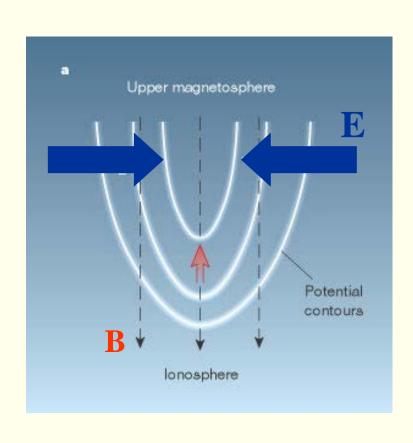
# Why particle acceleration?

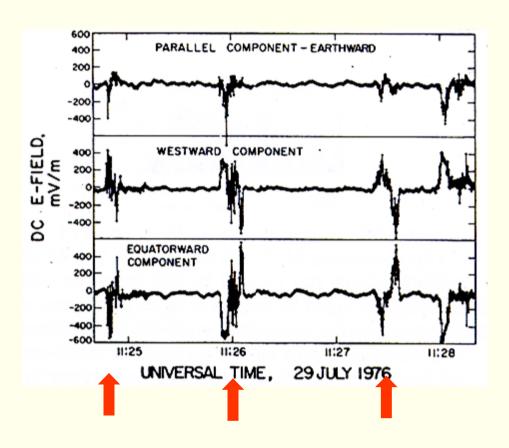


- Electrons are accelerated downwards by upward E-field.
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## Satellite signatures of U potential

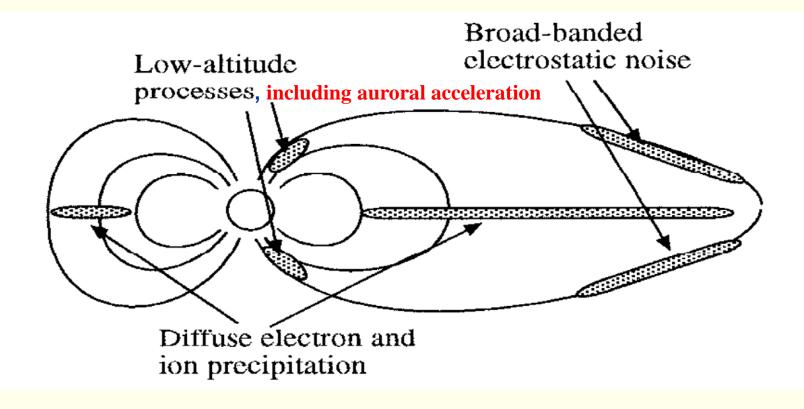




Measurements made by the ISEE satellite (Mozer et al., 1977)



# **Acceleration regions**

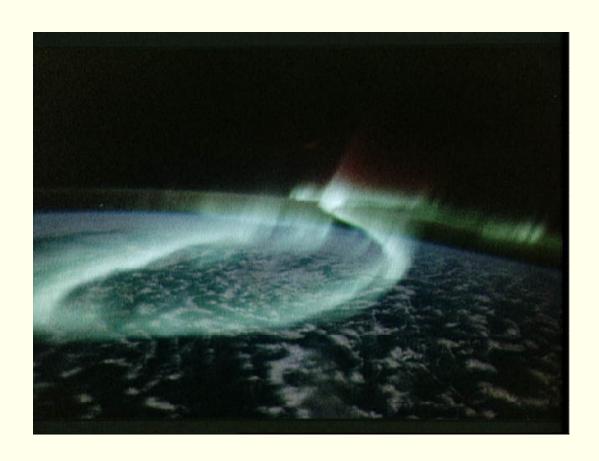


Auroral acceleration region typically situated at altitude of 1-3 R<sub>E</sub>



# **Auroral spirals**

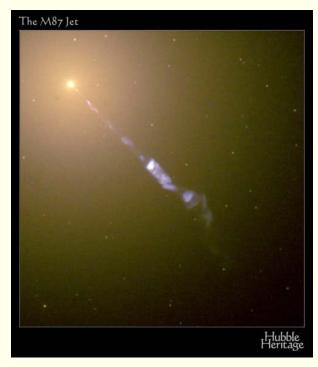




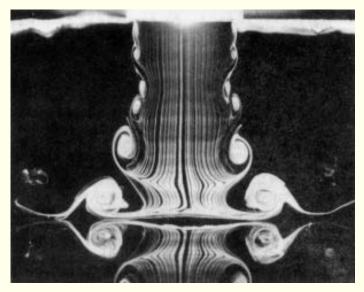
Develop when arcs become unstable



## Kelvin-Helmholzinstability – a general phenomenon



Extragalactic jet (M87)



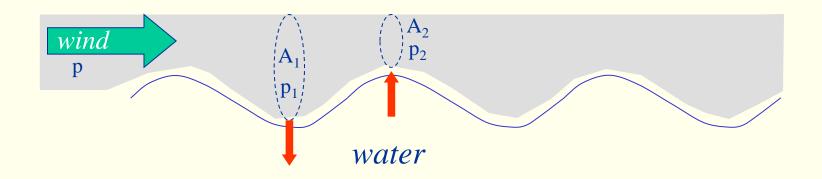
Aero- and fluid dynamics



Cluds



## Kelvin-Helmholz instability Example: water waves



Continuity equation:

$$A_1 v_1 = A_2 v_2$$

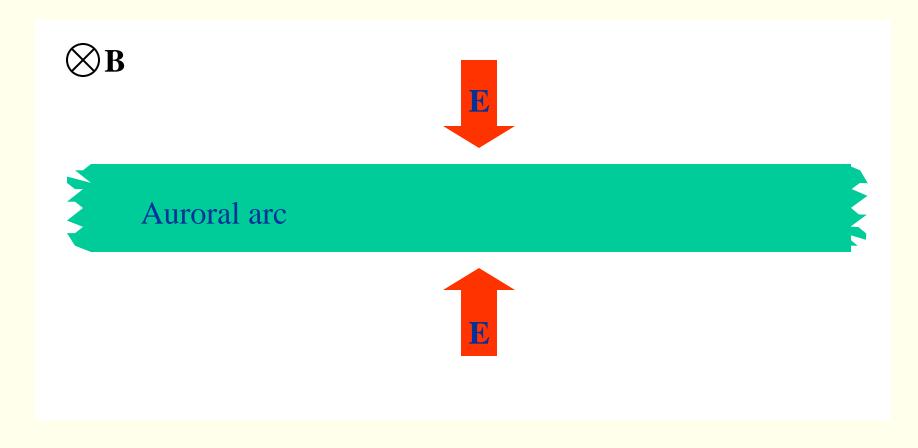
Bernoulli's equation:

$$p_1 + \rho v_1^2 = p_2 + \rho v_2^2 = const.$$

: 
$$p_1 > p > p_2$$

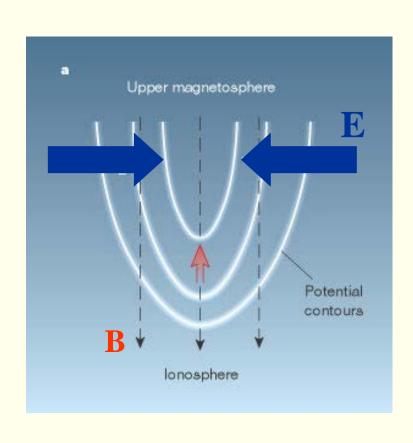


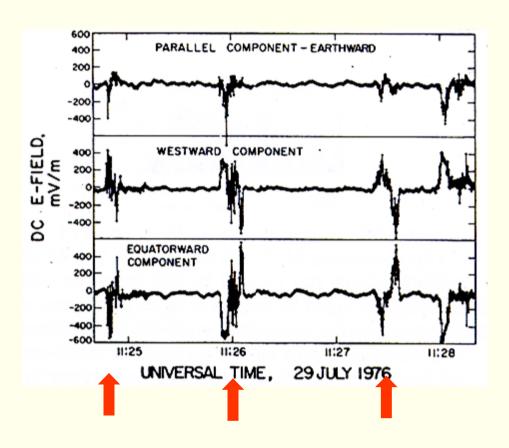
# Spirals – Kelvin-Helmholz instability





## Satellite signatures of U potential

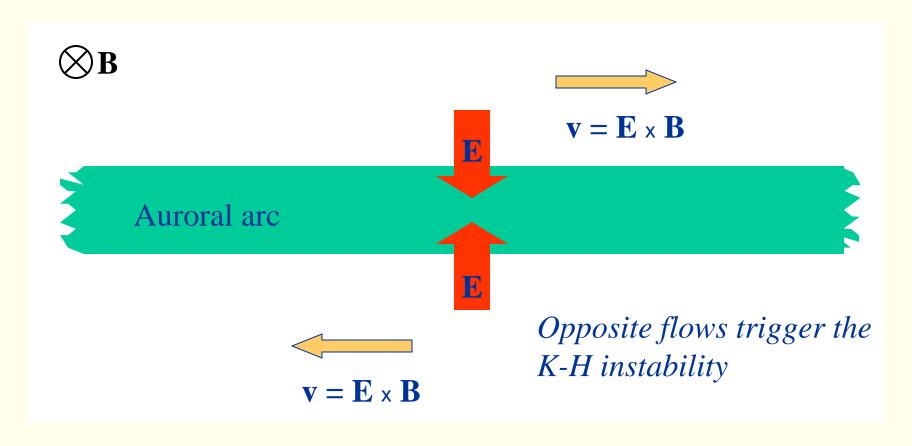




Measurements made by the ISEE satellite (Mozer et al., 1977)

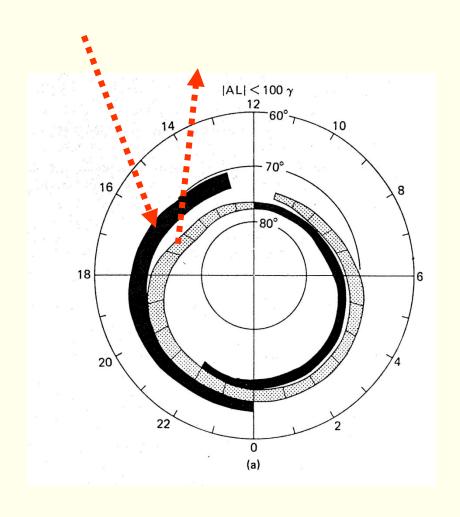


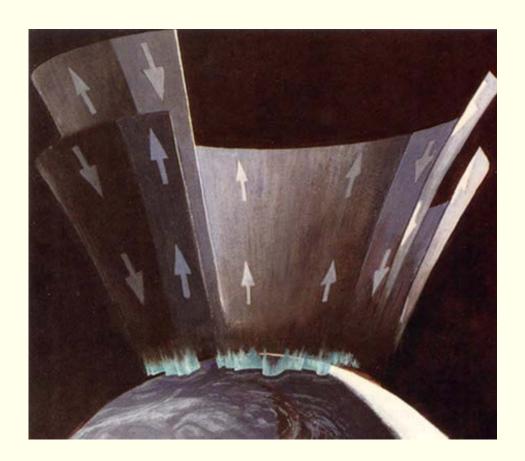
# Spirals – Kelvin-Helmholz instability





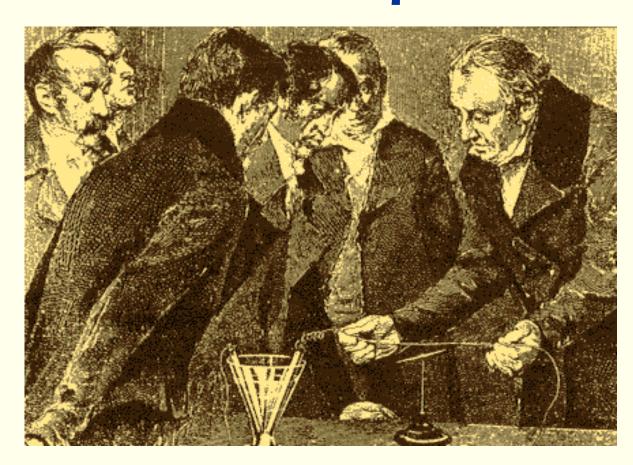
### Birkeland currents in the auroral oval





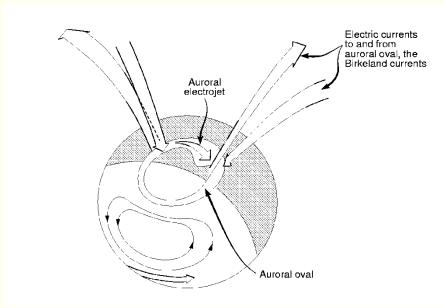


# How can you measure currents in space?

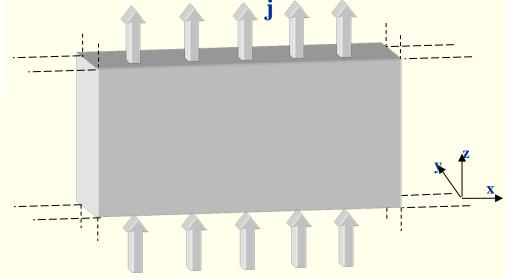




# **Current sheet approximation**

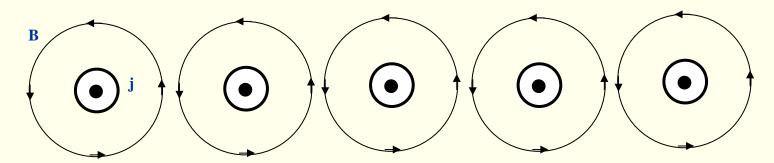


Approximate currents by thin current sheets with infinite size in the x- och z-directions.

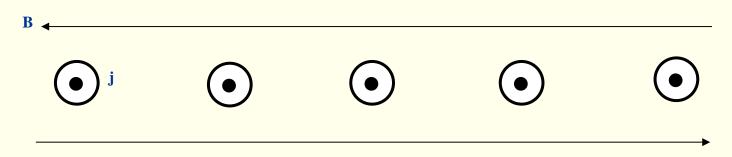




# **Current sheet approximation**



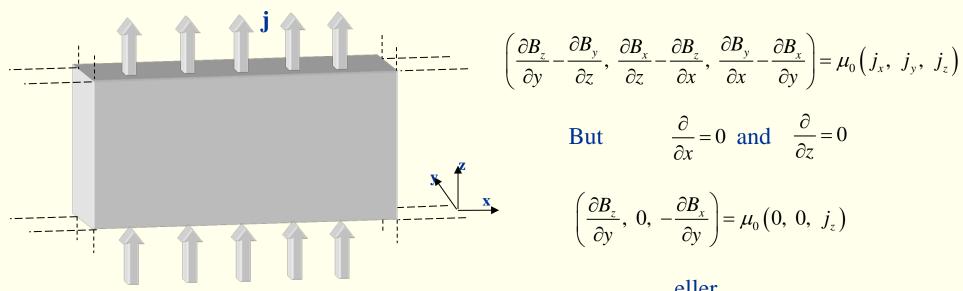
What will the magnetic field around such a current configuration be? Start by approximating with line currents to get a qualitative picture.



The closer you place the line currents, the more the magnetic fields between the line currents will cancel



## **Current sheet approximation and** Ampére's law



$$\left(\frac{\partial B_{z}}{\partial y} - \frac{\partial B_{y}}{\partial z}, \frac{\partial B_{x}}{\partial z} - \frac{\partial B_{z}}{\partial z}, \frac{\partial B_{y}}{\partial x} - \frac{\partial B_{y}}{\partial x} - \frac{\partial B_{x}}{\partial y}\right) = \mu_{0} \left(j_{x}, j_{y}, j_{z}\right)$$

But 
$$\frac{\partial}{\partial x} = 0$$
 and  $\frac{\partial}{\partial z} = 0$ 

$$\left(\frac{\partial B_z}{\partial y}, 0, -\frac{\partial B_x}{\partial y}\right) = \mu_0 \left(0, 0, j_z\right)$$

eller

Ampére's law (no time dependence):

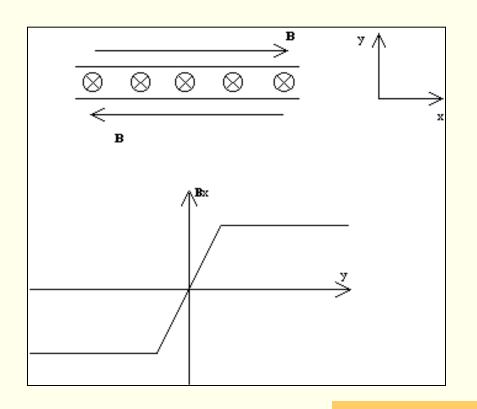
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

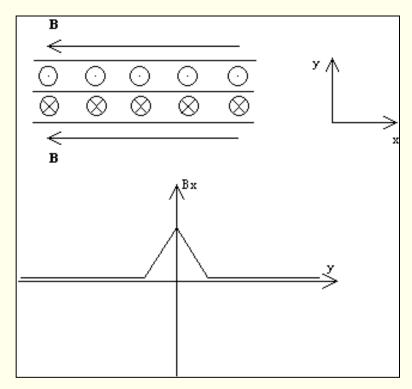


$$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$$

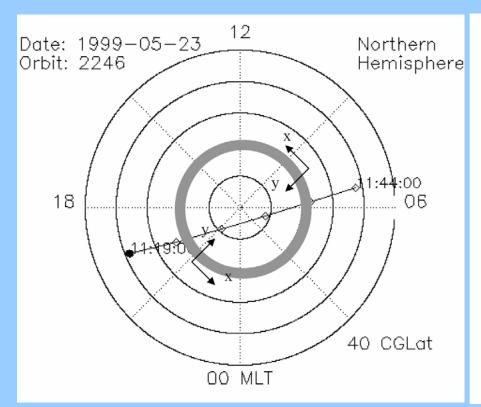


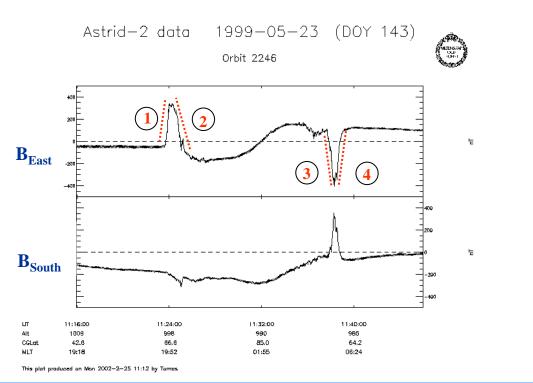
## Current sheet - example





$$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$$



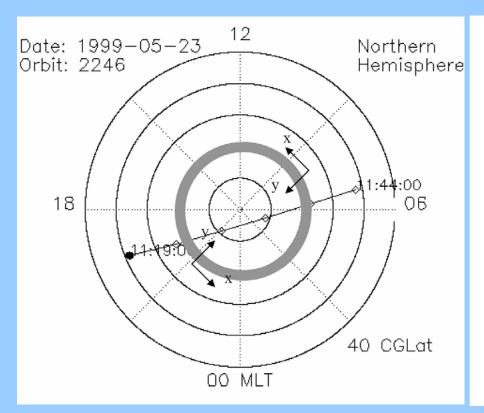


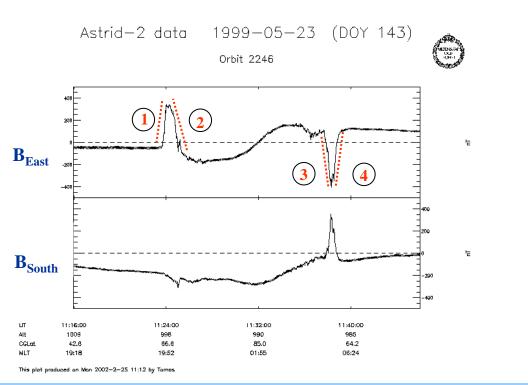
$$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$$

#### What is the direction of the current in current sheet 1?

Blue Into the ionosphere

Red Out of the ionosphere





#### What is the direction of the current in current sheet 1?

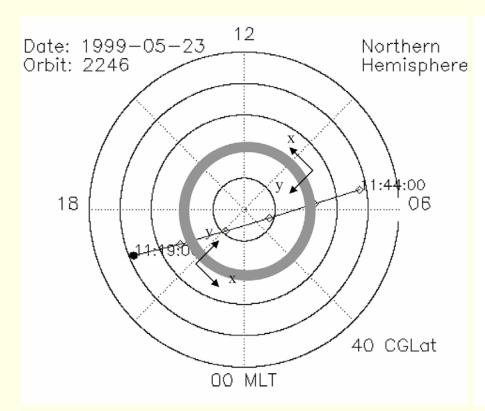
$$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$$

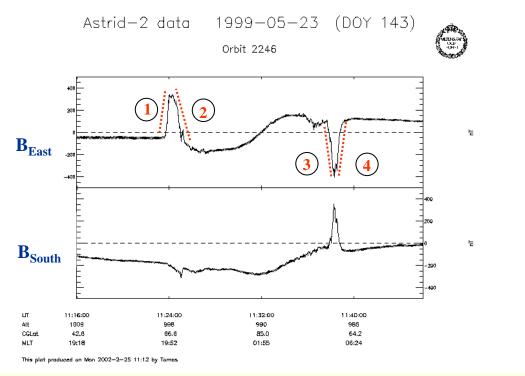
$$\frac{\partial B_{x}}{\partial y} = \frac{\partial B_{East}}{\partial y} > 0$$

$$\Rightarrow$$

Blue

Into the ionosphere





$$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$$

1) 
$$\frac{\partial B_x}{\partial y} > 0$$
  $\Rightarrow$   $j_z < 0$  Into the ionosphere

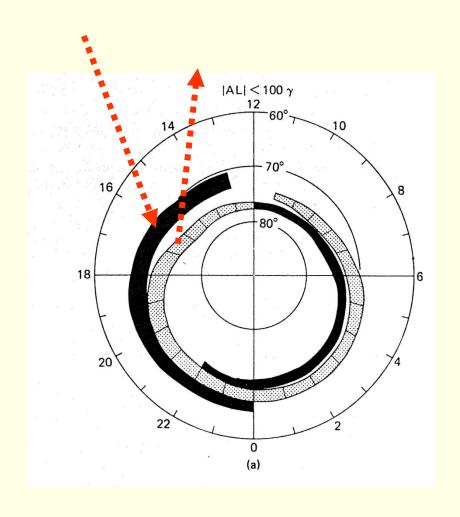
2)  $\frac{\partial B_x}{\partial y} < 0$   $\Rightarrow$   $j_z > 0$  Out of the ionosphere

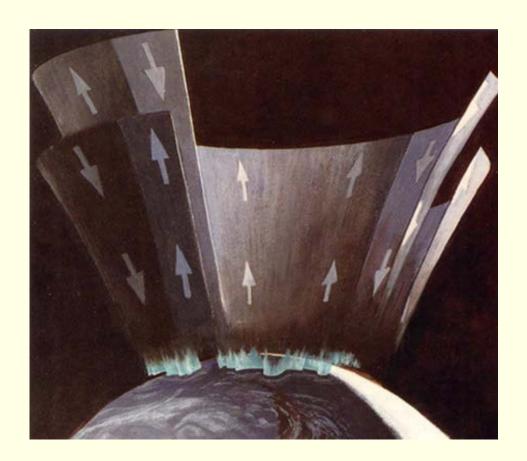
3)  $\frac{\partial B_x}{\partial y} > 0$   $\Rightarrow$   $j_z < 0$  Into the ionosphere

4)  $\frac{\partial B_x}{\partial y} < 0$   $\Rightarrow$   $j_z > 0$  Out of the ionosphere



## Birkeland currents in the auroral oval







## At what planets do you expect aurora to exist?

Blue

Earth, Mercury, Jupiter, Saturn

Yellow

Earth, Venus, Jupiter, Saturn, Uranus, Neptune

Green

Earth, Mars, Jupiter, Saturn, Uranus, Neptune

Red

Earth, Jupiter, Saturn, Uranus, Neptune



#### What do we need to have an aurora?

- Magnetic field (to guide the plasma particles towards the planet)
- Atmosphere (to create emissions)



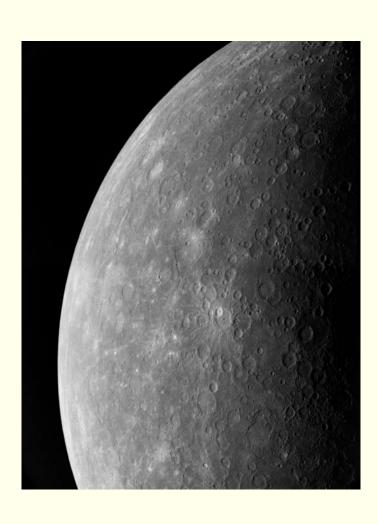
## At what planets do you expect aurora to exist?

Red

Earth, Jupiter, Saturn, Uranus, Neptune



## Mercury



- No atmosphere
- X-ray aurora???

  Can possibly be created by electrons colliding directly with the planetary surface and lose their energy in one single collision.



## Jupiter aurora

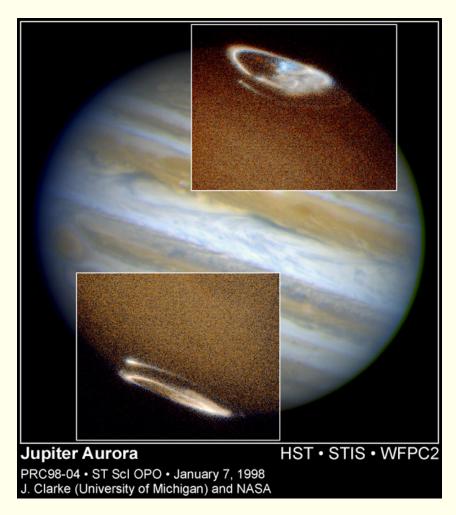
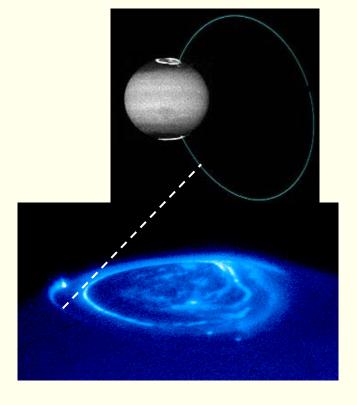


Foto från Hubble Space Telescope

• Jupiter's aurora has a power of ~1000 TW (compare Earth: ~100 GW, nuclear power plant: ~1 GW)

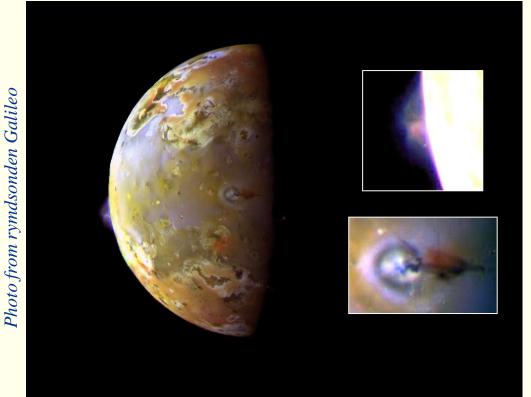
• Note the "extra" oval on Io's flux

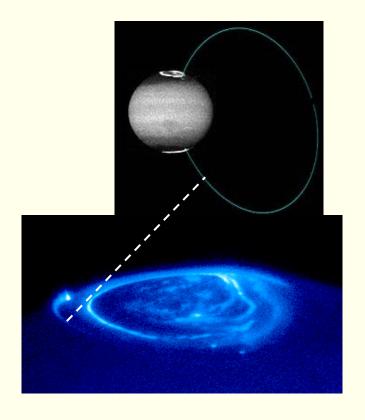
tube!





## Jupiter and lo



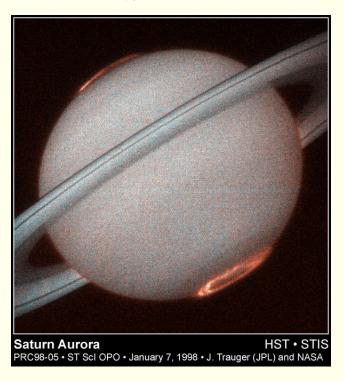


The Jupiter moon Io is very volcanically active, and deposes large amounts of dust and gas in Jupiter's magnetosphere. This is ionized by the sunlight, and the charged plasma partícles follow Jupiter's magnetic field lines towards the atmosphere and cause auroral emissions.



## Aurora of the other planets

#### Saturn



Saturnus' aurora: not noticeably different from Jupiter's, but much weaker. (Total power about the same as Earth's aurora.) Uranus: Auora detected in UV.
Probably associated with Uranus' ring
current/radiotion belts and not very
dynamic.

Neptunus: weak UV aurora detected.

Mars, Venus: No aurora.



## Prerequisites for...



## Life

- Energy source (sun)
- Atmosphere
- Magnetic field
- Water



#### **Aurora**

- Energy source (sun)
- Atmosphere
- Magnetic field



# On space weather and viewing aurora

#### Some space weather sites

http://spaceweather.com/

http://www.esa-spaceweather.net/

http://sunearthday.nasa.gov/swac/

http://www.noaawatch.gov/themes/space.php

http://www.windows2universe.org/spac eweather/more\_details.html

#### **Kiruna**

Kiruna all-sky camera:

http://www.irf.se/allsky/rtasc.php

http://sunearthday.nasa.gov/swac/tutorials/aur\_kiruna.php

#### Forecasts:

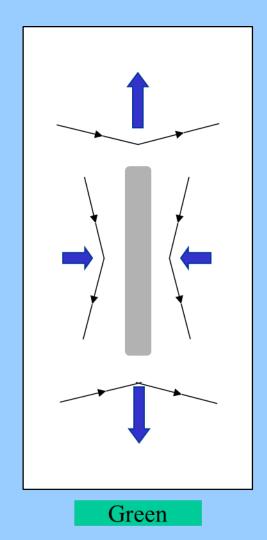
http://flare.lund.irf.se/rwc/aurora/

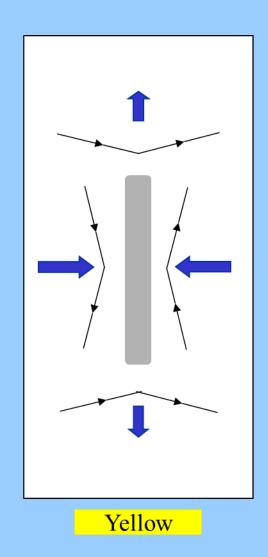
http://www.irf.se/Observatory/?link[All-

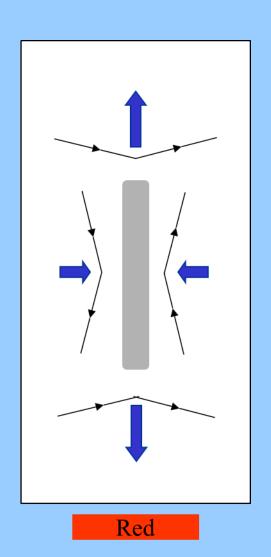
skycamera]=Aurora\_sp\_statistics



## **Magnetic reconnection**

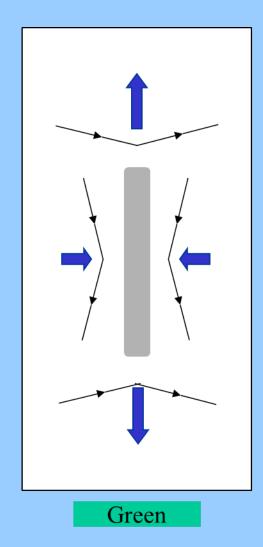


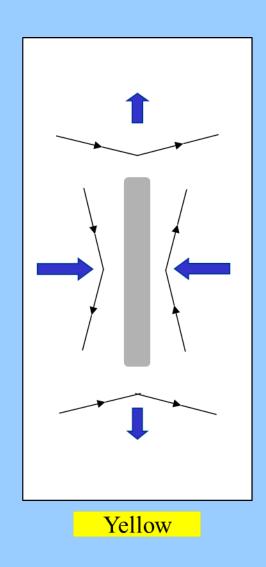


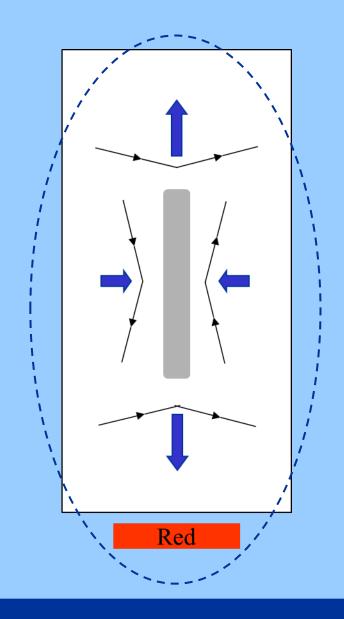




## **Magnetic reconnection**

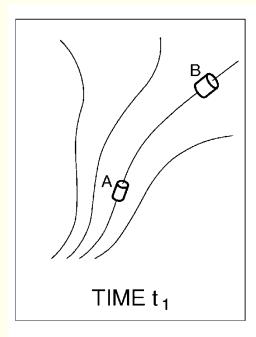


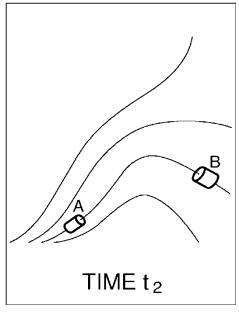






## Frozen in magnetic field lines





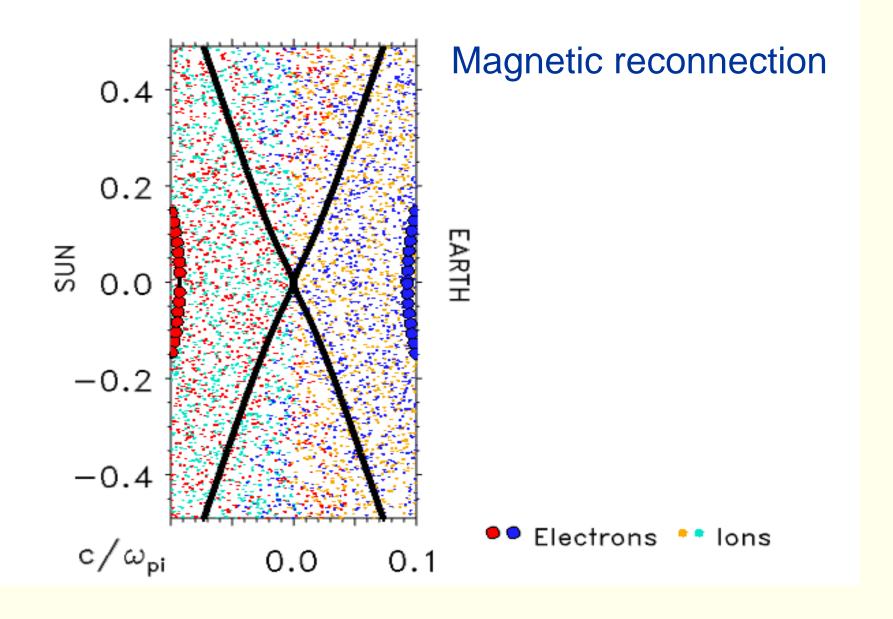
In fluid description of plasma two plasma elements that are connected by a common magnetic field line at time  $t_1$  will be so at any other time  $t_2$ .

This applies if the magnetic Reynolds number is large:

$$R_m = \mu_0 \sigma l_c v_c >> 1$$

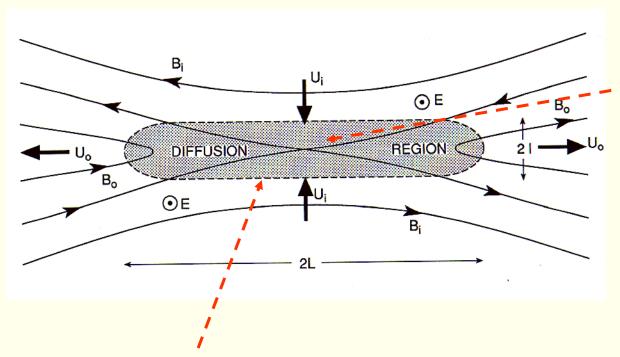
An example of the collective behaviour of plasmas.







#### Reconnection



- Field lines are "cut" and can be reconnected to other field lines
- Magnetic energy is transformed into kinetic energy  $(U_o >> U_i)$

In 'diffusion region':

$$R_{\rm m} = \mu_0 \sigma l v \sim 1$$

Thus: condition for frozen-in magnetic field breaks down.

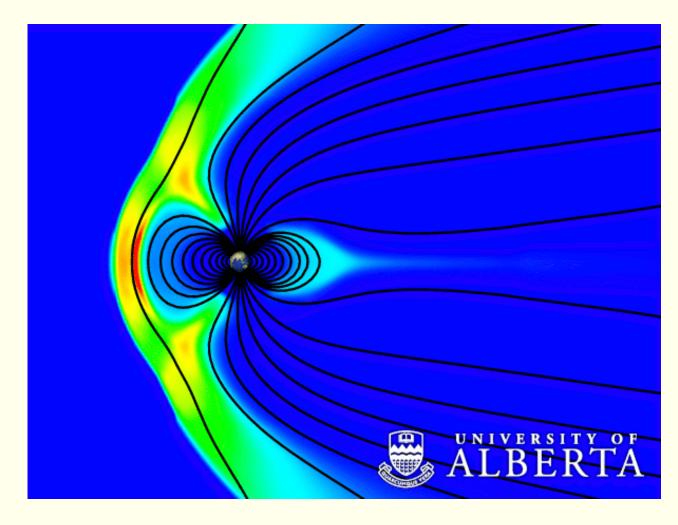
A second condition is that there are two regions of magnetic field pointing in opposite direction:

 Plasma from different field lines can mix



#### Reconnection and plasma convection

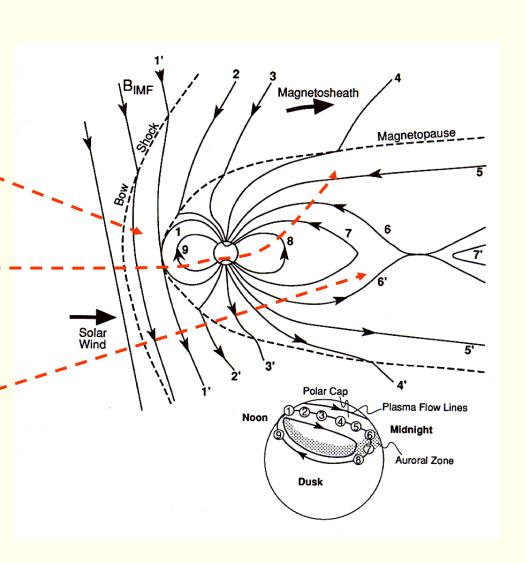






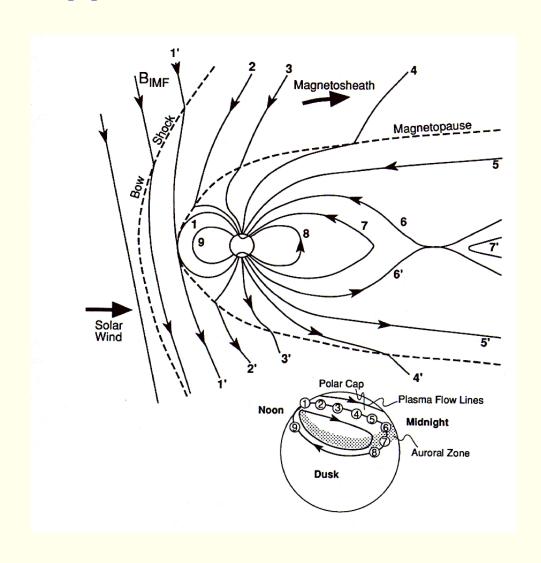
#### Reconnection och plasma convection

- Reconnection on the dayside "re-connects" the solar wind magnetic field and the geomagnetic field
- In this way the plasma convection in the outer magnetosphere is driven
- In the night side a second reconnection region drives the convection in the inner magnetosphere.
   The reconnection also heats the plasmasheet plasma.



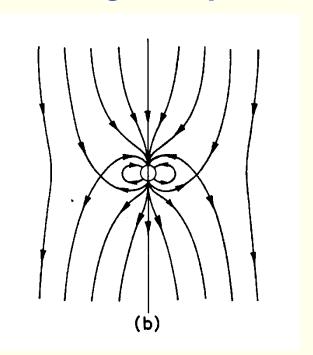


### What happens if IMF is northward instead?

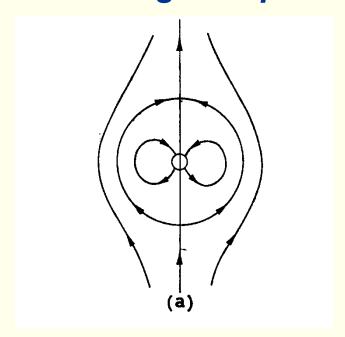




#### open magnetosphere



#### closed magnetosphere



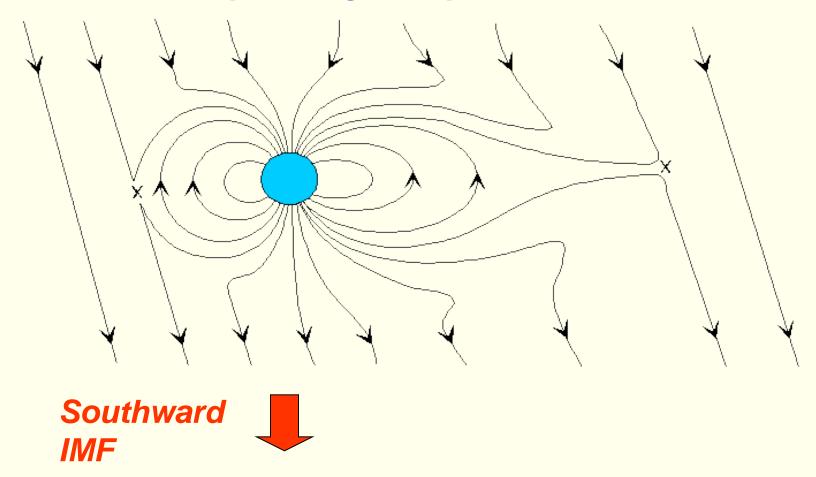


Interplanetary magnetic field (IMF)



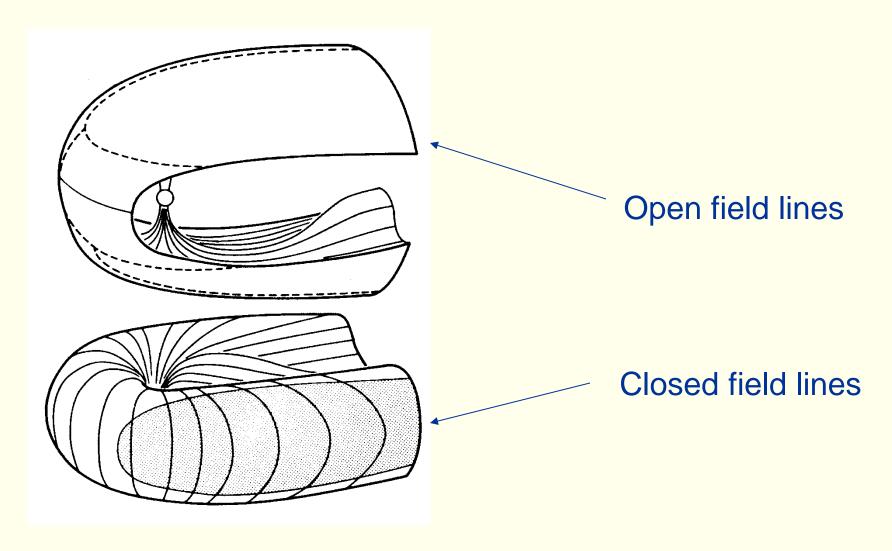


#### open magnetosphere



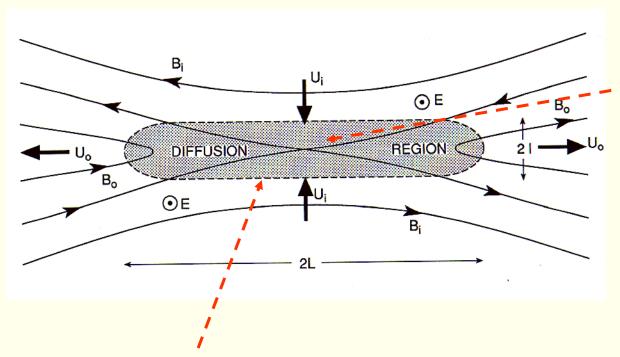


## Magnetospheric topology





#### Reconnection



- Field lines are "cut" and can be reconnected to other field lines
- Magnetic energy is transformed into kinetic energy  $(U_o >> U_i)$

In 'diffusion region':

$$R_{\rm m} = \mu_0 \sigma l v \sim 1$$

Thus: condition for frozen-in magnetic field breaks down.

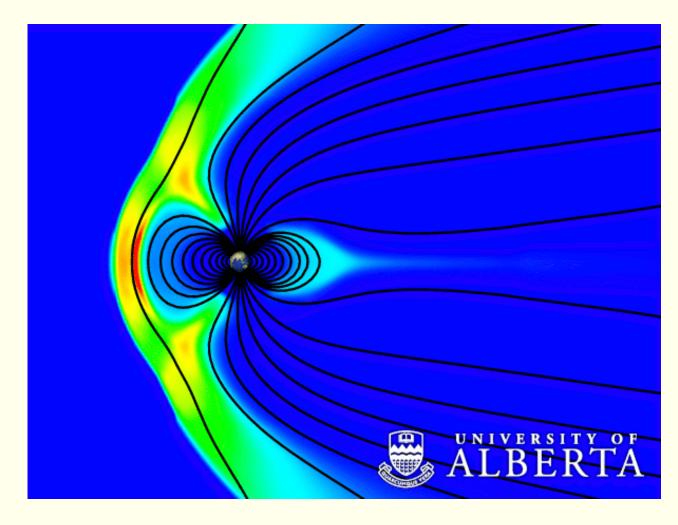
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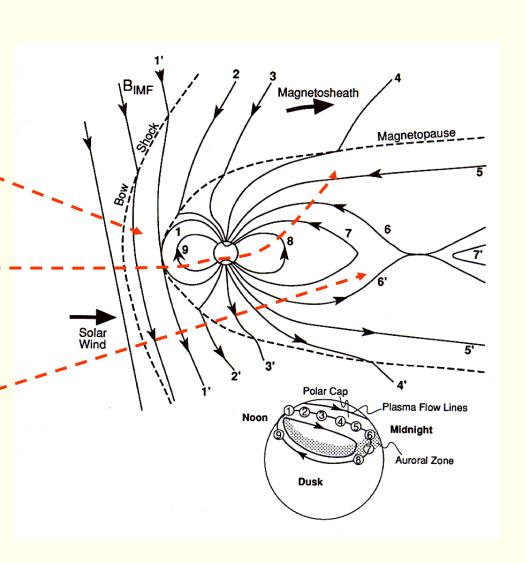






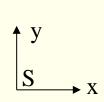
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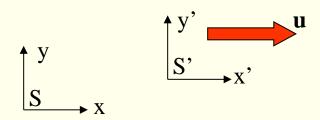
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   The reconnection also heats the plasmasheet plasma.





## Field transformations (relativistic)





Relativistic transformations (perpendicular to the velocity u):

$$\mathbf{E'} = \frac{\mathbf{E} + \mathbf{u} \times \mathbf{B}}{\sqrt{1 - u^2/c^2}}$$

$$\mathbf{B} = \frac{\mathbf{B} - (\mathbf{u}/c^2) \times \mathbf{E}}{\sqrt{1 - u^2/c^2}}$$

*For u* << *c*:

$$\mathbf{E'} = \mathbf{E} + \mathbf{u} \times \mathbf{B}$$

induced electric field

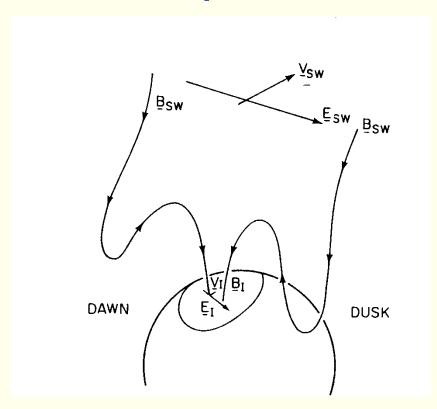
$$\mathbf{E} = \mathbf{E}' - \mathbf{u} \times \mathbf{B}$$

$$\mathbf{B} = \mathbf{B}$$



#### open magnetosphere

#### Viewpoint 1



The solar wind generates an electric field

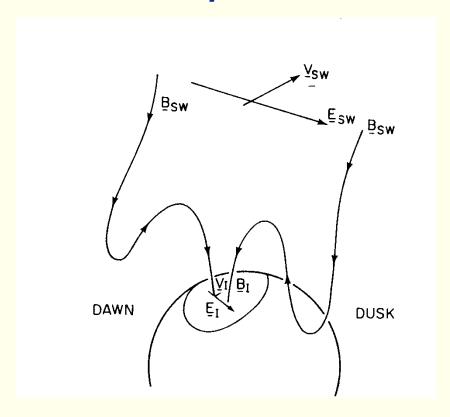
$$\mathbf{E}_{\mathrm{SW}} = -\mathbf{v}_{\mathrm{SW}} \times \mathbf{B}_{\mathrm{SW}}$$

which maps down to the ionosphere, since the field lines are very good conductors



#### open magnetosphere

#### Viewpoint 2



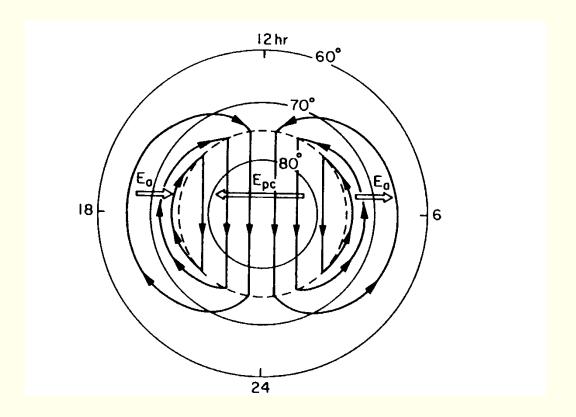
The solar wind magnetic field draws the ionospheric plasma with it, since the field is frozen into the plasma. This motion induces an ionospheric electric field

$$\mathbf{E}_{\scriptscriptstyle \rm I} = - \mathbf{v}_{\scriptscriptstyle \rm I} \times \mathbf{B}_{\scriptscriptstyle \rm I}$$



#### Plasma convection in the ionosphere

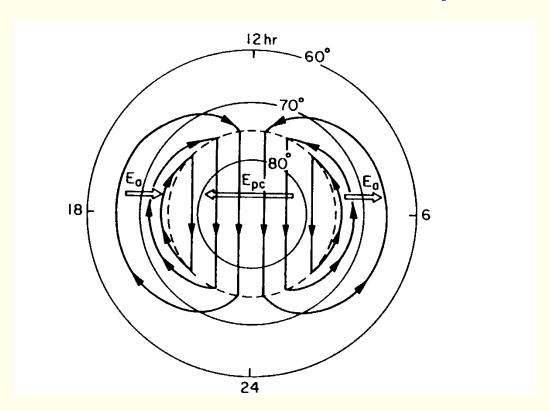
The electric field "propagates" to the ionosphere, since the field lines are good conductors, and thus equipotentials





# Do you recognize this pattern?

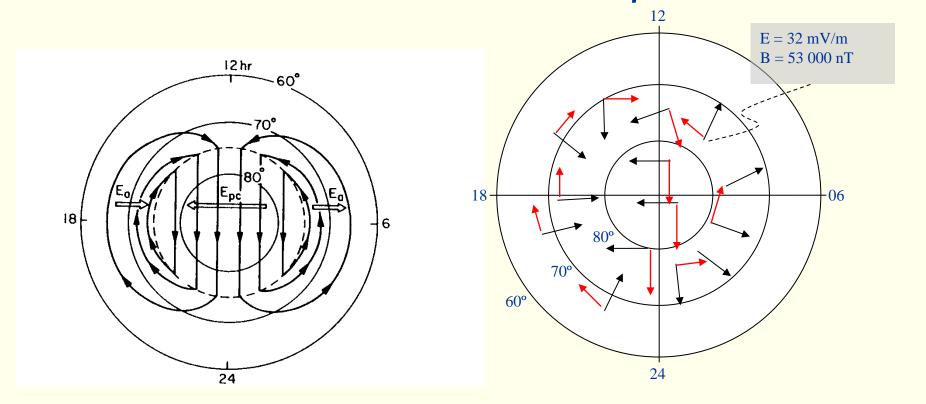
#### Plasma convection in the ionosphere





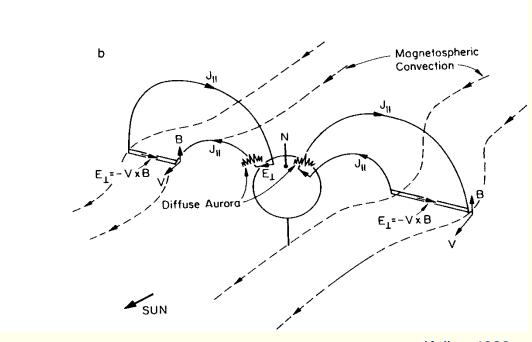
# Do you recognize this pattern?

#### Plasma convection in the ionosphere



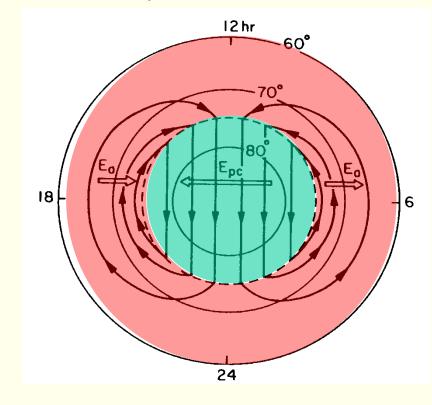
## Static, large-scale MI-coupling

#### Magnetospheric and ionospheric convection



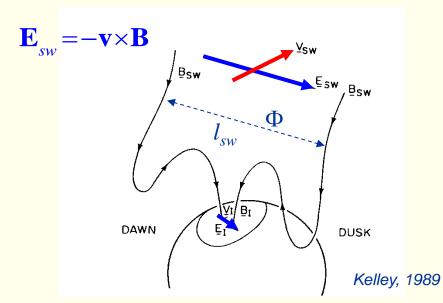
Kelley, 1989

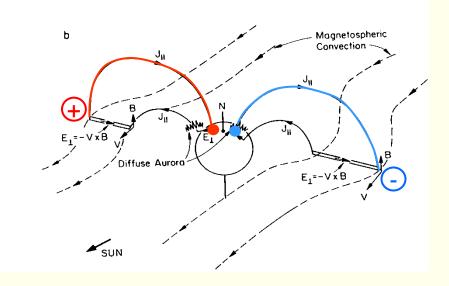
#### Ionospheric convection



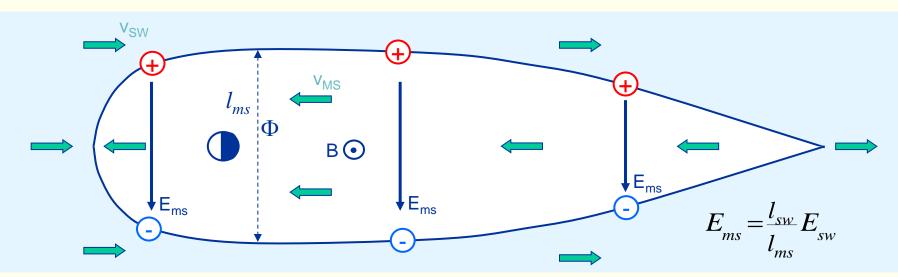


## Magnetospheric plasma convection



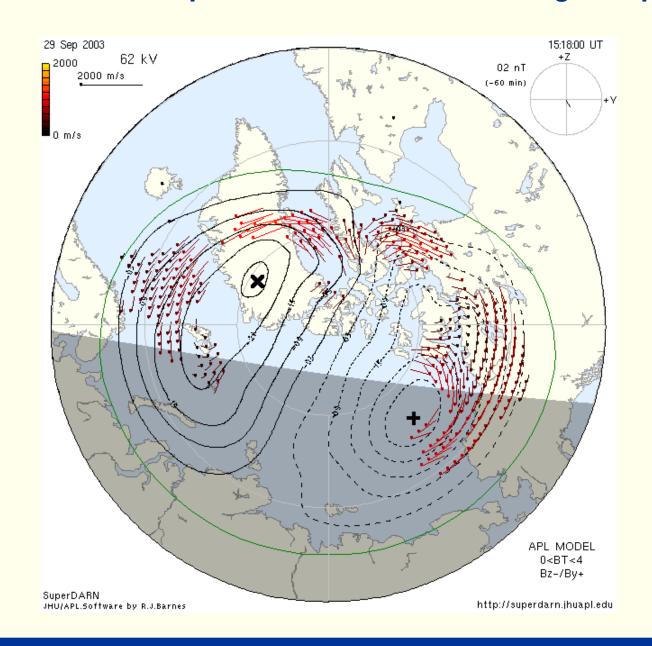






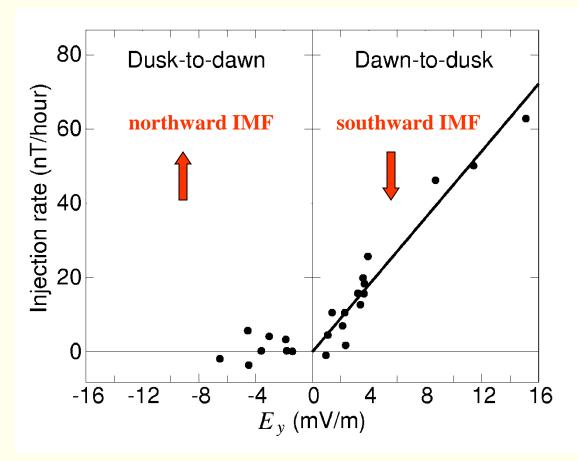


#### Measurements of plasma convection in the magnetosphere





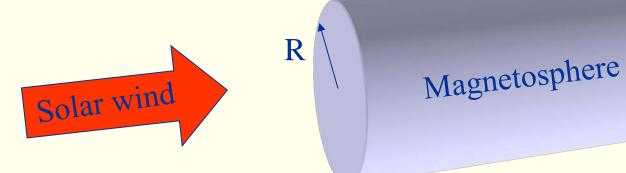
## Energy input Plasma convection in the magnetosphere



- Solar wind generates electric field E = - v × B.
- Depending on direction of B, sign of E changes
- Energy input only for open magnetosphere
- The magnetosphere works like a diode!



## Energy budget (1)



$$W_{kin} = \rho v^2/2 = 0.63 \cdot 10^{-9} \text{ Jm}^{-3}$$

$$W_{term} = n_e k_b T_e = 1.4 \cdot 10^{-11} \text{ Jm}^{-3}$$
  $A = \pi R^2 = \pi (10 R_E)^2$ 

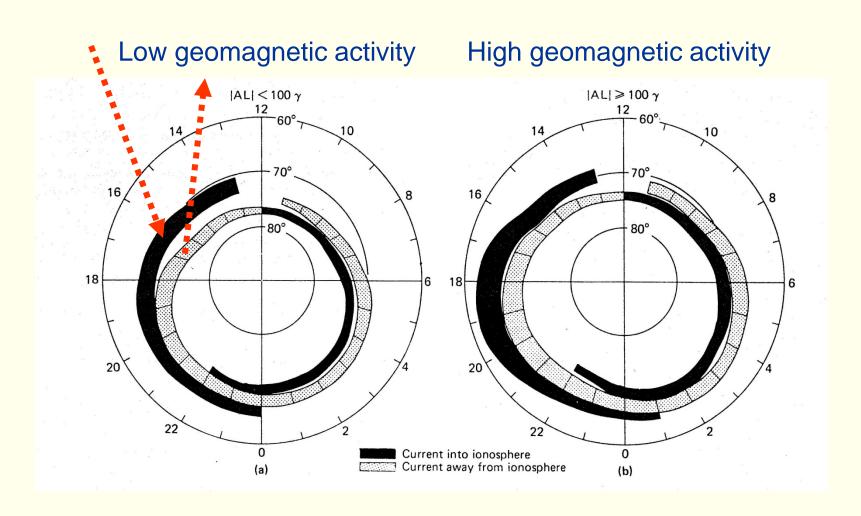
$$A = \pi R^2 = \pi (10R_E)^2$$

$$\Phi_{\rm kin} = v_{\rm SW} W_{\rm kin} = 0.2 \cdot 10^{-3} \ {\rm Wm^{-2}}$$

$$P_{sw} = \Phi_{kin} A = 3.10^{12} W$$



## Birkeland currents in the auroral oval





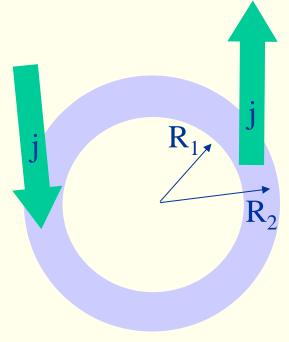
## **Energy budget (2)**

$$A = \pi (R_2^2 - R_1^2) = 2.10^{13} \text{ m}^2$$

$$I = jA/2 = \frac{1}{2} \cdot 0.1 \cdot 10^{-6} \text{ Am}^{-2} \cdot 2 \cdot 10^{13} \text{ m}^2$$
  
= 10 MA

$$U = ?$$

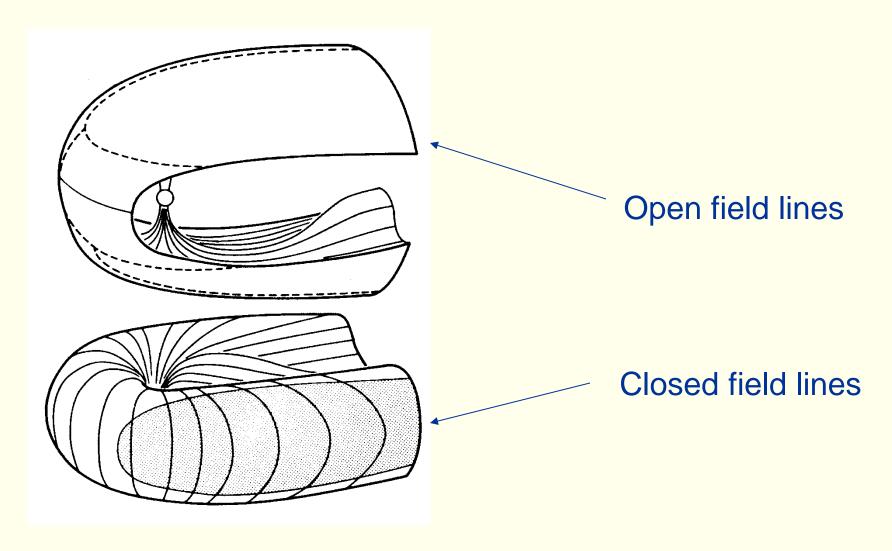
$$P = UI = ?$$



Auroral oval



## Magnetospheric topology



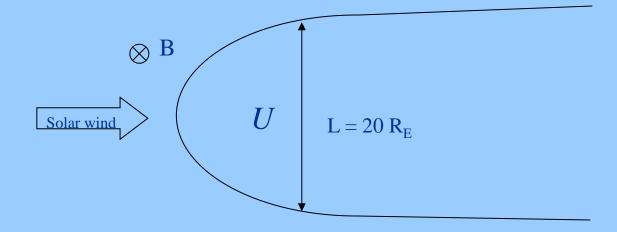


# What is the potential drop over the magnetosphere?

$$\mathbf{E} = -\mathbf{v}_{SW} \times \mathbf{B}_{SW}$$

$$v_{SW} = 300 \text{ km/s}$$

$$B_{SW} = 5 \text{ nT}$$



Blue

2 kV

Yellow

20 kV

Red

200 kV

Green

2 MV

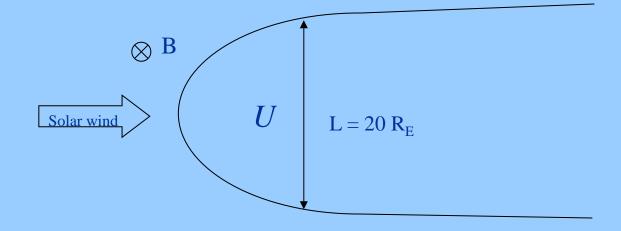


# What is the potential drop over the magnetosphere?

$$\mathbf{E} = -\mathbf{v}_{SW} \times \mathbf{B}_{SW}$$

$$v_{SW} = 300 \text{ km/s}$$

$$B_{SW} = 5 \text{ nT}$$



$$U = v_{SW}B_{SW}L = 300 \cdot 10^3 \cdot 5 \cdot 10^{-9} \cdot 20 \cdot 6378 \cdot 10^3 = 190 \text{ kV}$$

Red

200 kV

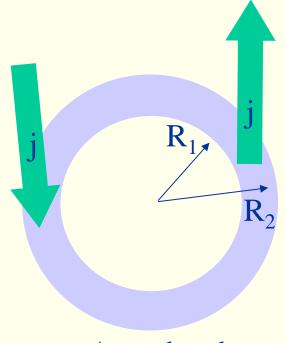


## **Energy budget (2)**

$$U = 200 \text{ kV}$$

$$A = \pi (R_2^2 - R_1^2) = 2.10^{13} \text{ m}^3$$

$$I = jA/2 = \frac{1}{2} \cdot 0.1 \cdot 10^{-6} \text{ Am}^{-2} \cdot 2 \cdot 10^{13}$$
  
 $m^2 = 10 \text{ MA}$ 



Auroral oval

$$P = UI = 2.10^{11} W = 6\% of P_{SW}$$