



Last lecture (5)

- Solar wind
 - magnetic structure
- Ionosphere
 - index of refraction
 - reflection of radio waves
 - particle drift motion in magnetized plasma

Today's lecture (6)

- Electrical conductivity in ionosphere
- Magnetosphere, introduction
- Magnetospheric size (standoff distance)
- Particle motion in the magnetosphere



Today

Activity	Date	Time	Room	Subject	Litterature
L1	28/8	15-17	Q21	Course description, Introduction, The Sun 1	CGF Ch 1.1,1.2, 1.4, 5, (p 110-113), 6.3
L2	29/8	13-15	Q2	The Sun 2, Plasma physics 1	CGF Ch 1.3, 5 (p 114-121)
L3	4/9	10-12	E2	Solar wind, The ionosphere and atmosphere 1, Plasma physics 2	CGF Ch 6.1, 2, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	6/9	8-10	Q21	Mini-group work 1	
L4	6/9	15-17	Q2	The ionosphere 2, Plasma physics 3	CGF Ch 3.4, 3.7, 3.8
T2	10/9	15-17	Q21	Mini-group work 2	
L5	11/9	10-12	E3	The Earth's magnetosphere 1, Plasma physics 4	CGF 4-1-4.3, LL Ch I, II, IV.A
T3	17/9	8-10	Q21	Mini-group work 3	
L6	18/9	13-15	Q33	The Earth's magnetosphere 2, Other magnetospheres	CGF Ch 4.6-4.9, LL Ch V.
L7	19/9	13-15	Q2	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
T4	24/9	8-10	Q2	Mini-group work 4	
L8	24/9	15-17	V3	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
T5	2/10	8-10	Q31	Mini-group work 5	
L9	2/10	13-15	Q2	Alfvén waves, Interstellar and intergalactic plasma, Cosmic radiation	CGF Ch 7-9, Extra material
T6	8/10	15-17	Q21		
L10	9/10	10-12	Q2	Guest Lecture by Swedish astronaut Christer Fuglesang	
Written examination	16/10	14-19	L21, L22, L31		

EF22445 Space Physics II

7.5 ECTS credits, P2

- shocks and boundaries in space
- solar wind interaction with magnetized and unmagnetized bodies
- reconnection
- sources of magnetospheric plasma
- magnetospheric and ionospheric convection
- auroral physics
- storms and substorms
- global oscillations of the magnetosphere

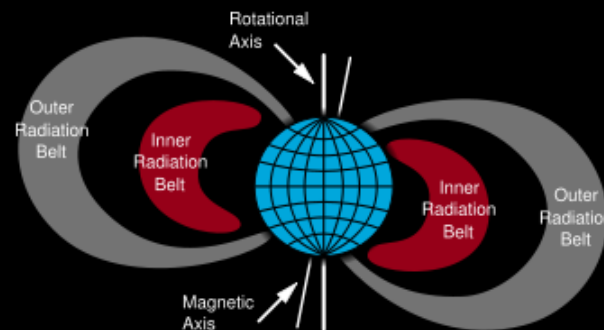
Courses at the Alfvén Laboratory

EF2260 SPACE ENVIRONMENT AND SPACECRAFT ENGINEERING , 6 ECTS credits, period 2

- environments spacecraft may encounter in various orbits around the Earth, and the constraints this places on spacecraft design
- basic operation principles underlying the thermal control system and the power systems in spacecraft
- measurements principles in space



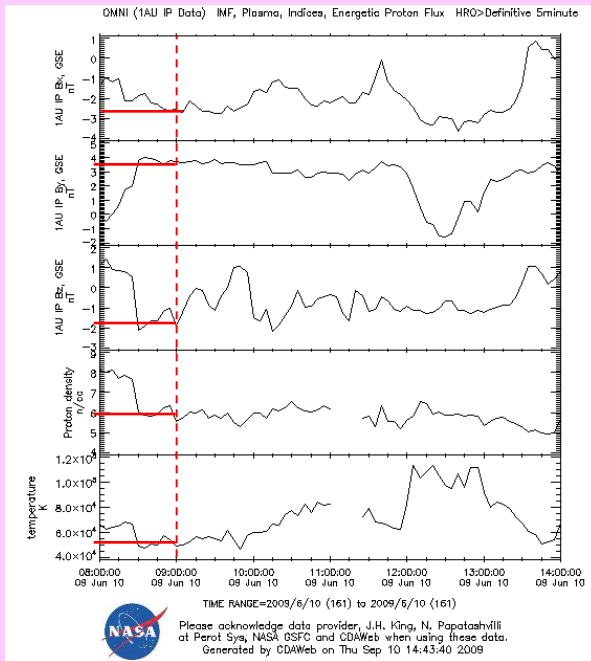
The Astrid-2 satellite



Radiation environment in near-earth space

Projects:

- Design power supply for spacecraft
- Study of radiation effects on electronics



Mini-groupwork 2

a)

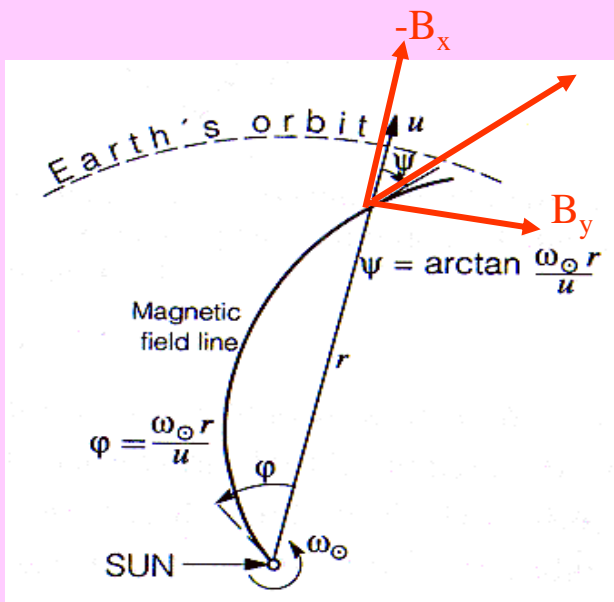
$$\psi = \arctan \frac{\omega_{sun} r}{u_{sw}} \quad \Rightarrow \quad u_{sw} = \frac{\omega_{sun} r}{\tan \psi}$$

$$\omega_{sun} = 2\pi/T = 2.9 \cdot 10^{-6} \text{ s}^{-1} \quad (T = 25 \text{ days at equator})$$

$$r = 1 \text{ A.U.}$$

$$\tan \psi = |B_y/B_x| \approx 3.6/2.6 \quad (\text{from figure}) \quad (\psi = 41^\circ)$$

With these figures I get $u_{sw} = 313 \text{ km/s}$



Mini-groupwork 2

b)

The magnetic Reynolds number is calculated by using typical plasma flow velocities v_c and typical length scales of magnetic field variations l_c

Use solar wind velocity obtained in a) for typical flow velocity. To obtain l_c , multiply the time t it takes the magnetic field structure (indicated in the figure), to pass over the satellite and use $l_c = vt$. I get $l_c = 2.8 \cdot 10^8$ m.

Using a temperature of $5 \cdot 10^4$ K, we can evaluate the conductivity, remembering that the temperature should be given in eV. We get the conversion from

$$W = \frac{3}{2} k_B T$$

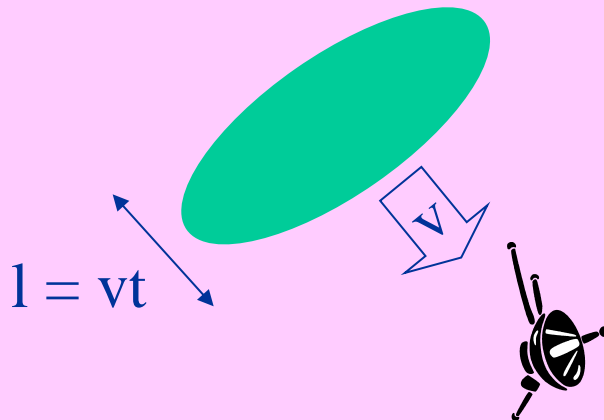
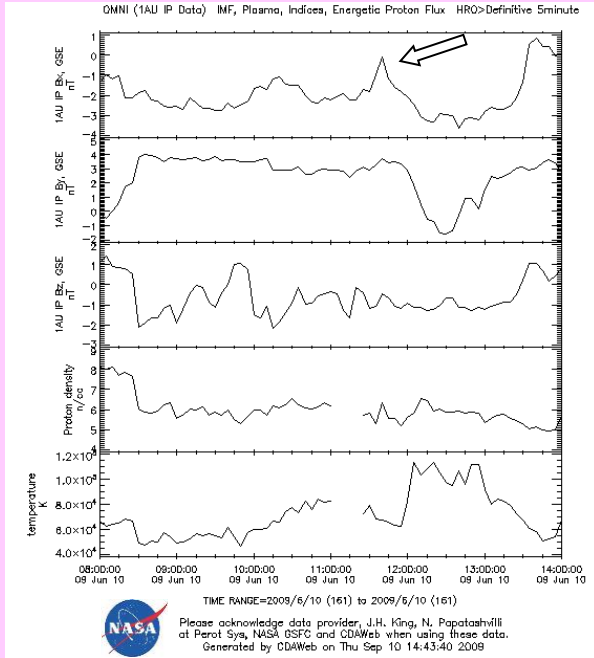
which gives the result that 1 eV corresponds to a temperature of 7729 K. We then get $T = 6.5$ eV, and

$$\sigma = 3.1 \cdot 10^4 \text{ S/m}$$

Putting in the numbers I get

$$R_m = \mu_0 \sigma v_c l_c \approx 9.8 \cdot 10^{14} \gg 1$$

So the solar wind magnetic field is frozen into the plasma to a very good approximation.



Energy - temperature

Average energy of molecule/atom:

$$E = \frac{3}{2} k_B T \Rightarrow$$

$$T = \frac{2E}{3k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$T = \frac{2E}{3k_B} = \frac{2 \cdot 1.6 \cdot 10^{-19} \text{ J}}{3 \cdot 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}} = 7729 \text{ K}$$

But beware!

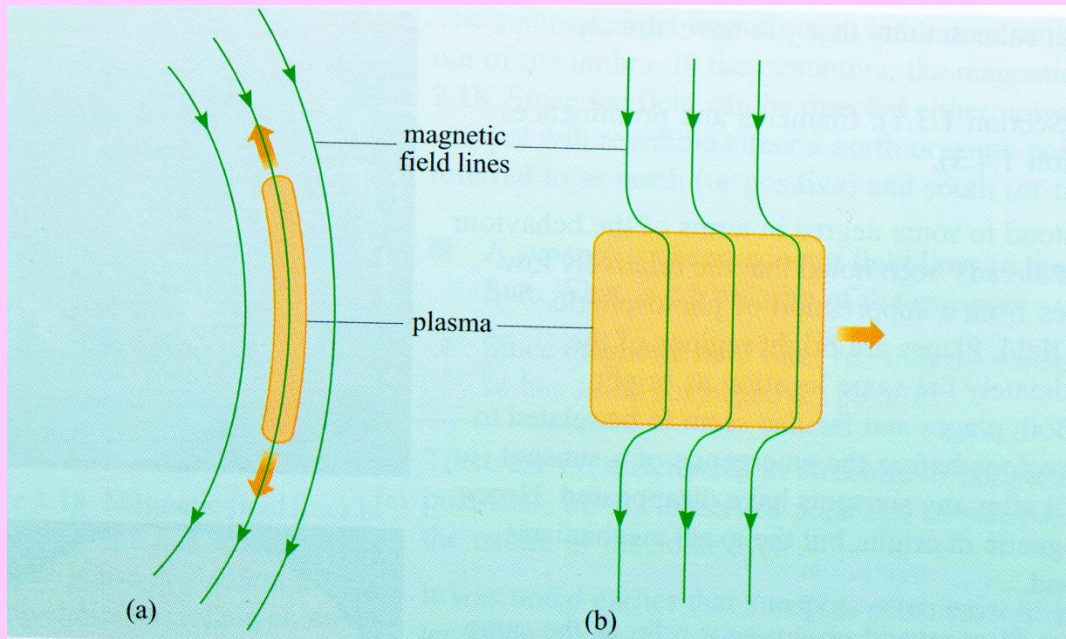
In plasma physics, usually:

$$E = \cancel{\frac{3}{2}} k_B T \Rightarrow$$
$$T = \frac{E}{k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$E = k_B T = \frac{1.6 \cdot 10^{-19} \text{ J}}{1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}} = 11594 \text{ K}$$

Does the plasma follow the magnetic field (a) or the other way around (b)?



$$\beta \ll 1$$

$$\beta \gg 1$$

Depends on relative energy density (pressure)

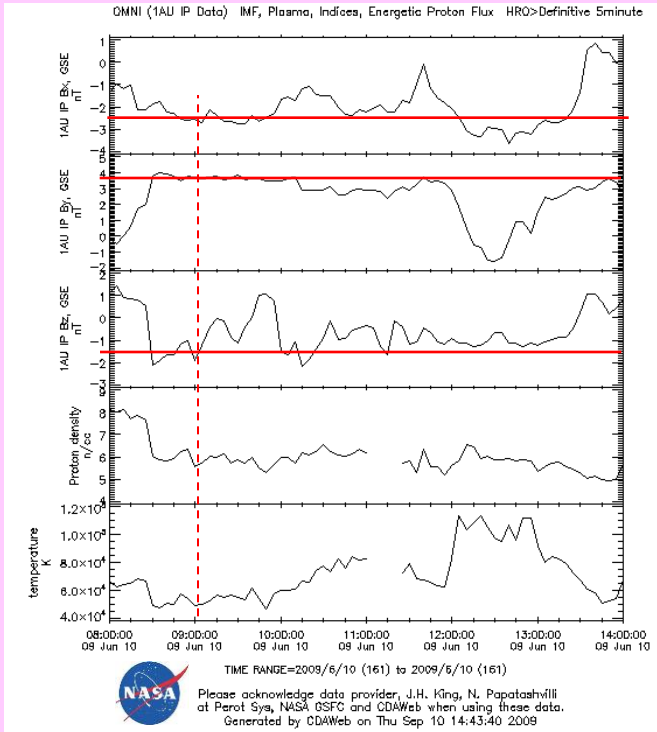
$$p_{pl} = nk_B T$$

$$p_B = \frac{B^2}{2\mu_0}$$

$$\beta = \frac{p_{pl}}{p_B}$$

Mini-groupwork 2

c)



$$\rho = n_e m_p = 6.1 \cdot 10^6 \cdot 1.67 \cdot 10^{-27} = 1.02 \cdot 10^{-20}$$

Then the kinetic energy density is ($v = 313$ km/s):

$$\rho v^2 / 2 = 5.0 \cdot 10^{-10} \text{ Jm}^{-3}$$

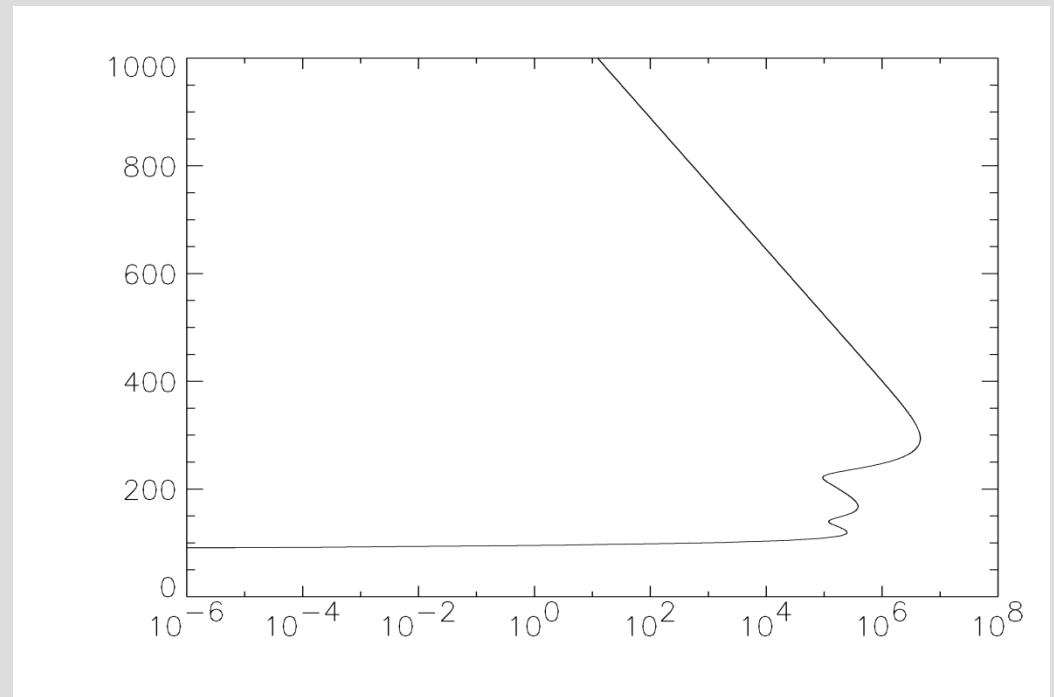
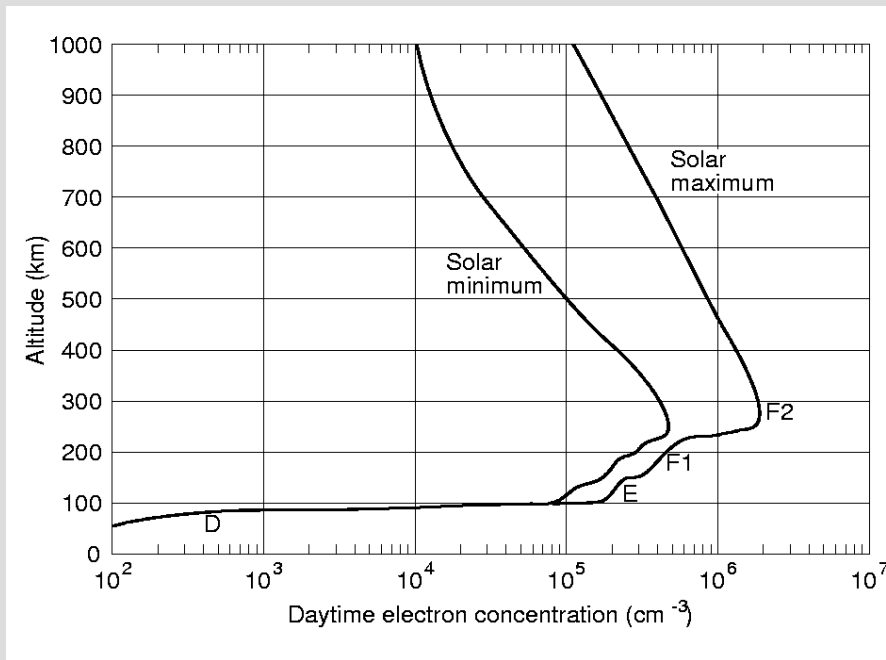
The magnetic energy density is (using values of figure)

$$\frac{B^2}{2\mu_0} = \frac{(B_x^2 + B_y^2 + B_z^2)}{2\mu_0} = (2.6^2 + 3.6^2 + 1.7^2) \cdot (10^{-9})^2 / 2\mu_0 = 9 \cdot 10^{-12} \text{ Jm}^{-3}$$

The ratio between the kinetic and magnetic energy densities is approximately **50**, thus the plasma motion determines the magnetic field configuration, and not the other way around.

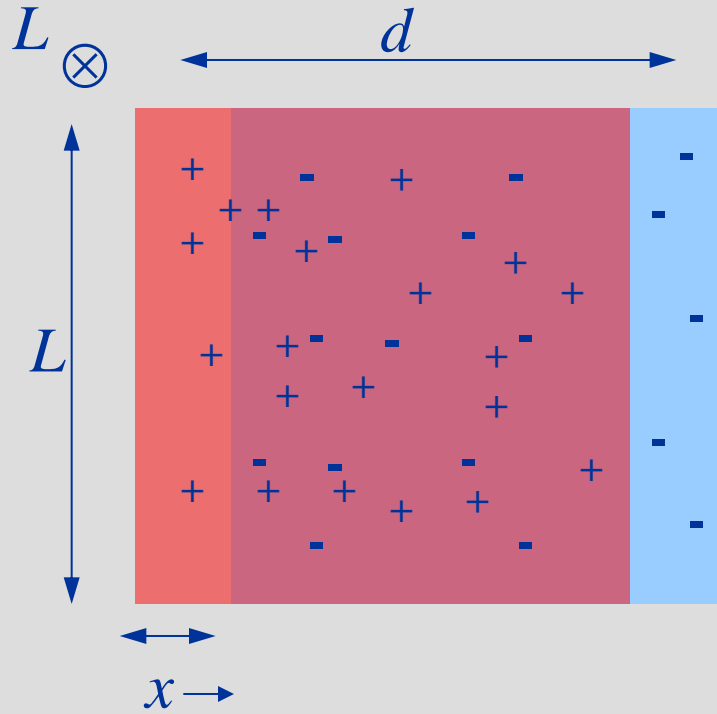
Measurements

"E" + "F1" + "F2"



Ionospheric layers

Layer	D	E	F ₁	F ₂
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm ⁻³)	<10 ²	2 · 10 ³	—	2 - 5 · 10 ⁵
Daytime electron density (cm ⁻³)	10 ³	1 - 2 · 10 ⁵	2 - 5 · 10 ⁵	0.5 - 2 · 10 ⁶
Ion species	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺ O ⁺	O ⁺ He ⁺ H ⁺
Cause of ionization	Lyman _α (1215 Å) + cosmic radiation	Lyman _β (1025 Å) X-rays	UV	UV

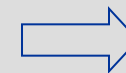


$$F = m_e a$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$F = -eE$$

$$\sigma = en_e x$$



$$-\frac{n_e e^2 x}{\epsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

Index of refraction for electromagnetic waves in a plasma (corrected)

$$(1) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(3) \quad \mathbf{j} = -en_e \mathbf{v}_e$$

$$(4) \quad m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency ω , e.g.:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$(1) \Rightarrow \nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \Rightarrow \nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\therefore \nabla \times (\nabla \times \mathbf{E}) = +\mu_0 \frac{\partial}{\partial t} (en_e \mathbf{v}_e) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

\Rightarrow

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = +\mu_0 en_e \frac{\partial \mathbf{v}_e}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

\Rightarrow

Index of refraction for electromagnetic waves in a plasma (corrected)

~~$$ik(\mathbf{k} \cdot \mathbf{E}) - k^2 \mathbf{E} = \mu_0 (-i\omega) en_e \mathbf{v}_e - \frac{1}{c^2} (-i\omega)^2 \mathbf{E}$$~~

Does not represent E.M. wave

(4) \Rightarrow

$$k^2 \mathbf{E} = -\mu_0 en_e \frac{e\mathbf{E}}{m_e} + \frac{1}{c^2} \omega^2 \mathbf{E}$$

\Rightarrow

$$c^2 k^2 = -c^2 \frac{\mu_0 n_e e^2}{m_e} + \omega^2 = \frac{-1}{\cancel{\mu_0 \epsilon_0}} \frac{\cancel{\mu_0} n_e e^2}{m_e} + \omega^2$$

$$\therefore \omega^2 = c^2 k^2 + \omega_p^2$$

$$n^2 = \frac{c^2}{v_{ph}^2} = \frac{c^2 k^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

\therefore

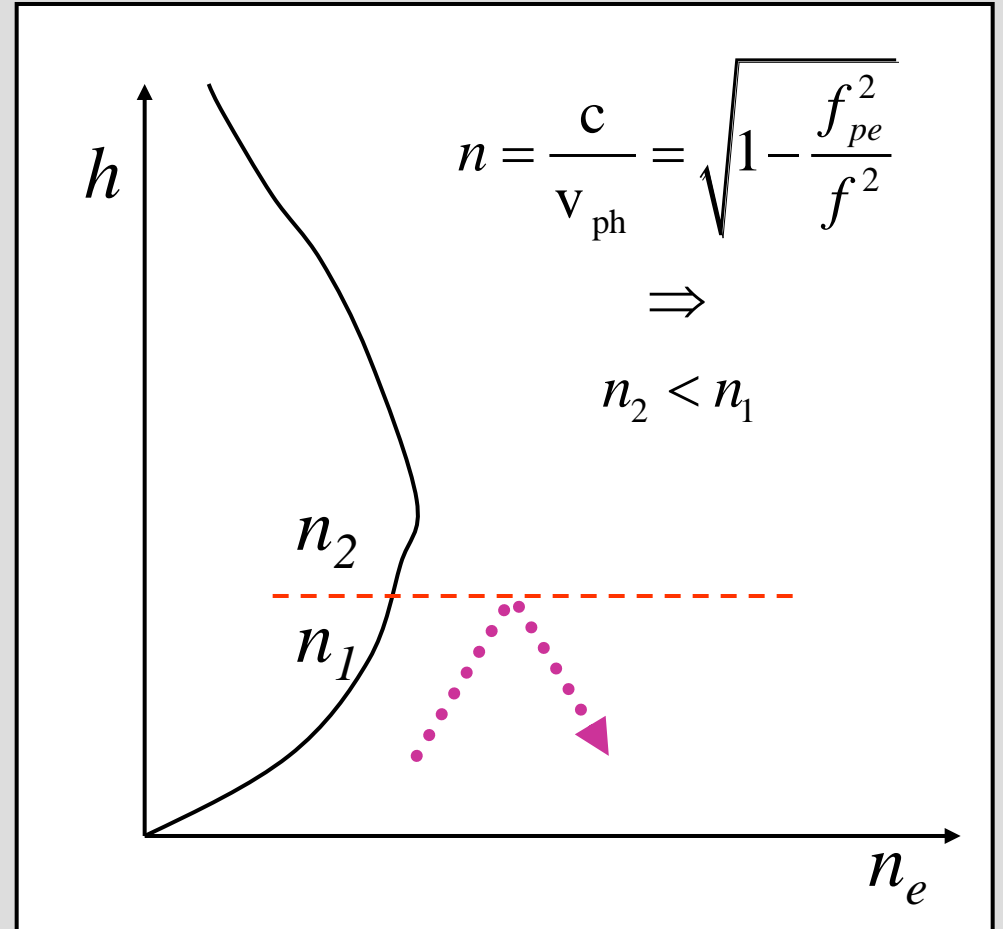
$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{f_p^2}{f^2}}$$

Where does the total reflection take place?

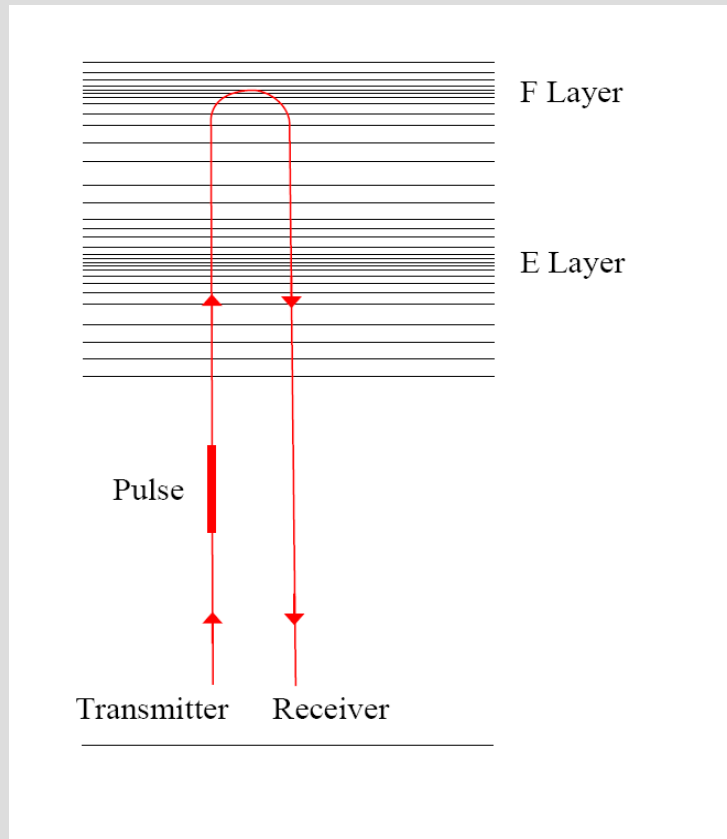
Large gradient when

$$f \approx f_{pe}$$

Higher frequencies \rightarrow higher $f_{pe}(n_e)$



Ionosonde



The pulse will be reflected where

$$f = f_{pe}$$

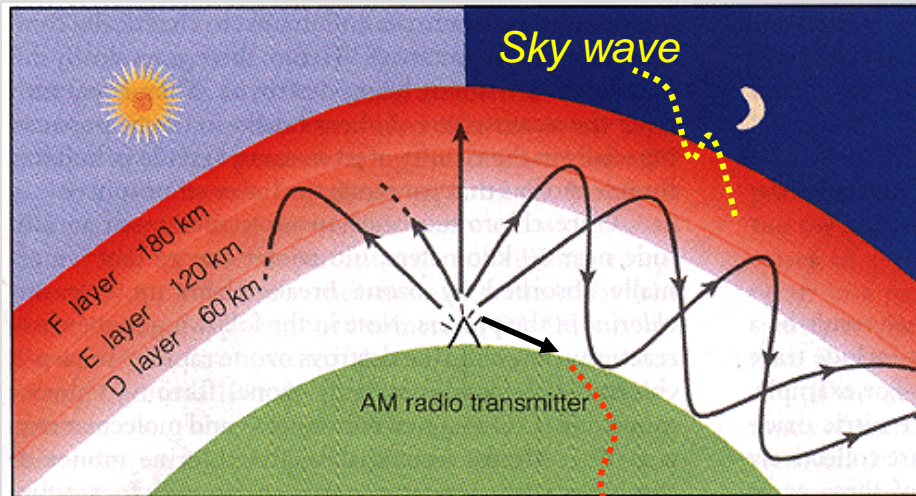
The altitude will be determined by

$$2h = ct$$

Where t is the time between when the pulse is sent out and the registered again.

Reflection of radio waves

F2-layer during night:

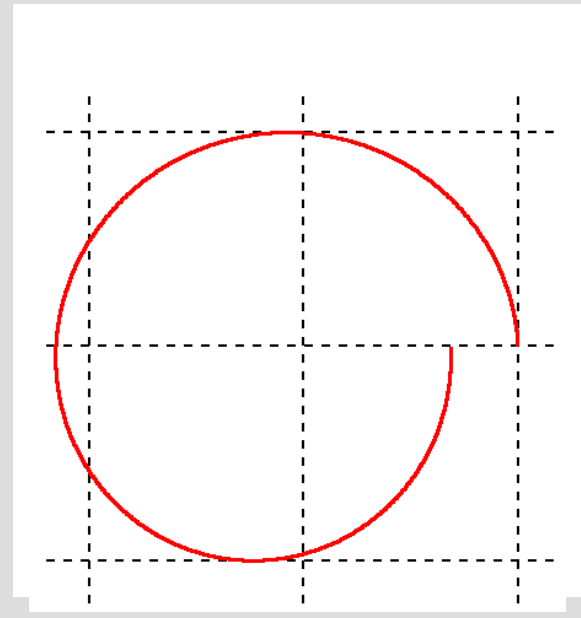
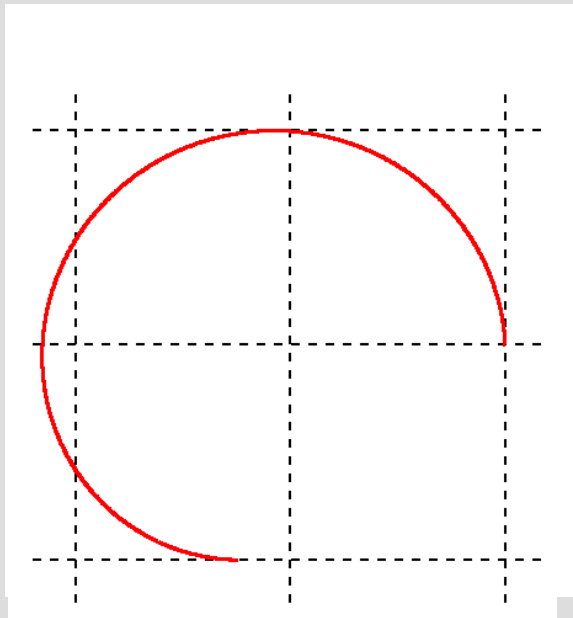
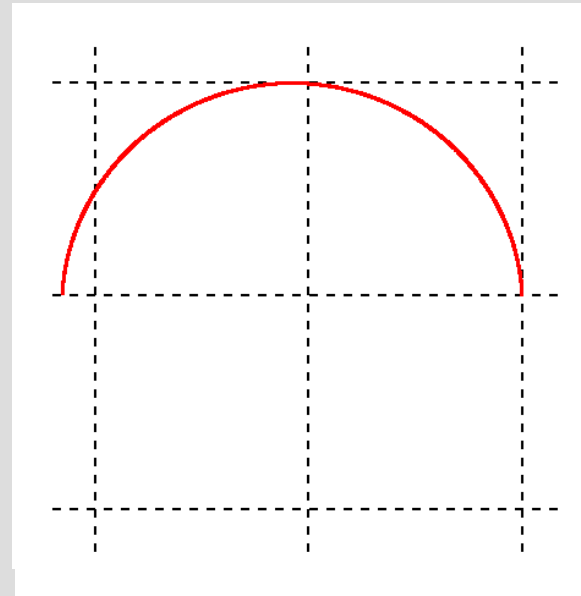
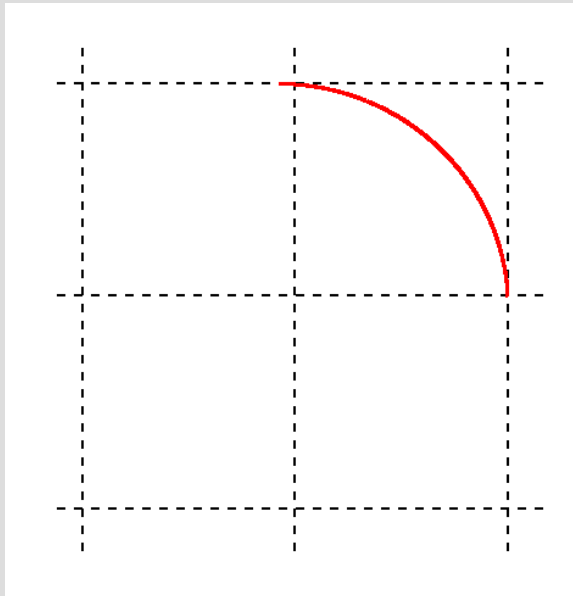


Ground wave

$$n_e = 5 \cdot 10^{11} \text{ m}^{-3} \Rightarrow$$

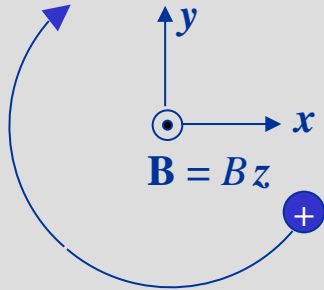
$$f_{pe} = 10^7 \text{ Hz} = 10 \text{ MHz}$$

= HF/short wave



Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

$$\mathbf{E} = (E_x, 0, E_z)$$

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B} + \mathbf{E}) \implies$$

$$\begin{cases} m \frac{dv_x}{dt} = qv_y B + qE_x \\ m \frac{dv_y}{dt} = -qv_x B \\ m \frac{dv_z}{dt} = qE_z \end{cases} \quad \text{Constant acceleration along } z$$



$$\begin{cases} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$



Drift motion

$$\begin{cases} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$

∴

$$\begin{cases} \frac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \frac{d^2 \left(v_y + \frac{E_x}{B} \right)}{dt^2} = -\omega_g^2 \left(v_y + \frac{E_x}{B} \right) \end{cases}$$



$$\begin{cases} v_x = v_{\perp} e^{i\omega_g t + \delta_x} \\ v_y = -\frac{E_x}{B} + v_{\perp} e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

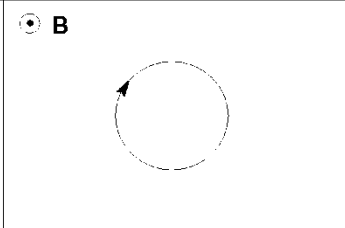
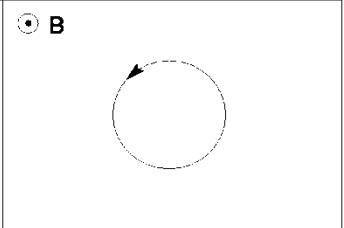
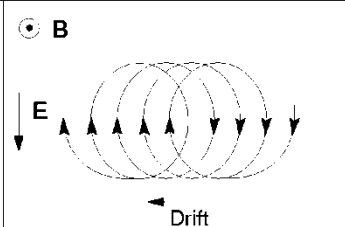
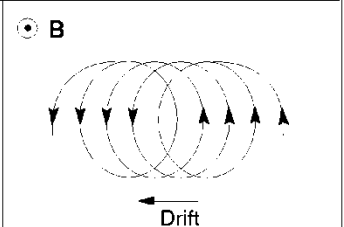
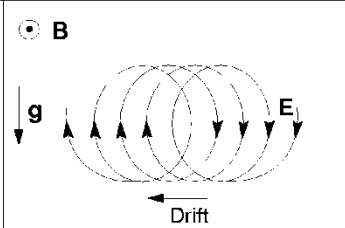
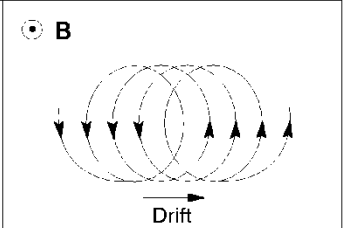
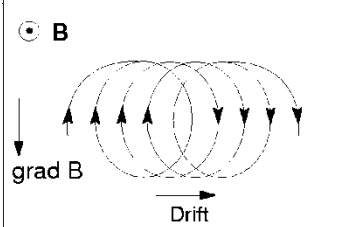
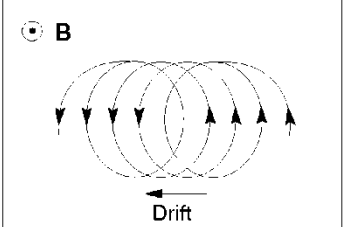
$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E} \times \mathbf{B})_y}{B^2}$$

In general:

$$\mathbf{v}_{drift} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{q\mathbf{E} \times \mathbf{B}}{qB^2} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

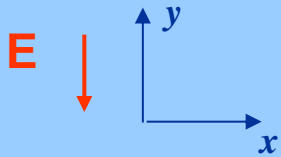
Drift motion

$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = m\mathbf{g}$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{ grad } B$		



Suppose you apply an electric field \mathbf{E} in the direction showed in the figure, and that one electron and one ion (charge $-e$ and e) is present. What will the resulting current be?



$$\mathbf{I} \equiv e\mathbf{u}_i - e\mathbf{u}_e$$

Yellow

$$\mathbf{I} = -e \frac{E}{B} \hat{\mathbf{x}}$$

Blue

$$\mathbf{I} = 0$$

Red

$$\mathbf{I} = \frac{1}{2} e \frac{E}{B} \hat{\mathbf{x}} - \frac{1}{2} e \frac{E}{B} \hat{\mathbf{y}}$$

Green

$$\mathbf{I} = e \frac{E}{B} \hat{\mathbf{y}}$$

	$\mathbf{u} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$	
	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = m\mathbf{g}$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{grad } B$		

$$\mathbf{I} \equiv e\mathbf{u}_i - e\mathbf{u}_e$$

$$\mathbf{u}_i = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{e\mathbf{E} \times \mathbf{B}}{eB^2} = -\hat{\mathbf{x}} \frac{EB}{B^2} = -\hat{\mathbf{x}} \frac{E}{B}$$

$$\mathbf{u}_e = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{-e\mathbf{E} \times \mathbf{B}}{-eB^2} = -\hat{\mathbf{x}} \frac{EB}{B^2} = -\hat{\mathbf{x}} \frac{E}{B}$$

$$\mathbf{I} \equiv e\mathbf{u}_i - e\mathbf{u}_e = e(\mathbf{u}_i - \mathbf{u}_e) = 0$$

Blue



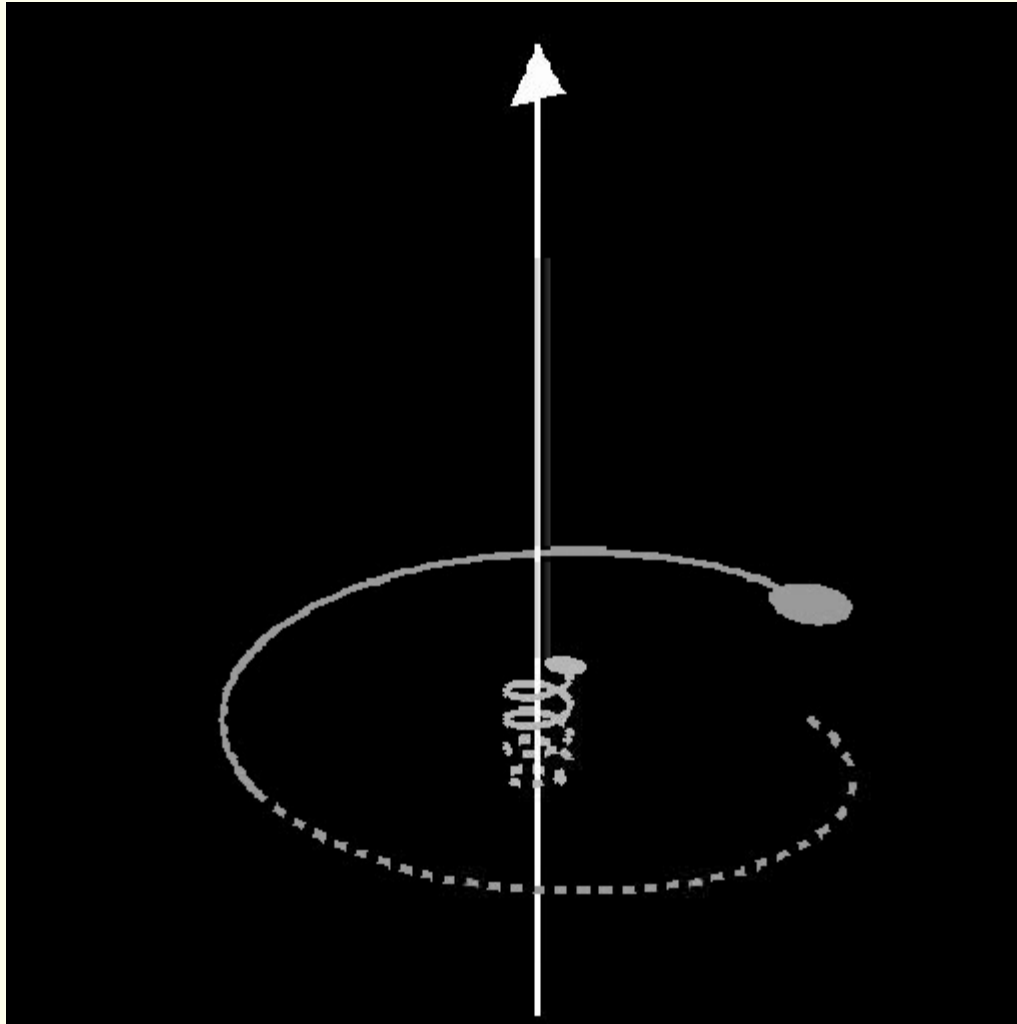
So, if there is no current when you apply an electric field, is the conductivity of the ionospheric plasma zero ?



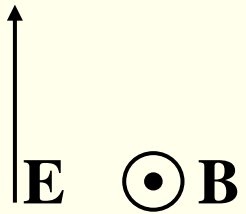
What is the electron
density at 100 km?

What is the neutral
density at 100 km?

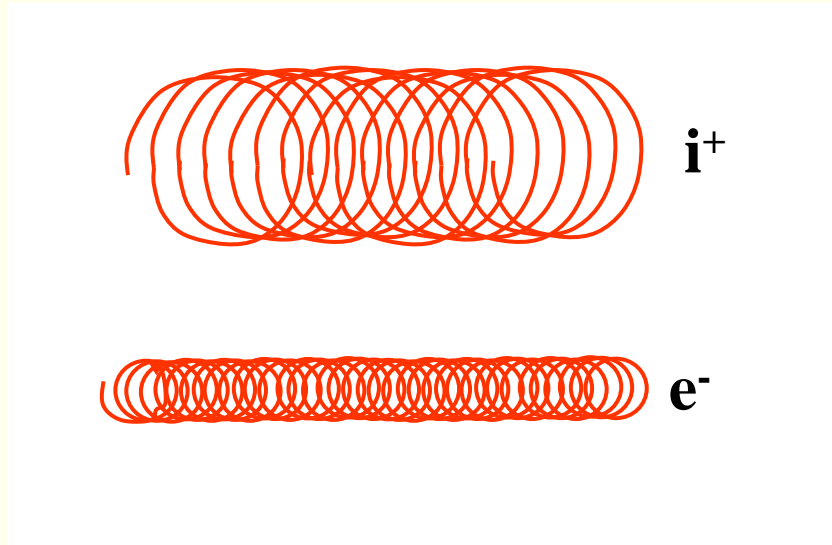
Gyro motion



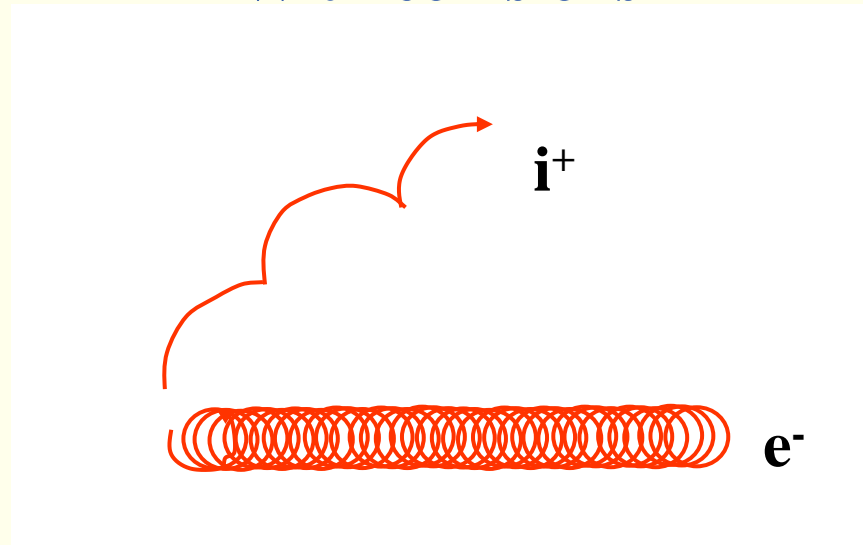
ExB-drift



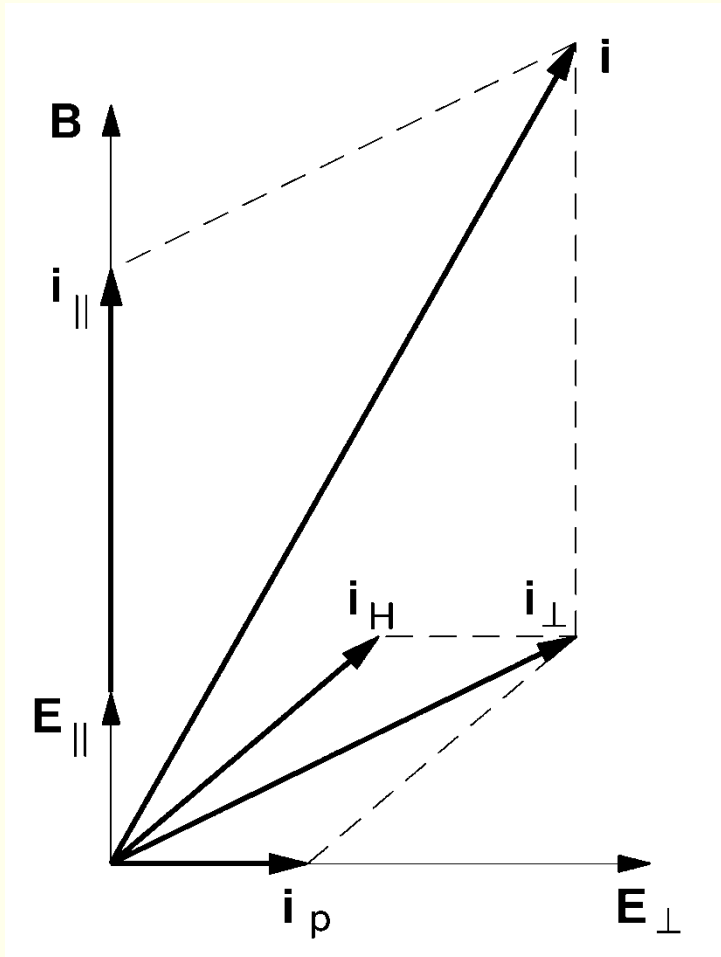
Without collisions



With collisions



Electric conductivity in a magnetized plasma



- $i_{||}$ = parallel current
- i_p = Pedersen current
- i_H = Hall current

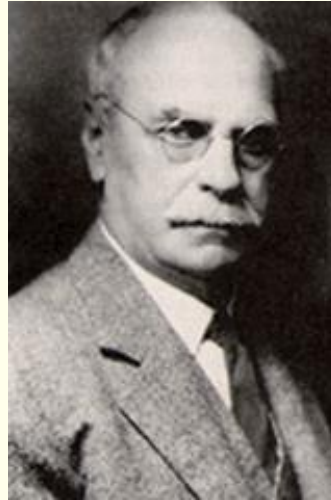
Birkeland, Hall, Pedersen



Kristian Birkeland

1867-1917

Norwegian
scientist



Edwin Hall

1855-1938

American
physicist



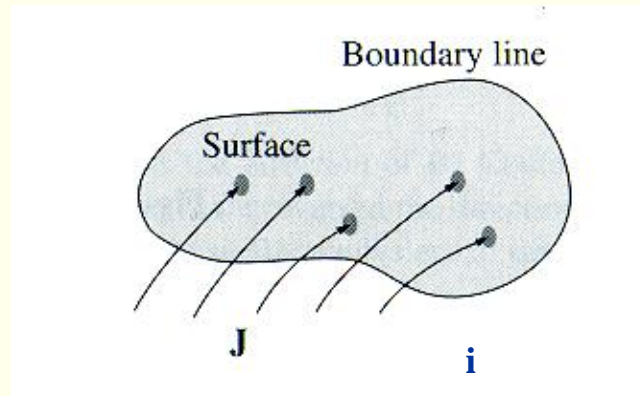
Peder Oluf Pedersen

1874-1941

Danish engineer
and physicist

Current density

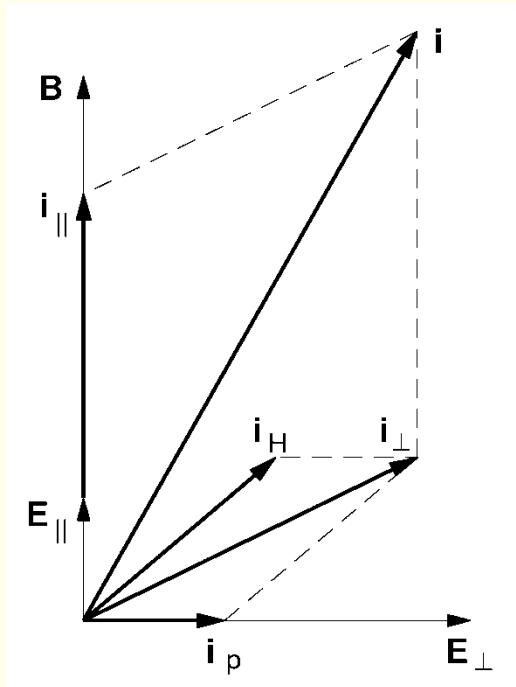
The current density \mathbf{j} is a vector field with dimension $[\mathbf{j}] = \text{Am}^{-2}$.



The total current I through the surface S is

$$I = \int_S \mathbf{j} \cdot d\mathbf{S}$$

Electric conductivity in a magnetized plasma II



$$\sigma_P = \sigma_e \frac{1}{1 + \omega_{ge}^2 \tau_e^2} + \sigma_i \frac{1}{1 + \omega_{gi}^2 \tau_i^2}$$

$$\sigma_H = \sigma_e \frac{\omega_{ge} \tau_e}{1 + \omega_{ge}^2 \tau_e^2} - \sigma_i \frac{\omega_{gi} \tau_i}{1 + \omega_{gi}^2 \tau_i^2}$$

$$\sigma_{//} = \sigma_e + \sigma_i$$

$$\sigma_e = e^2 n \tau_e / m_e$$

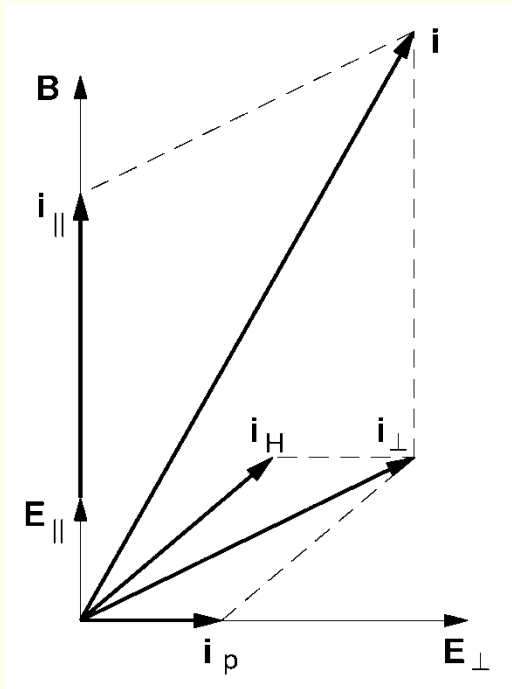
$$\sigma_i = e^2 n \tau_i / m_i$$

$$i_{//} = \sigma_{//} E_{//}$$

$$\left. \begin{aligned} i_P &= \sigma_P E_{\perp} \\ i_H &= \sigma_H E_{\perp} \end{aligned} \right\}$$

$$\text{or } \mathbf{i}_{\perp} = \sigma_P \mathbf{E}_{\perp} + \sigma_H \frac{\mathbf{B} \times \mathbf{E}_{\perp}}{B}$$

Electric conductivity in a magnetized plasma II



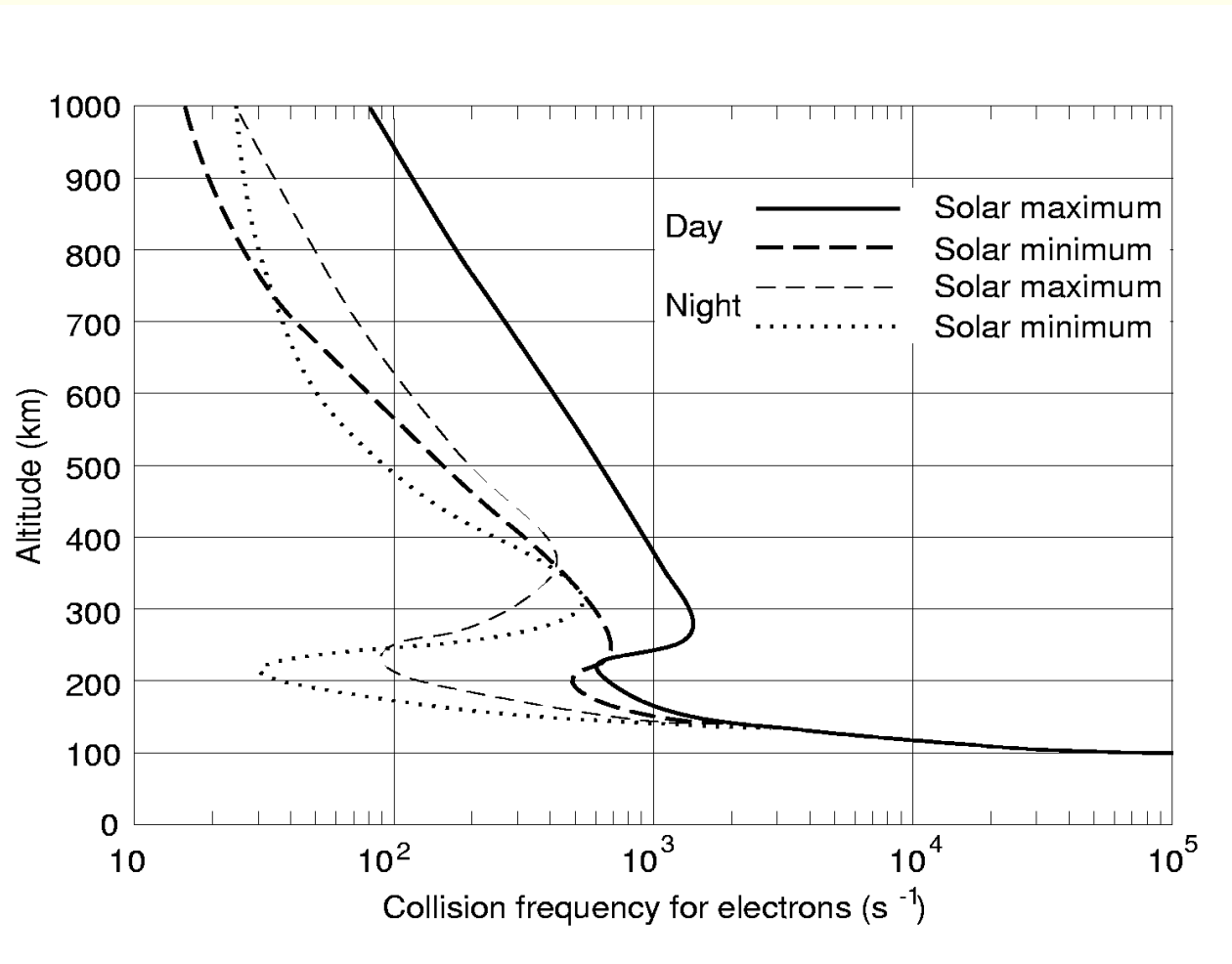
$$\mathbf{i} = \boldsymbol{\sigma} \cdot \mathbf{E}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix}$$

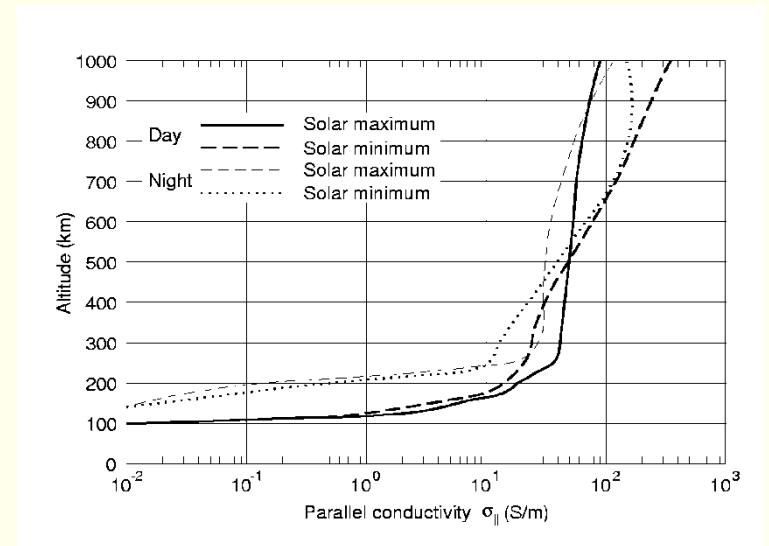
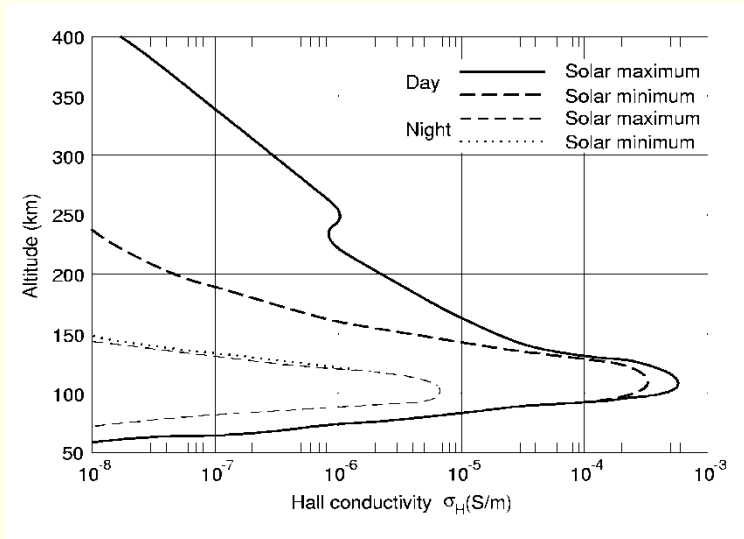
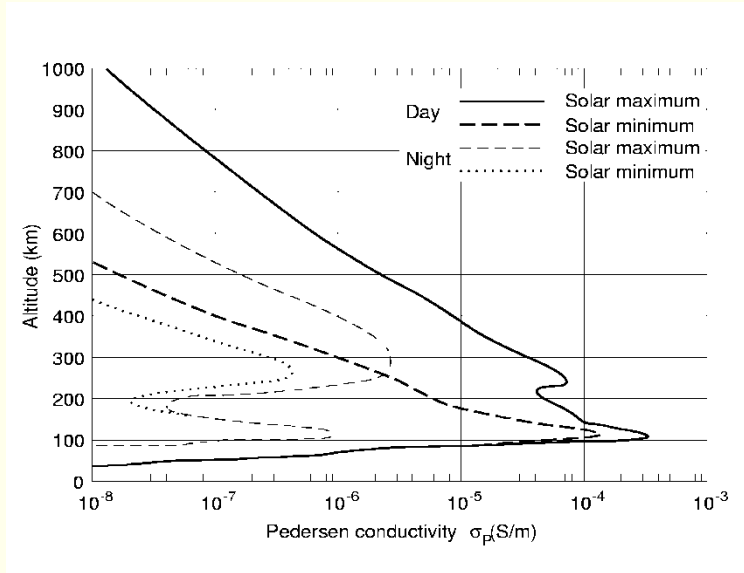
conductivity tensor

May be formulated as a tensor equation

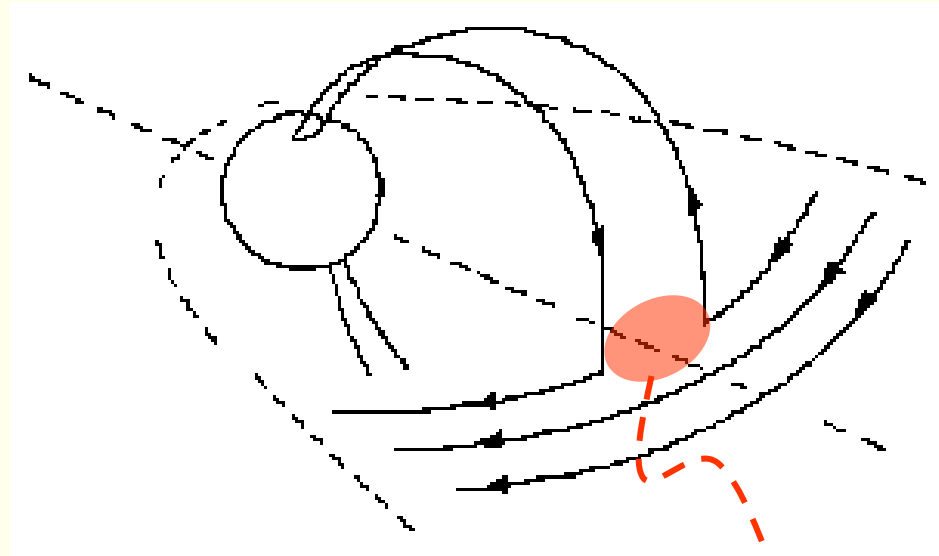
Collisional frequency



Ionospheric conductivities



Consequence: Birkeland currents



Region of low conductivity

When the conductivity out in the magnetosphere is low, it is easier for the current to close through the ionosphere via currents parallel to the geomagnetic field. Such currents are called *Birkeland* currents.

Exemple: Electric field **700 km** above the aurora.

$$\mathbf{E} = E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}}$$

$$E_x = 1 \text{ Vm}^{-1}$$

$$E_z = 1 \text{ } \mu\text{Vm}^{-1}$$

$$\left. \begin{aligned} j_P = j_x &= 0.01 \text{ } \mu\text{Am}^{-2} \\ j_{//} = j_z &= 40 \text{ } \mu\text{Am}^{-2} \end{aligned} \right\}$$

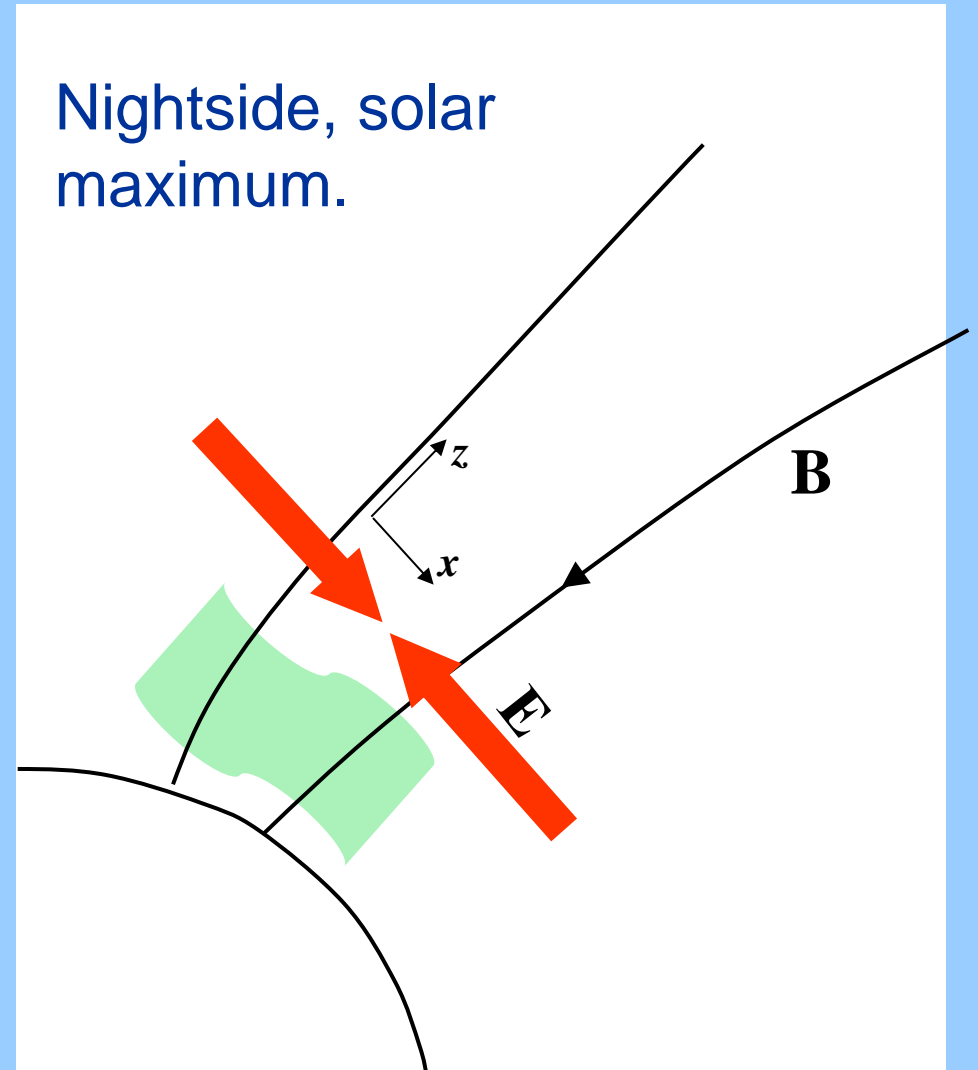
Yellow

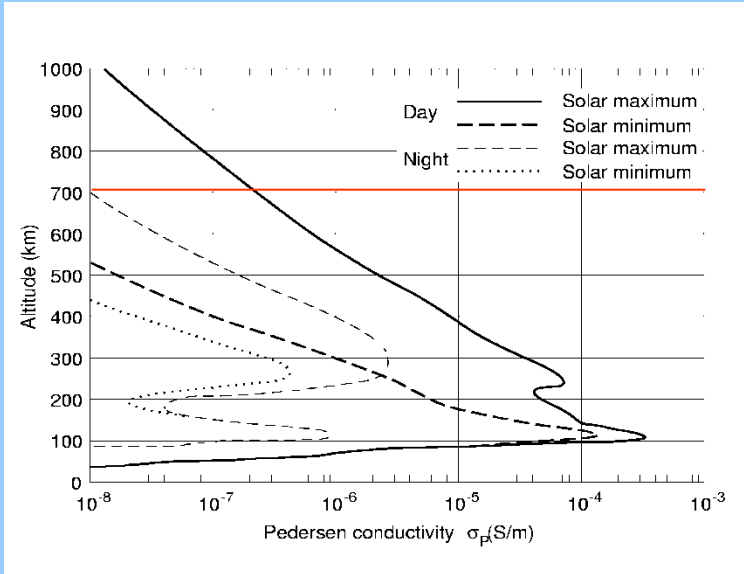
$$\left. \begin{aligned} j_P = j_x &= 10.0 \text{ } \mu\text{Am}^{-2} \\ j_{//} = j_z &= 4.0 \text{ } \mu\text{Am}^{-2} \end{aligned} \right\}$$

Red

$$\left. \begin{aligned} j_P = j_x &= 1.0 \text{ } \mu\text{Am}^{-2} \\ j_{//} = j_z &= 40 \text{ mA}\text{m}^{-2} \end{aligned} \right\}$$

Blue



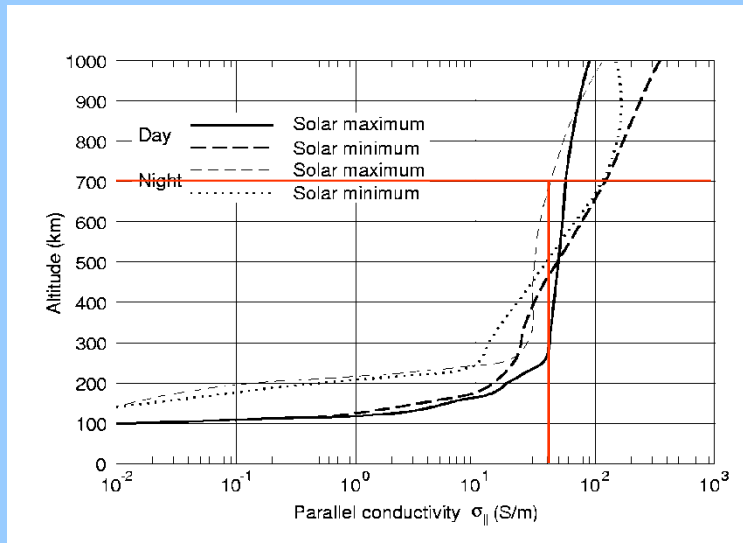


$$\sigma_P \approx 1 \cdot 10^{-8} \text{ Sm}^{-1}$$

$$\sigma_{//} \approx 40 \text{ Sm}^{-1}$$

$$j_P = j_x = \sigma_P E_x = 1 \cdot 10^{-8} \text{ Am}^{-2} = 0.01 \mu\text{Am}^{-2}$$

$$j_{//} = j_z = \sigma_{//} E_z = 40 \cdot 10^{-6} \text{ Am}^{-2} = 40 \mu\text{Am}^{-2}$$



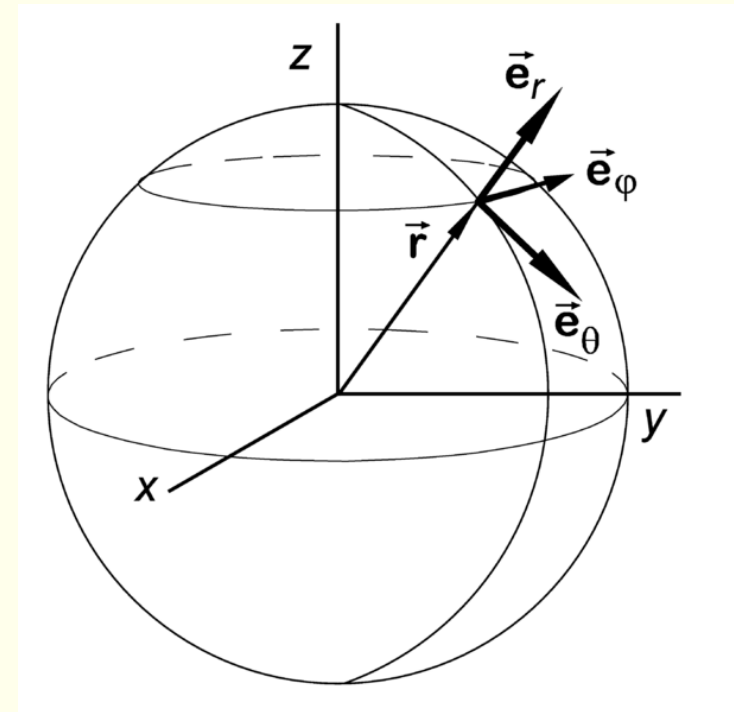
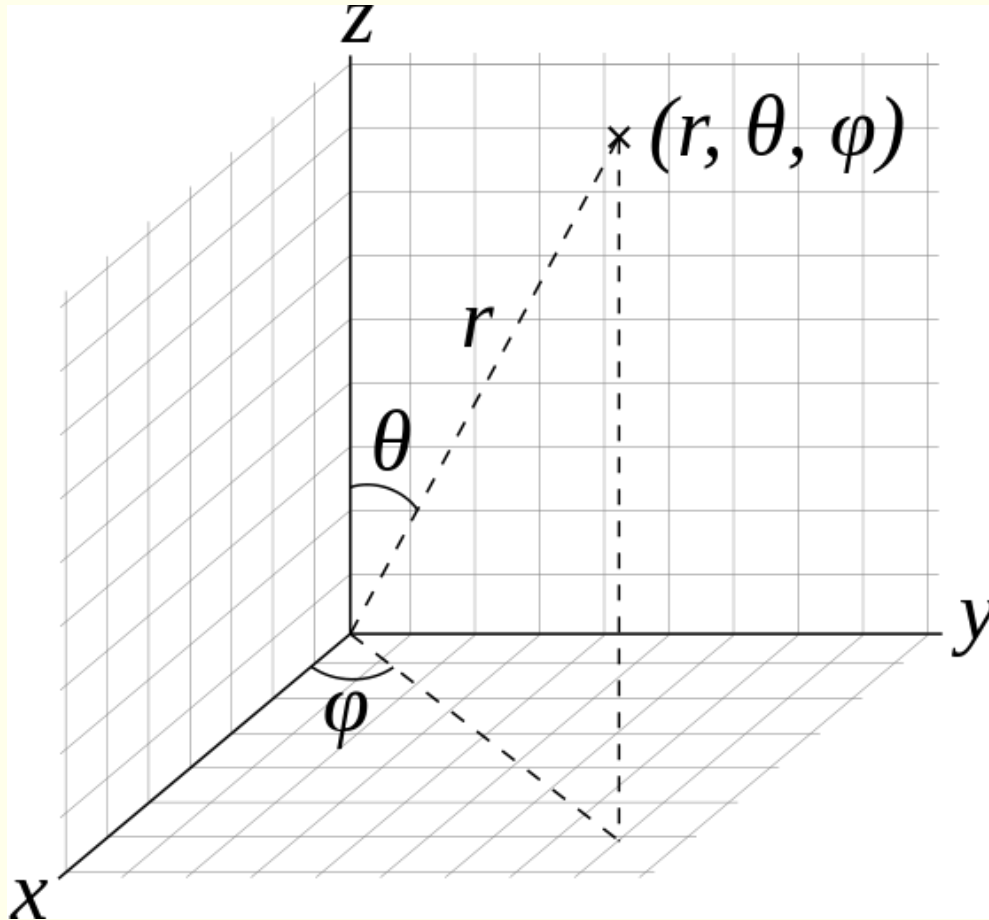
Yellow



How do we define "the magnetosphere"?

The region in space where the magnetic field is dominated by the geomagnetic field.

Polar (spherical) coordinates



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos\left(\frac{z}{r}\right)$$

$$\varphi = \arctan\left(\frac{y}{x}\right)$$

$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

Geomagnetic field

Approximated by a dipole close to Earth.

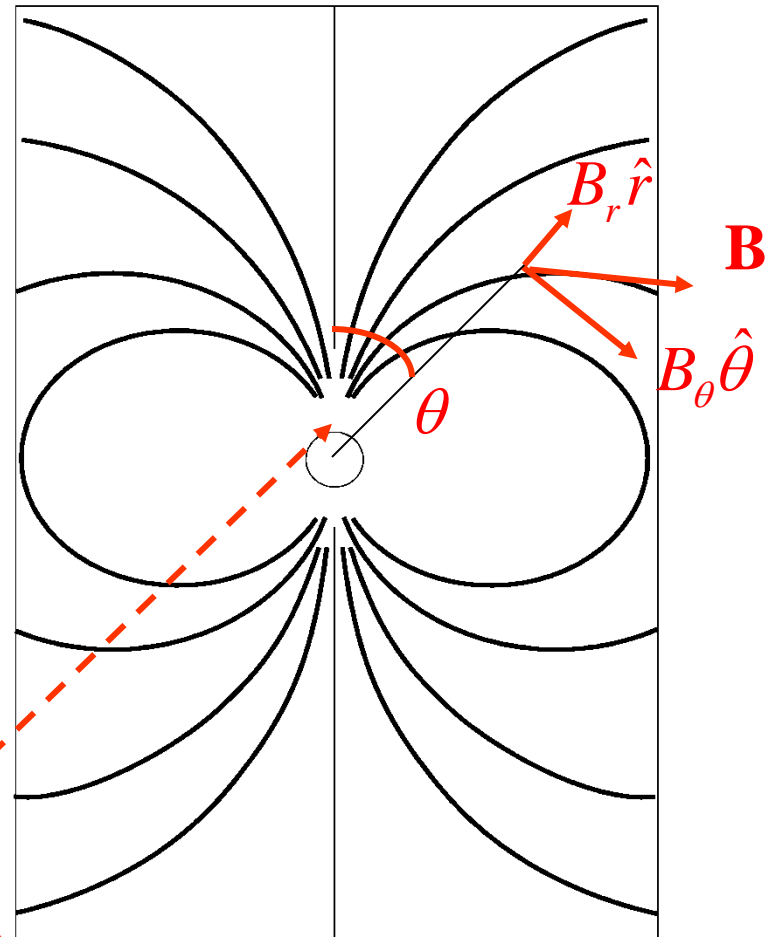
$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin \theta$$

$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Magnetic field at the "north pole"



Geomagnetic field

Alternative formulation of dipole field

$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin \theta$$

$$B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos \theta$$

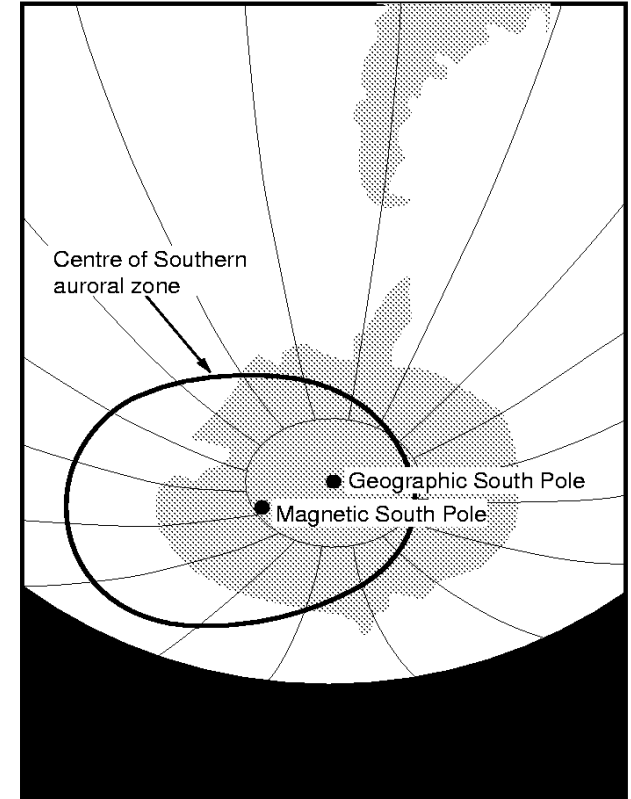
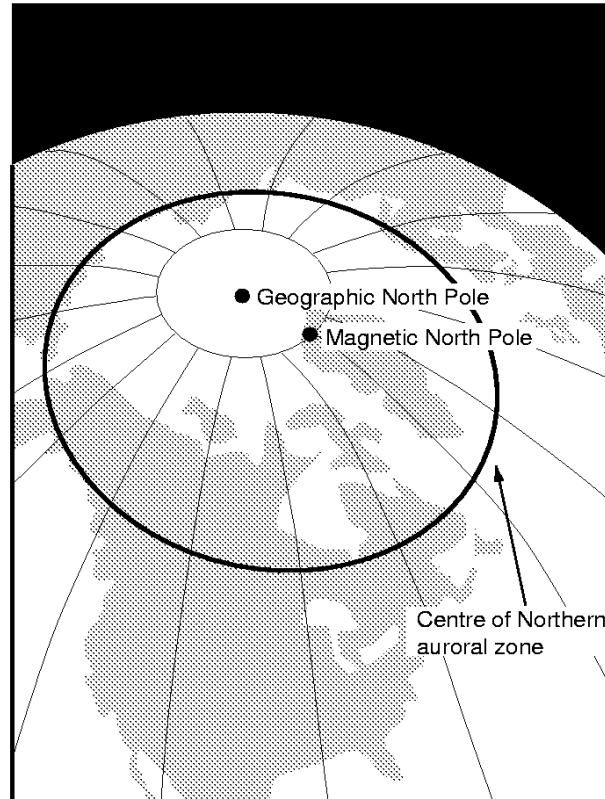
$$B_\theta = \frac{\mu_0 a}{2\pi} \cdot \frac{1}{2} \cdot \frac{1}{r^3} \sin \theta$$

$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

 magnetic dipole moment

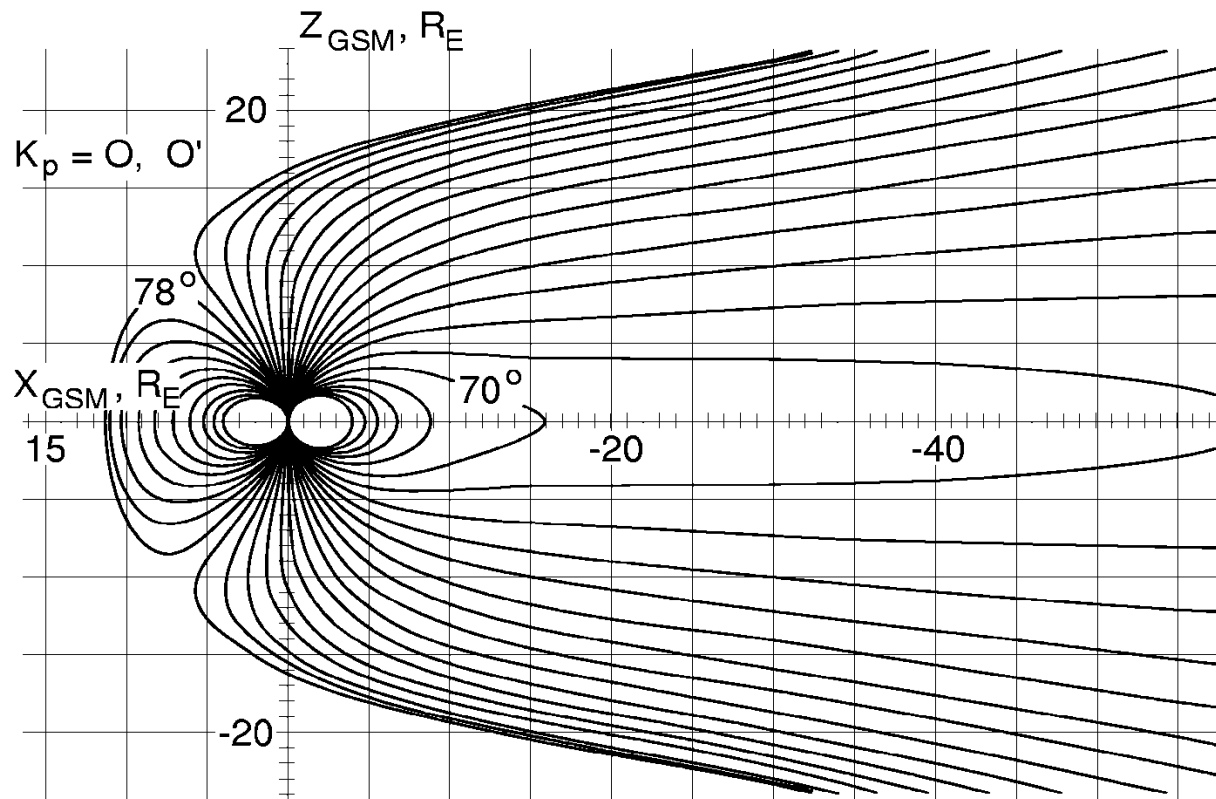
Geomagnetic field

- Angle between dipole axis and spin axis: $\approx 11^\circ$
- The geographic north pole is a magnetic south pole, and vice versa.
- $B_{equator} = 31 \mu\text{T}$,
 $B_{pole} = 62 \mu\text{T}$

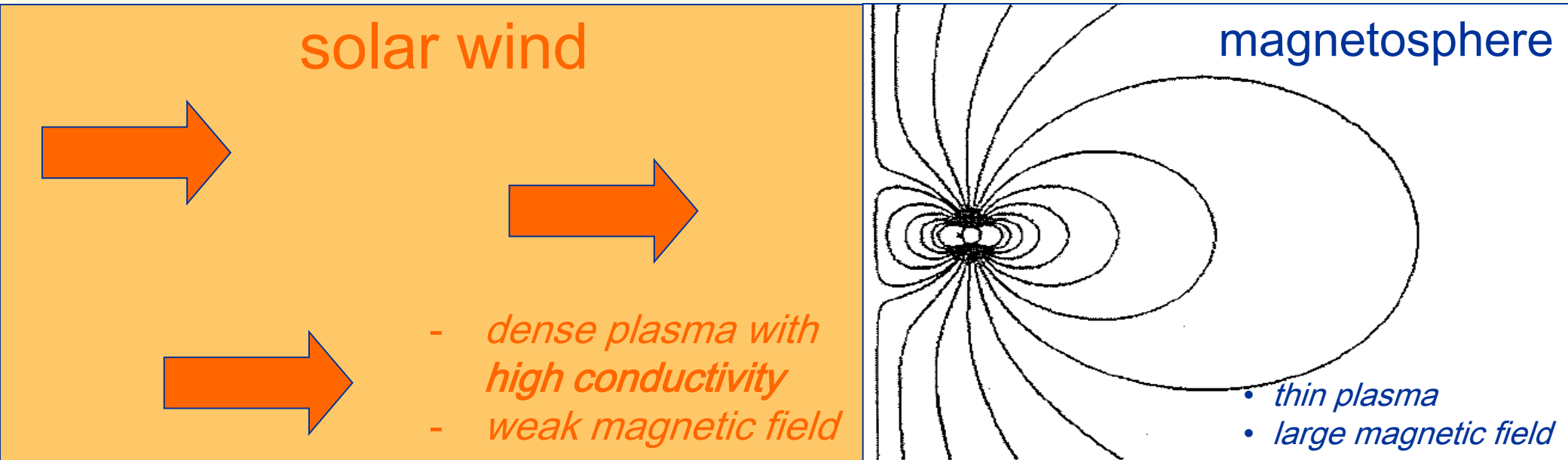


Geomagnetic field

Modified by solar wind into tail-like configuration



Stand-off distance from pressure balance



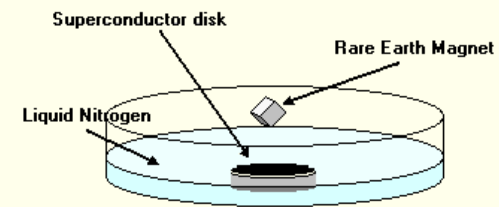
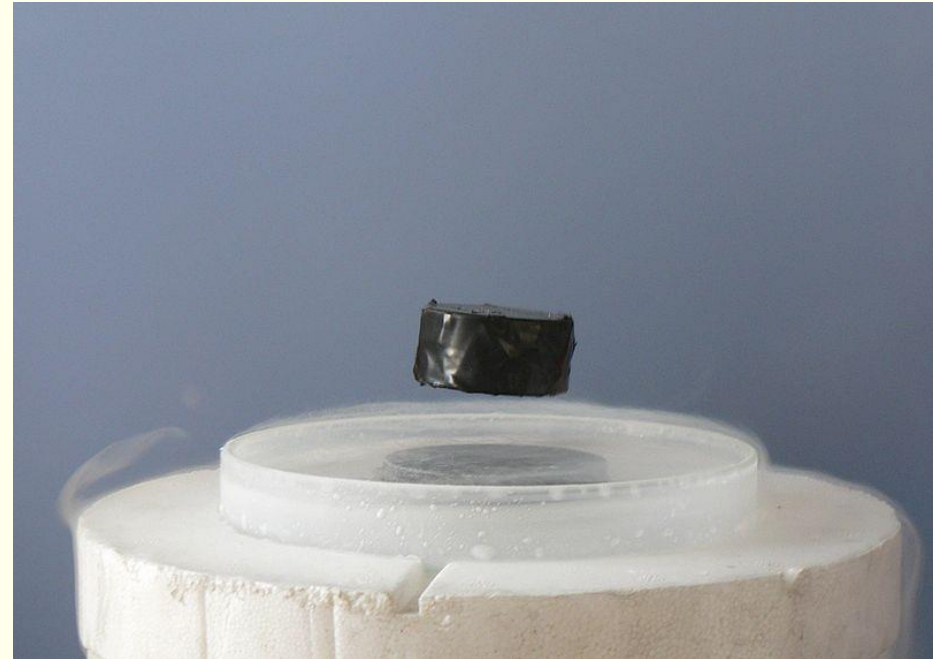
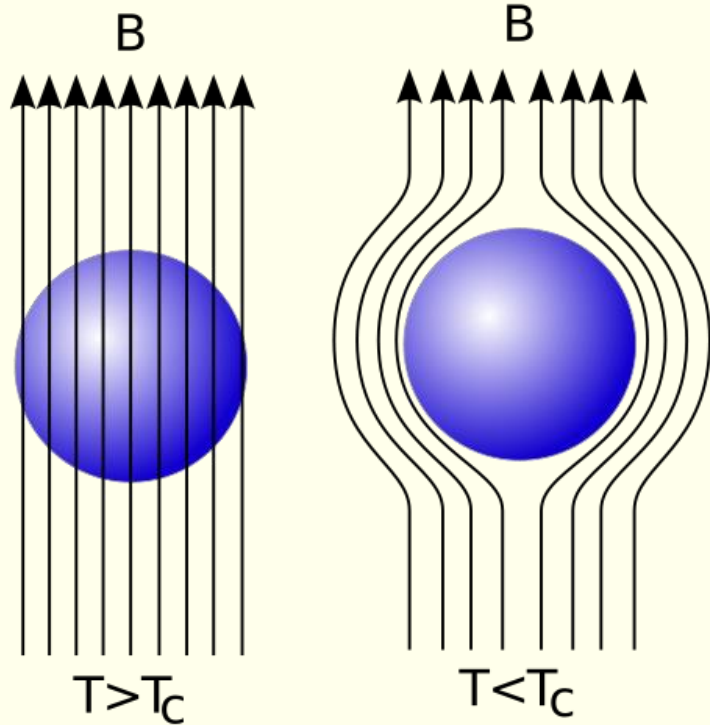
Dynamic pressure:

$$p_d = \rho_{SW} v_{SW}^2$$

Magnetic pressure:

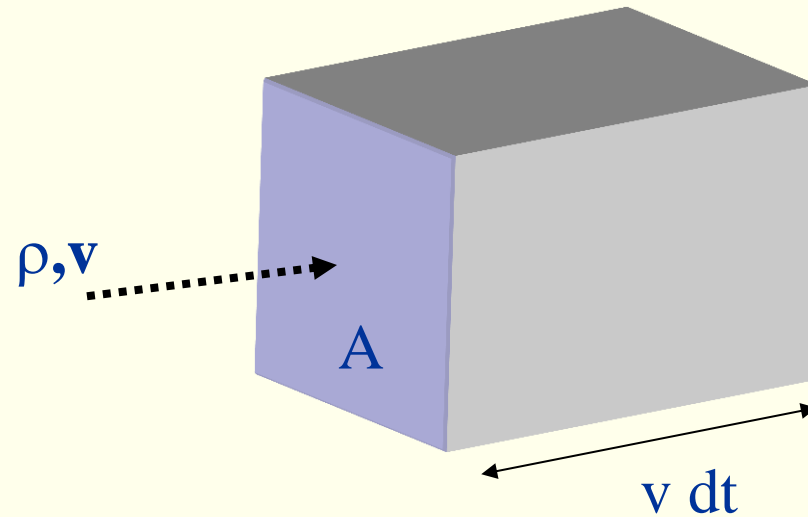
$$p_B = \frac{B^2}{2\mu_0}$$

Meissner effect in super-conductors



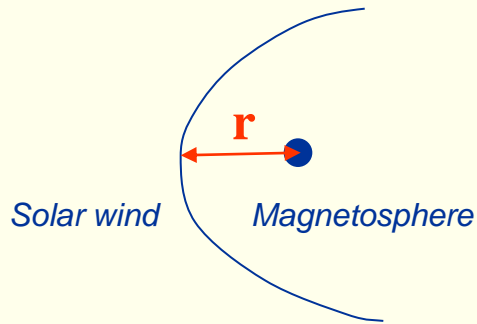
The Meissner Effect

Dynamic (kinetic) pressure



$$p_d = \frac{F}{A} = \frac{d(mv)}{dt} \frac{1}{A} \approx \frac{\Delta(mv)}{\Delta t} \frac{1}{A} = \frac{\rho \cdot Av \Delta t \cdot v}{\Delta t A} = \rho v^2$$

Magnetopause “stand-off distance”



Dynamic pressure: $p_d = \rho_{SW} v_{SW}^2$

Magnetic pressure: $p_B = \frac{1}{2\mu_0} B^2$

Dipole field strength
(in equatorial plane): $B = \frac{\mu_0 a}{4\pi} \frac{1}{r^3}$

$$p_d = p_B \Rightarrow \rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 \Rightarrow$$

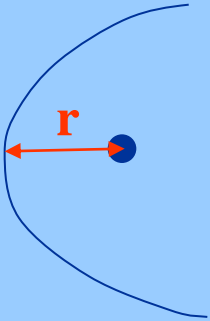
$$r = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

$a = 8 \times 10^{22} \text{ Am}^2$, $v = 500 \text{ km/s}$, $\rho_{SW} = 10^7 \times 1.7 \times 10^{-27} \text{ kg/m}^3$:

$r = 7 R_e$ (1 $R_e = 6378 \text{ km}$)

Standoff distance

$v=500$ km/s, $\rho_{SW}=10^7 \times 1.7 \times 10^{-27}$ kg/m³: $r = 7 R_e$



$$r = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

How will the standoff distance change if the magnetosphere is hit by a coronal mass ejection (CME)? ($\rho = 10\rho_{SW}$, $v = 1000$ km/s)

Blue

$r = 1.8 R_e$

Yellow

$r = 5.8 R_e$

Green

$r = 3.8 R_e$

Red

$r = 9.8 R_e$

Standoff distance

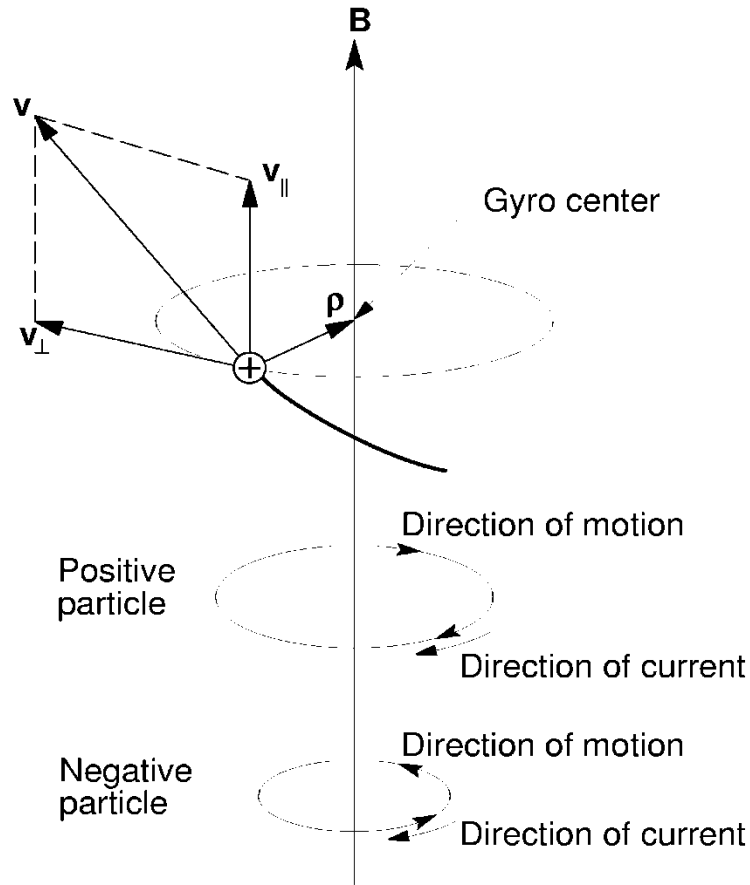
$$r = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \mathbf{10} \rho_{SW} (\mathbf{2}v)_{SW}^2 \right)^{-1/6} = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6} \mathbf{40}^{-1/6}$$

$$40^{-1/6} \cdot 7 = 0.54 \cdot 7 = 3.8$$

Green

$$r = 3.8 R_e$$

Particle motion in magnetic field



gyro radius

$$\rho = \frac{mv_{\perp}}{qB}$$

gyro frequency

$$\omega_g = \frac{qB}{m}$$

magnetic moment

$$\mu = IA = q f_g \pi \rho^2 = mv_{\perp}^2 / 2B$$



Adiabatic invariant

DEFINITION:

An **adiabatic invariant** is a property of a physical system which stays constant when changes are made slowly.

By 'slowly' in the context of charged particle motion in magnetic fields, we mean much slower than the gyroperiod.

'First adiabatic invariant' of particle drift:

$$\mu = \frac{mv_{\perp}^2}{2B}$$



Last Minute!



Last Minute!

- What was the most important thing of today's lecture? Why?
- What was the most unclear or difficult thing of today's lecture, and why?
- Other comments