Last lecture (1)

- Course info
- Definition of plasma
- Solar interior and atmosphere
- Plasma physics 1

Today's lecture (2)

- Plasma physics 2
- Solar activity

Examination

1. Written examination (open book*), 30/10

100 p

2. Continous examination (mini-group works)

25 p

Grades:

A:	111-125 p
B:	96-110 p
C:	81-95 p
D:	66-80 p
E:	50-65 p

(Fx)

Written examination,
30/10 2013, 14.00-19.00, B21, B22, B23, B24

(*) You may bring:

- all the course material
- any notes you have made
- pocket calculator
- mathematics and physics formula books or your favourite physics book
- formula sheet

(No computers are allowed, due to the possibility to communicate with the outside world.)

Approx. 5 different problems (which may contain sub-problems).

The character of the problems is such that to get a high score you will have to show that you have obtained a certain course goal, e.g. to make a reasonable order of magnitude estimate or figure out a simple model for some space physics phenomenon.

Continous examination
Mini-group works

5 mini-group works
(5×5 p = 25 p)

Approx. 1 h during Tutorials 1-5

- *A problem similar to those on the written examination is given*
- *Groups of 3 (randomized).*
- *Elect a secretary!*
- *Write down a solution!*

Litterature

- C-G. Fälthammar, ”Space Physics” (compendium), 2nd Ed, Third Printing, 2001.
- Larry Lyons, ”Space Plasma Physics”, from *Encyclopedia of Physical Science and Technology, 3rd edition, 2002.*
- Lecture notes and extra material handed out during lectures.

Course home page

KTH Social:

https://www.kth.se/social/course/EF2240/

At the home page I will post new information continuously. Here you can also find lecture notes, exercises (and some solutions), etc.

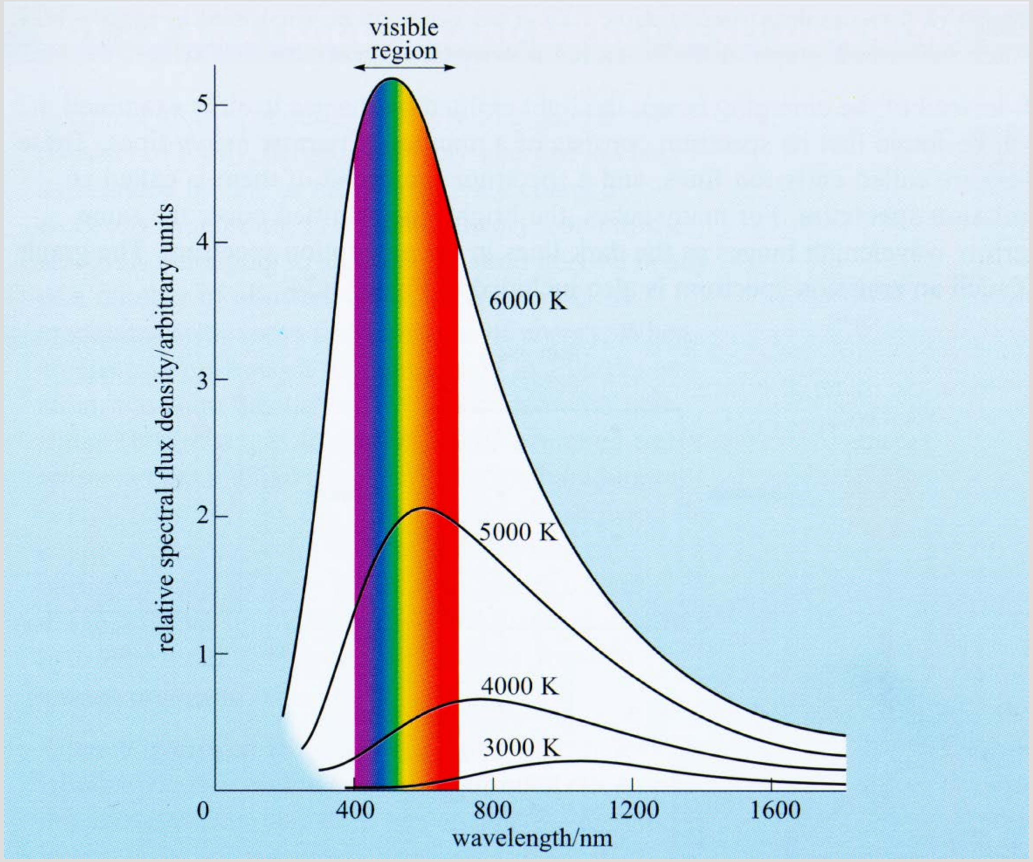
Preliminary guest lecturer

Swedish astronaut Christer Fuglesang
Lecture 10

Today

Activity	Date	Time	Room	Subject	Litterature
L1	2/9	10-12	Q33	Course description, Introduction, The Sun 1, Plasma physics 1	**CGF** Ch 1, 5, (p 110-113)
L2	3/9	15-17	Q31	The Sun 2, Plasma physics 2	**CGF** Ch 5 (p 114-121), 6.3
L3	9/9	10-12	Q33	Solar wind, The ionosphere and atmosphere 1, Plasma physics 3	**CGF** Ch 6.1, 2.1-2.6, 3.1-3.2, 3.5, **LL** Ch III, Extra material
T1	11/9	10-12	Q34	Mini-group work 1	
L4	16/9	15-17	Q33	The ionosphere 2, Plasma physics 4	**CGF** Ch 3.4, 3.7, 3.8
L5	18/9	15-17	Q21	The Earth's magnetosphere 1, Plasma physics 5	**CGF** 4.1-4.3, **LL** Ch I, II, IV.A
T2	23/9	10-12	Q34	Mini-group work 2	
L6	25/9	10-12	M33	The Earth's magnetosphere 2, Other magnetospheres	**CGF** Ch 4.6-4.9, **LL** Ch V.
L7	30/9	14-16	L51	Aurora, Measurement methods in space plasmas and data analysis 1	**CGF** Ch 4.5, 10**, LL** Ch VI, Extra material
T3	3/10	10-12	V22	Mini-group work 3	
L8	7/10	10-12	V22	Space weather and geomagnetic storms	**CGF** Ch 4.4, **LL** Ch IV.B-C, VII.A-C
T4	9/10	15-17	Q31	Mini-group work 4	
L9	11/10	10-12	M33	Interstellar and intergalactic plasma, Cosmic radiation, Swedish and international space physics research.	**CGF** Ch 7-9
T5	15/10	10-12	L51	Mini-group work 5	
L10	16/10	13-15	Q36	Guest lecture: Swedish astronaut Christer Fuglesang	
T6	17/10	15-17	Q31	Round-up	
Written examination	30/10	14-19	B21-24		

Black-body radiation



Wien's displacement law

$$\lambda_{peak} = \frac{2.90\times10^{-3}}{T}$$

Stefan-Bolzmanns law

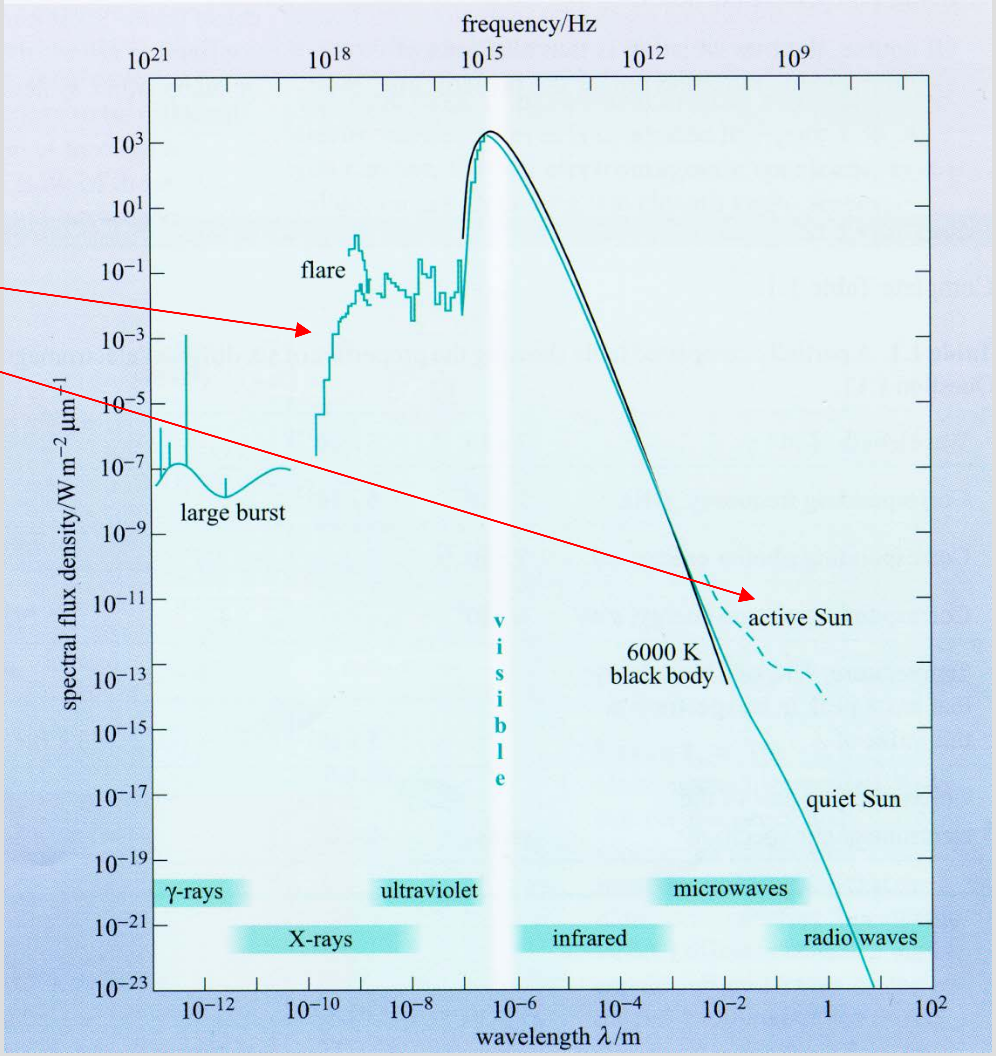
$$J = \sigma_{SB} T^4$$

(J = total energy radiated per unit area per unit time)

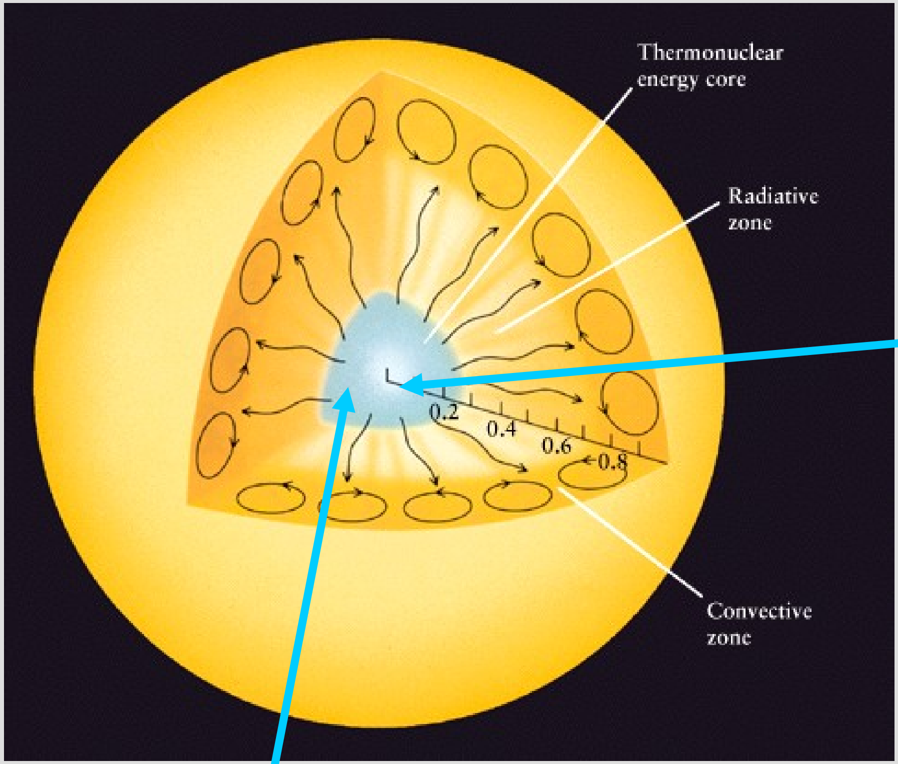
Black-body good approximation for opaque bodies where emitted light is much more likely to interact with the material of the source than to escape.

The solar spectrum

Non-blackbody contributions

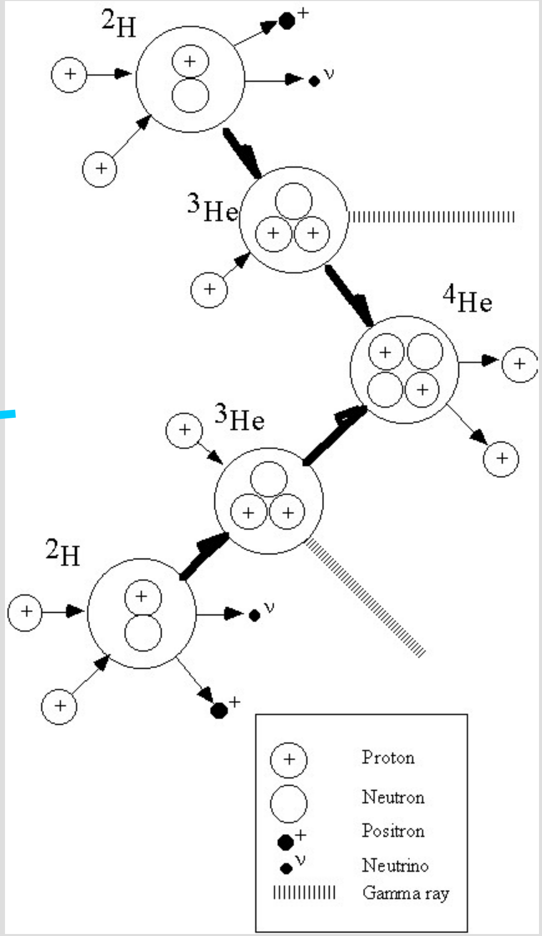


Sun´s interior



$$T = 15 \cdot 10^6 \text{ K}$$
$$P = 4 \cdot 10^{26} \text{ W}$$
$$(P/m \sim 1 mW/kg)$$

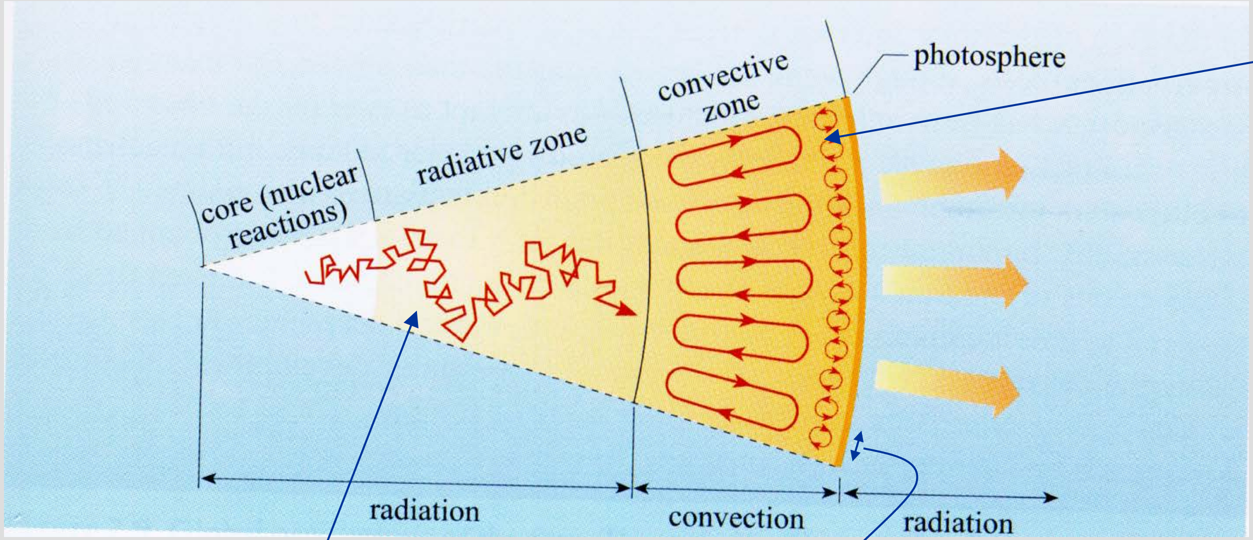
The proton cycle



$$4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2e^+ + 2\nu_e + 2\gamma$$

Energy transport in the sun

Transport by convection

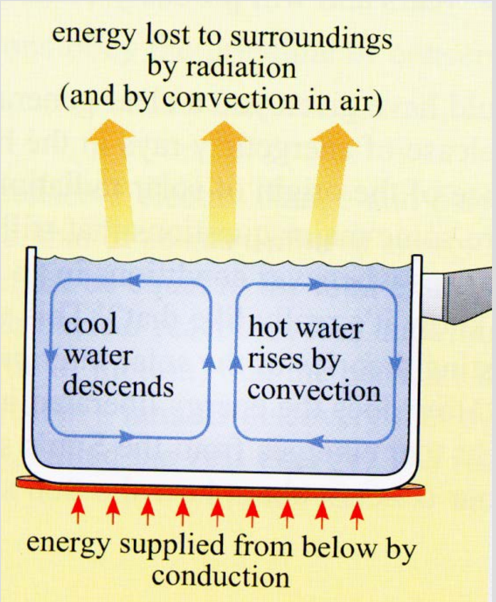


~1000 km

Transport by radiation, which interacts with the dense solar matter (scattering and absorption/re-emission).

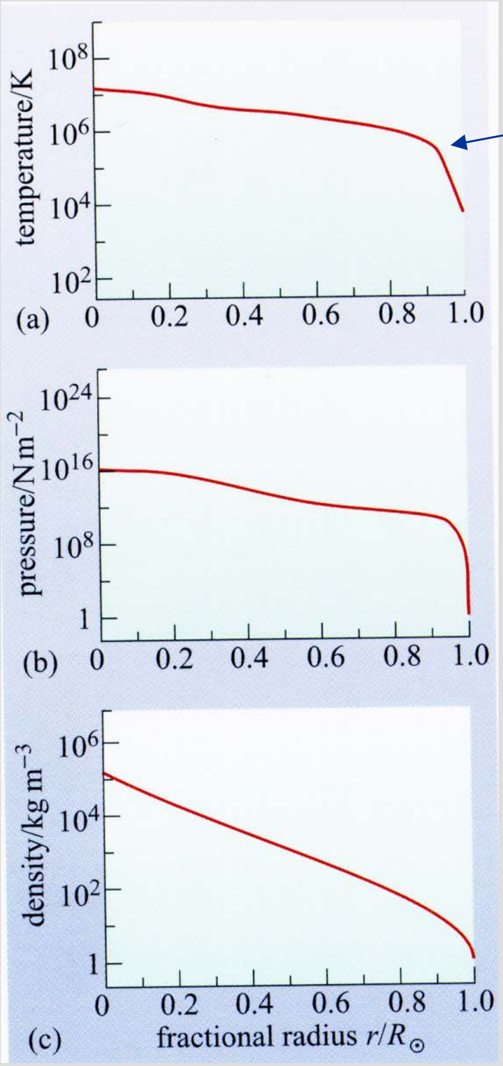
I takes on average 200 000 years for a photon to reach the photosphere!

These convection cells are called *granulation*.



At the photosphere the mean free path of the photons becomes so large that they can reach directly out into space.

Sun's interior



At the photosphere the mean free path of the photons becomes so large that they can reach directly out into space.

As a consequence also the temperature, and pressure drops.

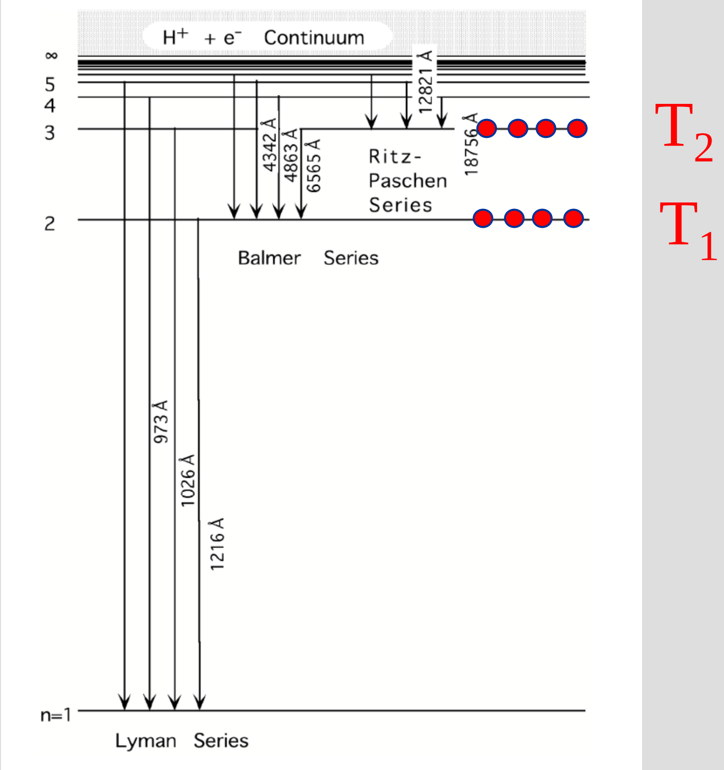
$$p_{pl} = nk_B T$$

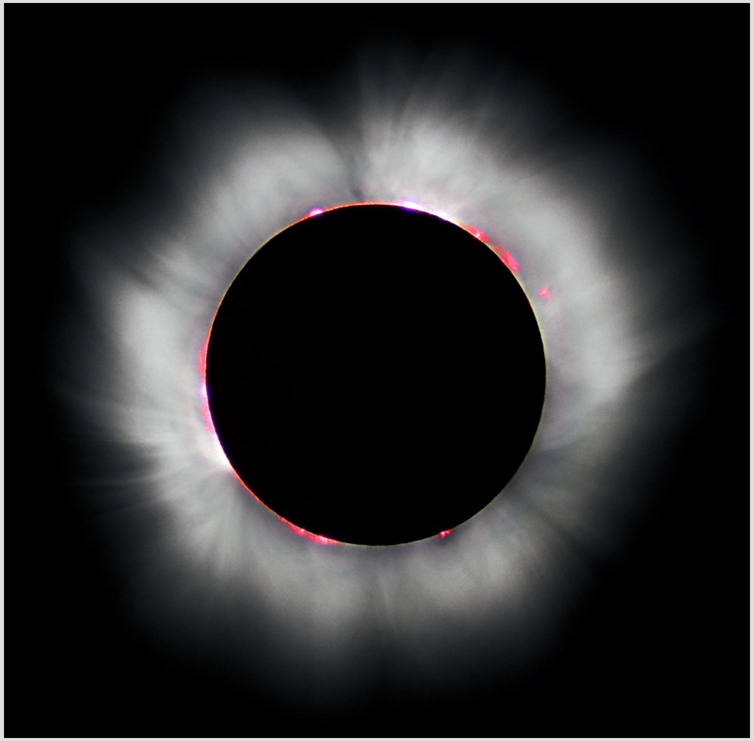
Example of exponential density variation in balance between pressure and gravity

$$\rho_m = const \cdot e^{-z/(k_B T/gm)} = const \cdot e^{-z/H}$$

Why is the chromosphere red?

Hydrogen spectrum

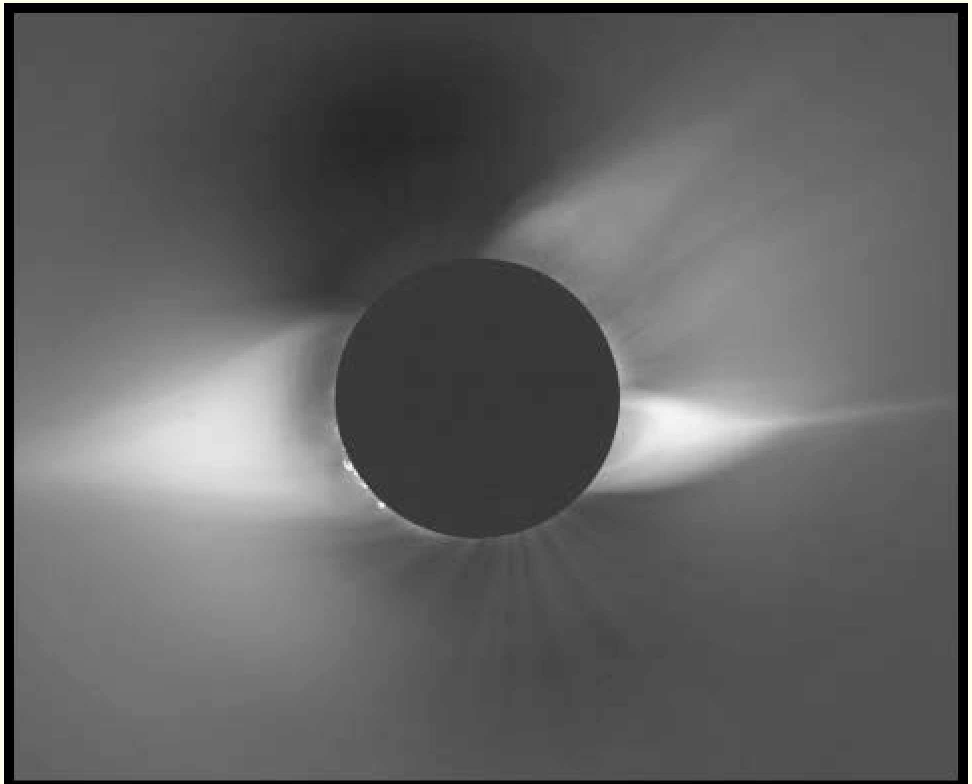




Hγ 434 nm Hβ 486 nm Hα 656 nm

Corona

- Temperature: up to 2 MK
- Density: 10^{-18} g/cm^3 – 10^{-24} g/cm^3
- Turns into the solar wind at high altitudes, without a sharp boundary.



Solar Corona at Eclipse, 3 Nov 1994, Putre, Chile.
High Altitude Observatory, NCAR, Boulder, Colorado, USA.

The layers of the solar atmosphere

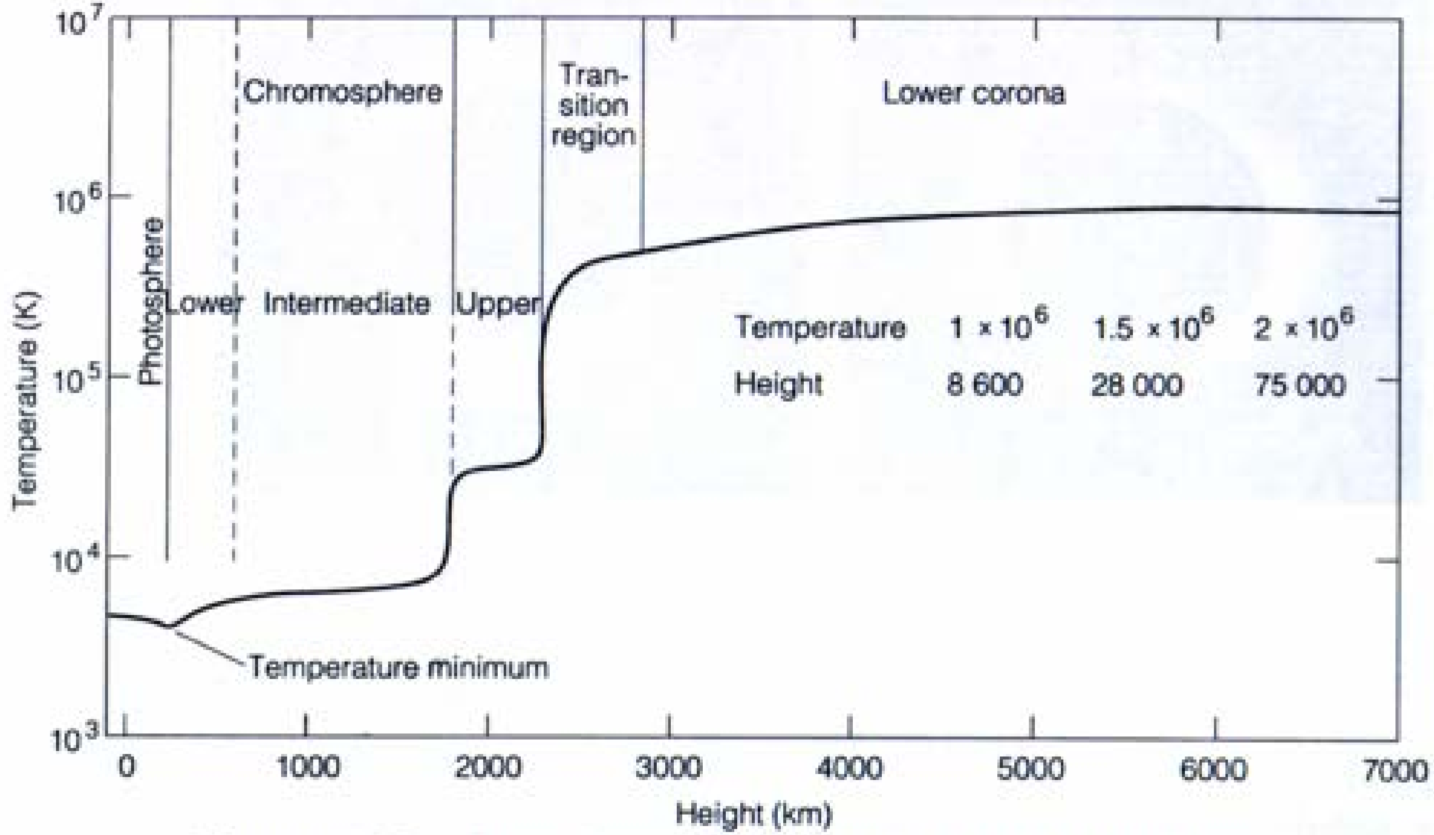
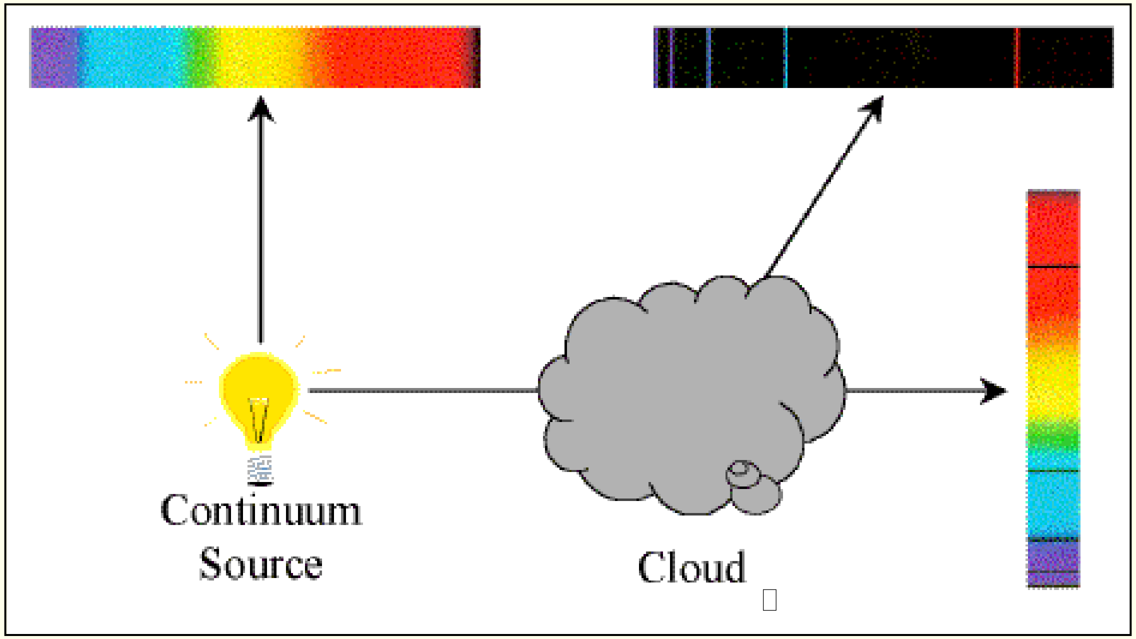


Figure 5.3. Distribution of average temperature in the solar atmosphere (Athay 1976).

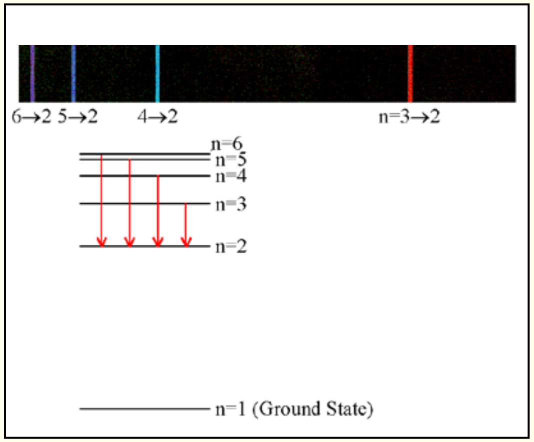


For non-blackbody thermal light emitter (for example a thin gas) it is more complicated. Spectrum depends e.g. chemical composition, and how many atoms/molecules happen to be in state with high probability to decay and cause emission.

Energy (and wavelength) of emitted quantum can still be approximated:

Black-body radiation

$$\lambda_{peak} = \frac{2.90\times10^{-3}}{T}$$

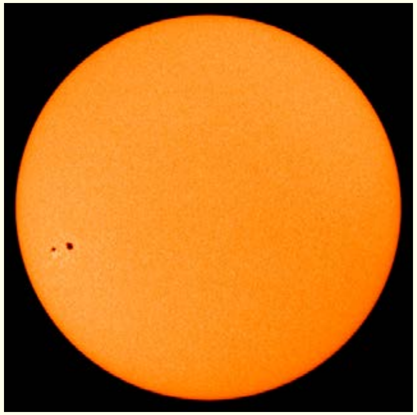


Atomic energy levels

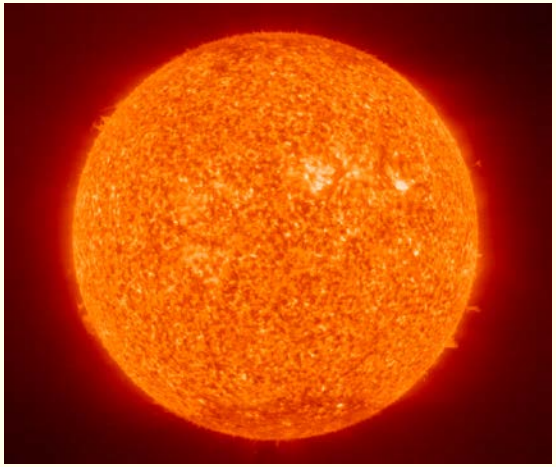
$$E \sim k_B T$$

$$E = hf$$

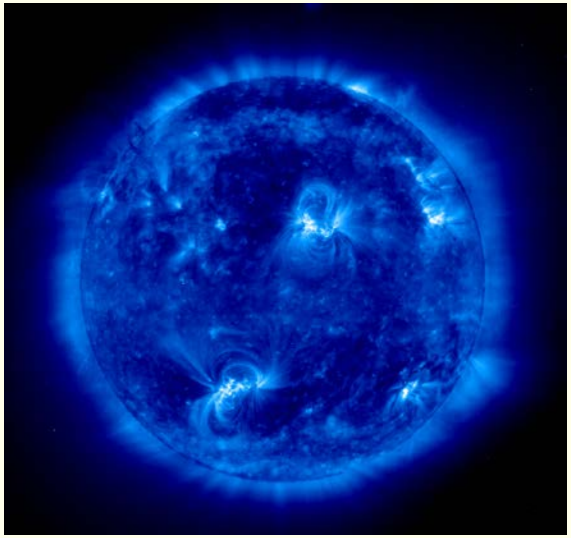
$$\lambda \sim \frac{hc}{k_B T}$$



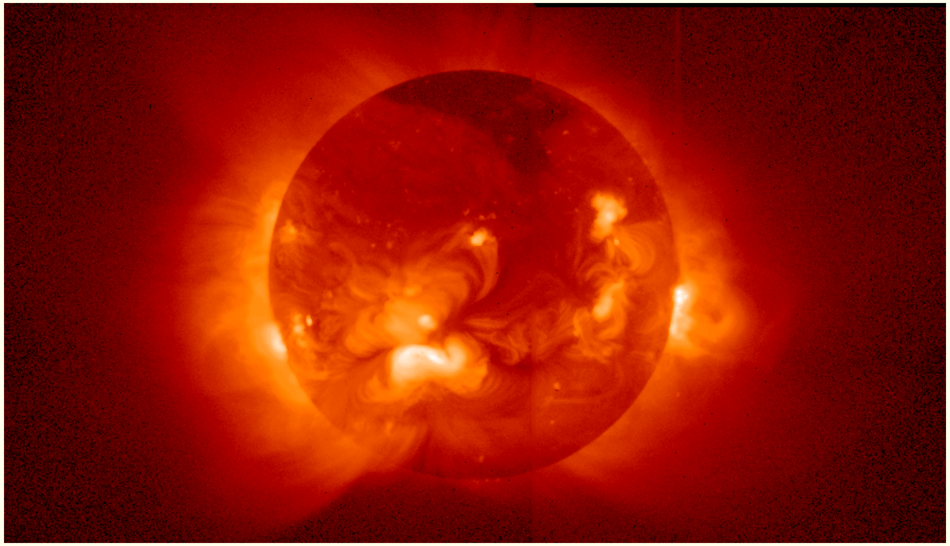
Visible light ~ 6768 Å



He II emission line at 304 Å

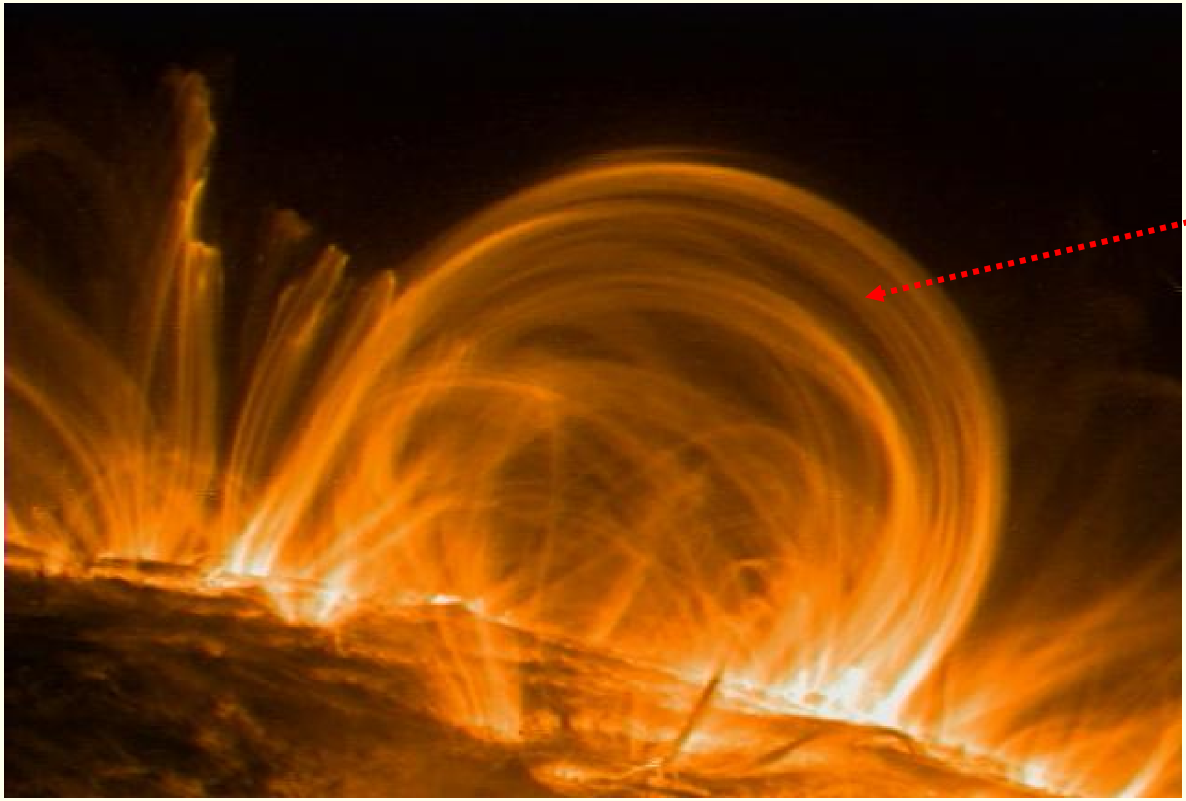


(Fe IX/X) at 171 Å



X-ray at 0.3-5 Å

Coronal loops



What gives the loops this structure???

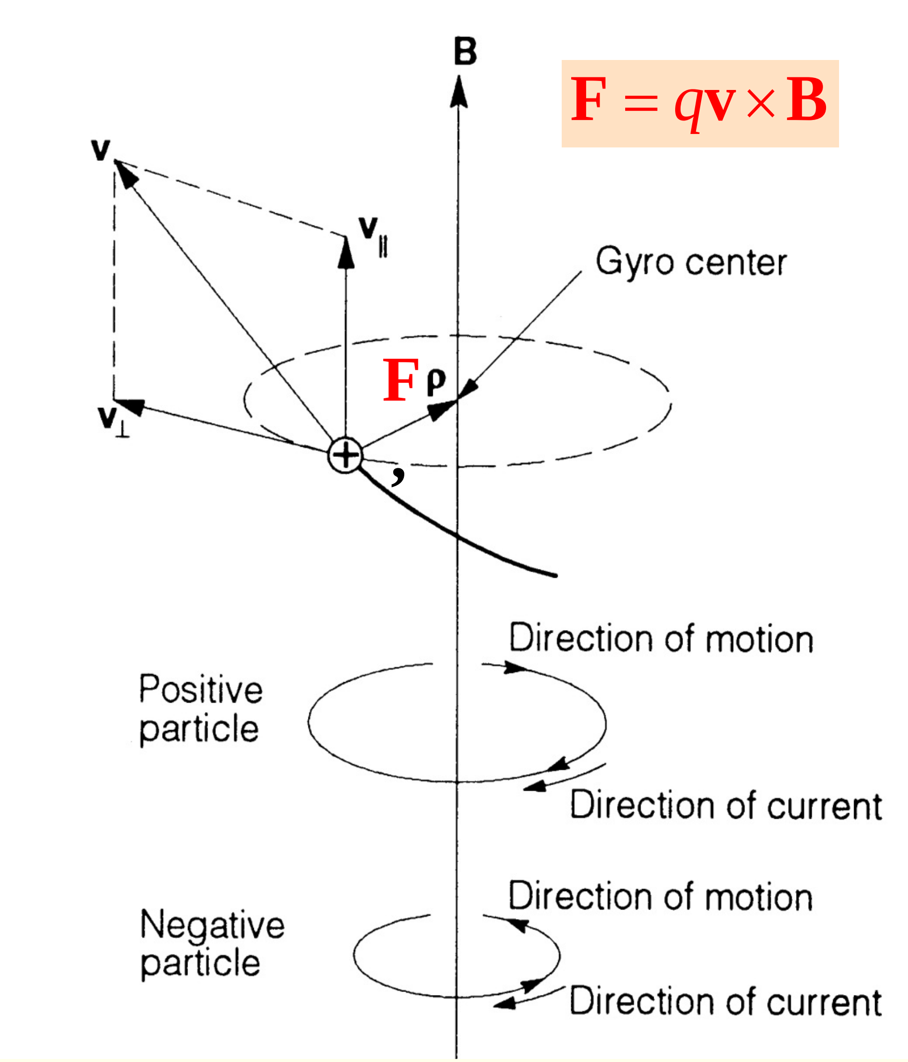
Coronal loops

Why does the plasma follow the magnetic field lines?

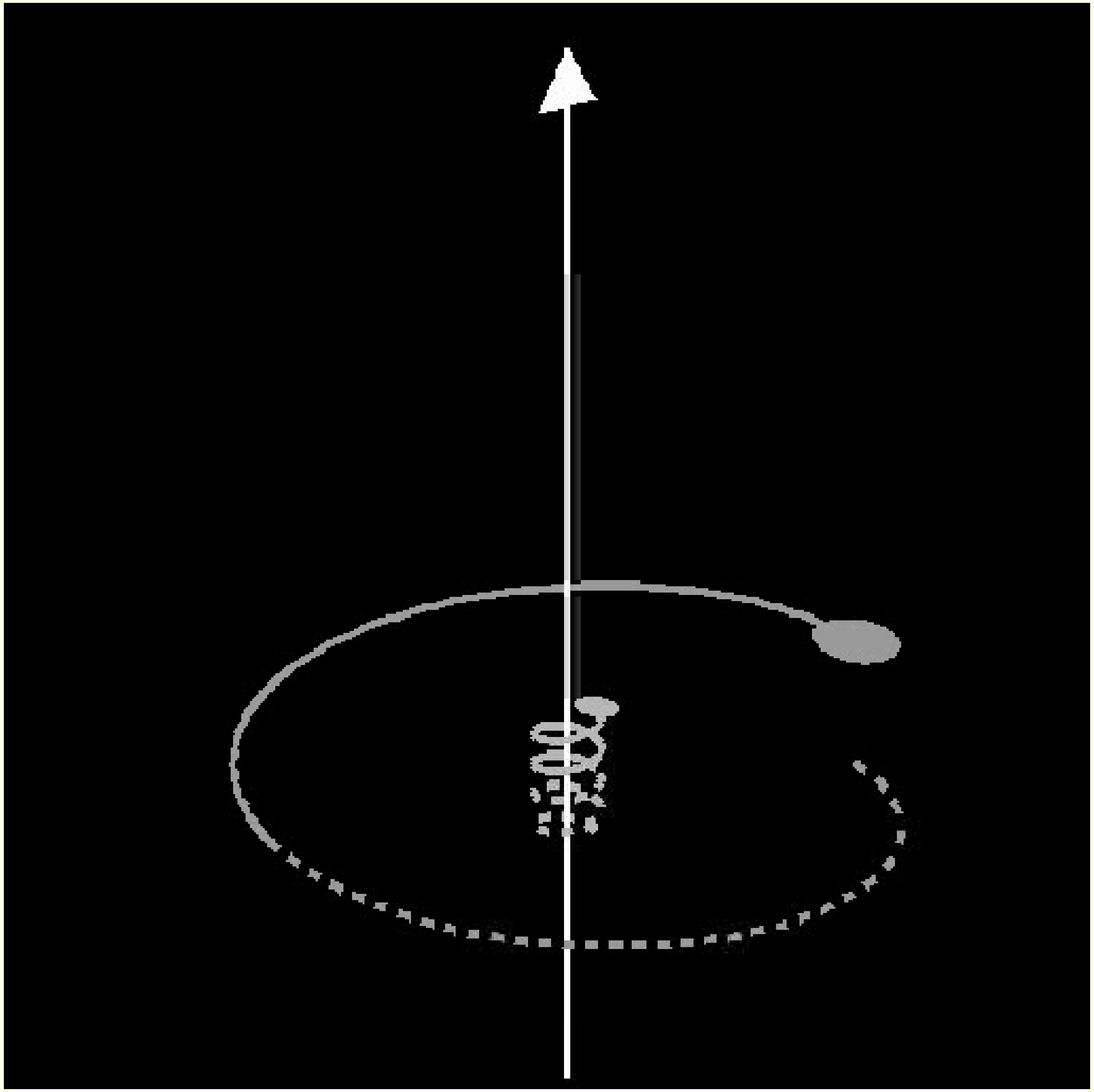
Magnetized plasma

Extremely common in space.

In single particle description of plasma, the particles gyrate in the plane perpendicular to **B**.

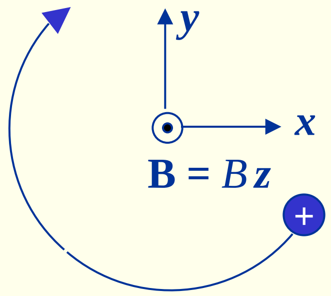


Gyro motion



Gyro motion

Consider a positively charged particle in a magnetic field.



Assume that the magnetic field is in the z-direction.

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v}\times\mathbf{B} \Rightarrow$$

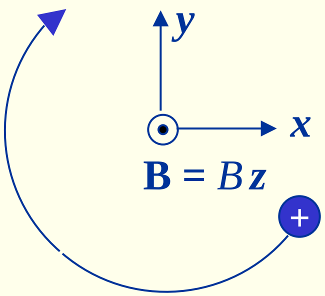
$$\begin{cases} m\dfrac{dv_x}{dt} = qv_yB \\ m\dfrac{dv_y}{dt} = -qv_xB \\ m\dfrac{dv_z}{dt} = 0 \end{cases} \Rightarrow$$

Constant velocity along ***z***

$$\begin{cases} \dfrac{d^2v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g\dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g\dfrac{dv_x}{dt} = -\omega_g^2 v_y \end{cases}$$

$\Rightarrow$

Gyro motion



$$\begin{cases} \dfrac{d^2 v_x}{dt^2} = -\omega_g^2 v_x \\ \dfrac{d^2 v_y}{dt^2} = -\omega_g^2 v_y \end{cases}$$

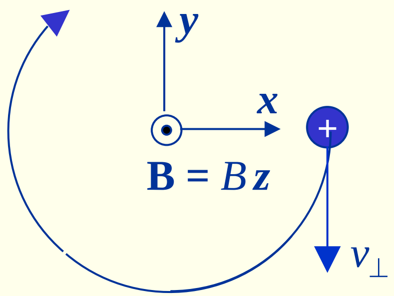
$\Rightarrow$

$$\begin{cases} v_x = Re\left(v_{0x} e^{i(\omega_g t + \delta_x)}\right) = v_{0x} cos(\omega_g t + \delta_x) \\ v_y = Re\left(v_{0y} e^{i(\omega_g t + \delta_y)}\right) = v_{0y} cos(\omega_g t + \delta_y) \end{cases}$$

and

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} sin(\omega_g t + \delta_x) \\ y = \dfrac{v_{0y}}{\omega_g} sin(\omega_g t + \delta_y) \end{cases}$$

Gyro motion



$$\begin{cases} v_x = v_{0x} cos(\omega_g t + \delta_x) \\ v_y = v_{0y} cos(\omega_g t + \delta_y) \end{cases}$$

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} sin(\omega_g t + \delta_x) \\ y = \dfrac{v_{0y}}{\omega_g} sin(\omega_g t + \delta_y) \end{cases}$$

For a particle starting at time t=0 at (x_0,0) with velocity (0,-$v_\perp$) we get (by definition v_{0x}, v_{0y}, $v_\perp$ > 0).

$$\begin{cases} v_y(0) = v_{0y} \cos \delta_y = -v_\perp & \Longrightarrow v_{0y} = v_\perp,\ \delta_y = \pi \\ v_x(0) = v_{0x} \cos \delta_x = 0 & \Longrightarrow 0 = 0 \end{cases}$$

and

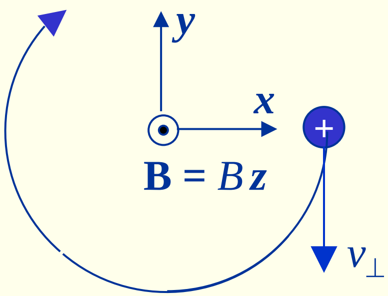
$$\begin{cases} x(0) = \dfrac{v_{0x}}{\omega_g} \sin \delta_x = x_0 & \Rightarrow\ \delta_x = \dfrac{\pi}{2},\ x_0 = \dfrac{v_{0x}}{\omega_g} \\ y(0) = \dfrac{v_\perp}{\omega_g} \sin \pi = 0 \end{cases}$$

So

$$\begin{cases} v_x = v_{0x} cos\left(\omega_g t + \dfrac{\pi}{2}\right) = -v_{0x} sin(\omega_g t) \\ v_y = v_\perp cos(\omega_g t + \pi) = -v_\perp cos(\omega_g t) \end{cases}$$

$$\begin{cases} x = \dfrac{v_{0x}}{\omega_g} sin\left(\omega_g t + \dfrac{\pi}{2}\right) = \dfrac{v_{0x}}{\omega_g} cos(\omega_g t) = \dfrac{v_{0x}}{\omega_g} cos(-\omega_g t) \\ y = \dfrac{v_\perp}{\omega_g} sin(\omega_g t + \pi) = -\dfrac{v_\perp}{\omega_g} sin(\omega_g) = \dfrac{v_\perp}{\omega_g} sin(-\omega_g t) \end{cases}$$

Gyro motion



$$\begin{cases} v_x = -v_{0x} sin(\omega_g t) \\ v_y = -v_\perp cos(\omega_g t) \end{cases}$$

$$\begin{cases} x = \frac{v_{0x}}{\omega_g} cos(-\omega_g t) \\ y = \frac{v_\perp}{\omega_g} sin(-\omega_g t) \end{cases}$$

Then (because the force is all the time perpendicular to the velocity)

$$v_x{}^2 + v_y{}^2 = v_{0x}{}^2 sin^2(\omega_g t) + v_\perp{}^2 cos^2(\omega_g t) = v_\perp{}^2$$

so

$$v_{0x} = v_\perp$$

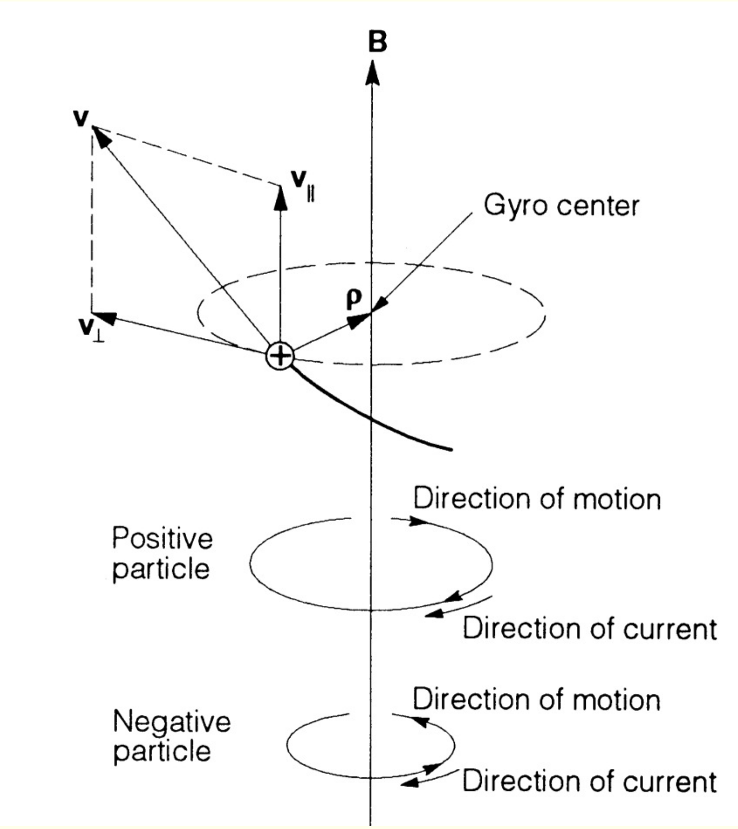
So

$$\begin{cases} x = \frac{v_\perp}{\omega_g} cos(-\omega_g t) \\ y = \frac{v_\perp}{\omega_g} sin(-\omega_g t) \end{cases}$$

and

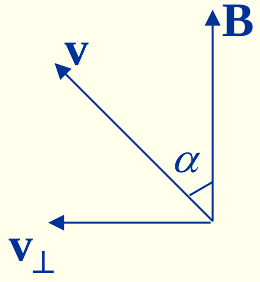
$$x^2 + y^2 = \frac{v_\perp{}^2}{\omega_g{}^2} \equiv r_L{}^2 = \varrho^2$$

Gyro (Larmor) radius



Magnetic force: $$\mathbf{F} = q\mathbf{v}_\perp \times \mathbf{B}$$

Centripetal force: $$\mathbf{F} = \frac{mv_\perp^2}{\rho}\hat{\boldsymbol{\rho}}$$

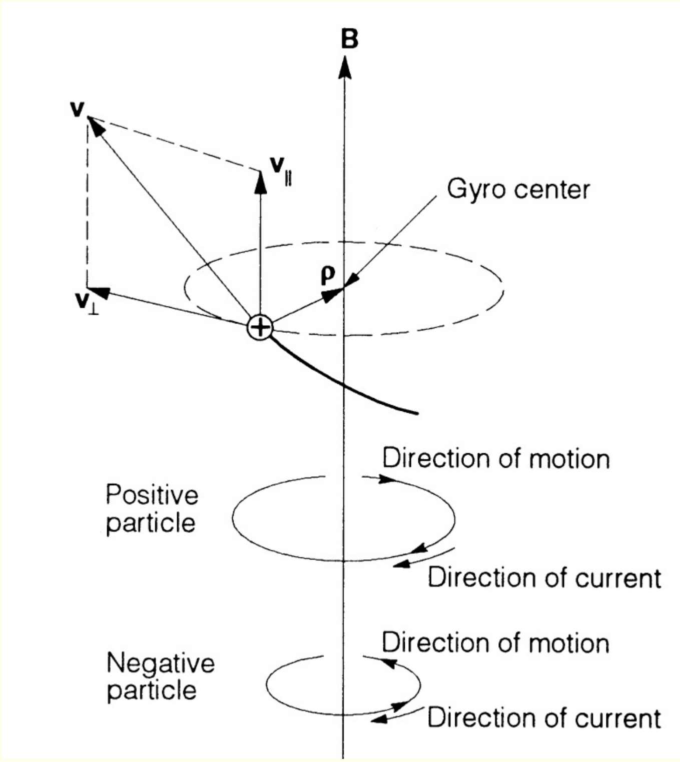


$$v_\perp = v \cdot \sin\alpha$$

$$\Leftrightarrow$$

$$\rho = \frac{mv_\perp}{qB}$$

Gyro frequency



$$\rho = \frac{mv_\perp}{qB}$$

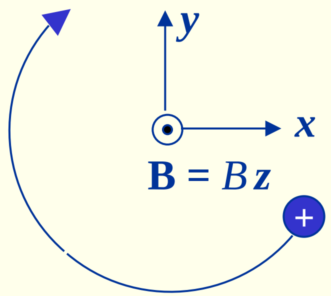
$$\omega\rho = v_\perp$$

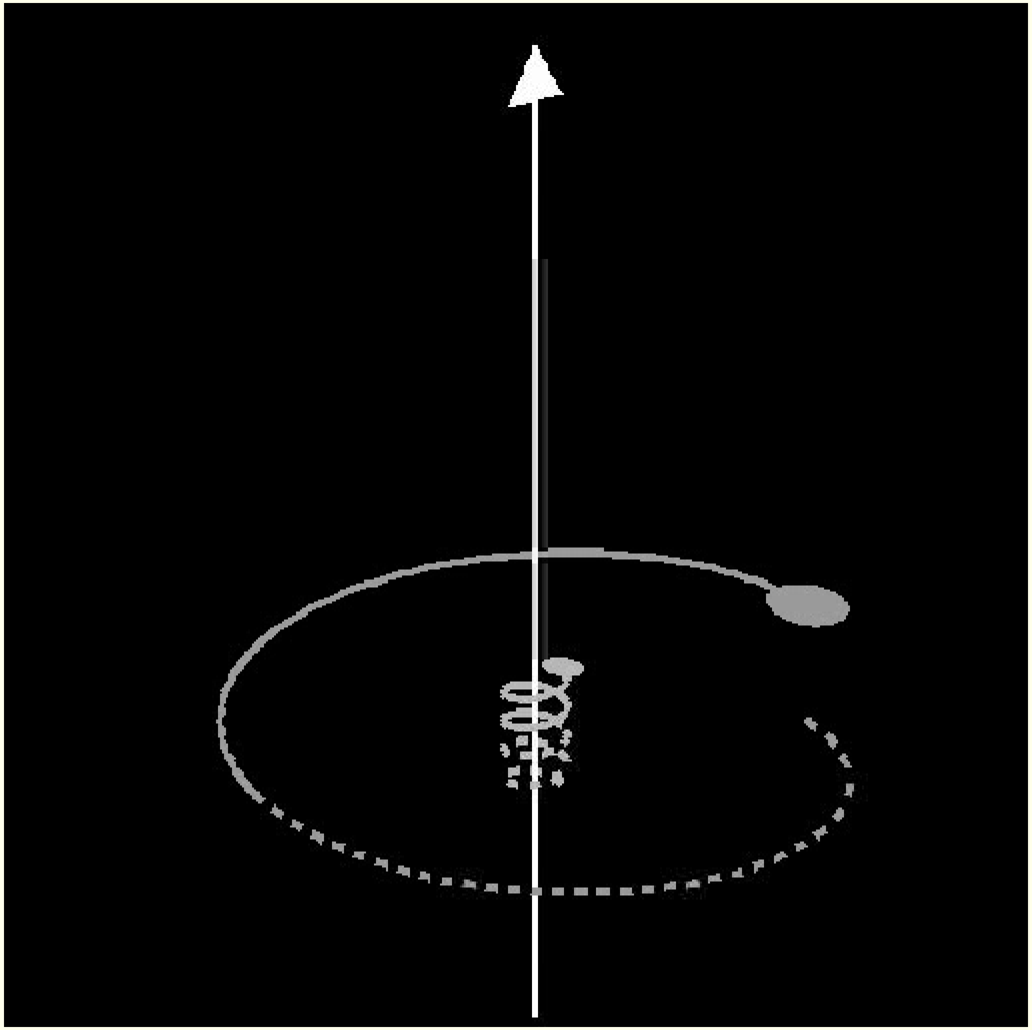
$$\Rightarrow$$

$$\omega_g = \frac{qB}{m}$$

$$\omega = 2\pi f$$

Drift motion





Then

$$x = r_L cos(-\omega_g t)$$
$$y = r_L sin(-\omega_g t)$$

$$\omega_g = \frac{qB}{m}$$

$$r_L = \frac{mv_\perp}{qB}$$

Magnetized plasma

A magnetic field drastically changes some of the plasma properties because the plasma particles are tightly bound to the magnetic field lines.

It is difficult for the particles to move perpendicular to **B**, but easy to move along **B**.

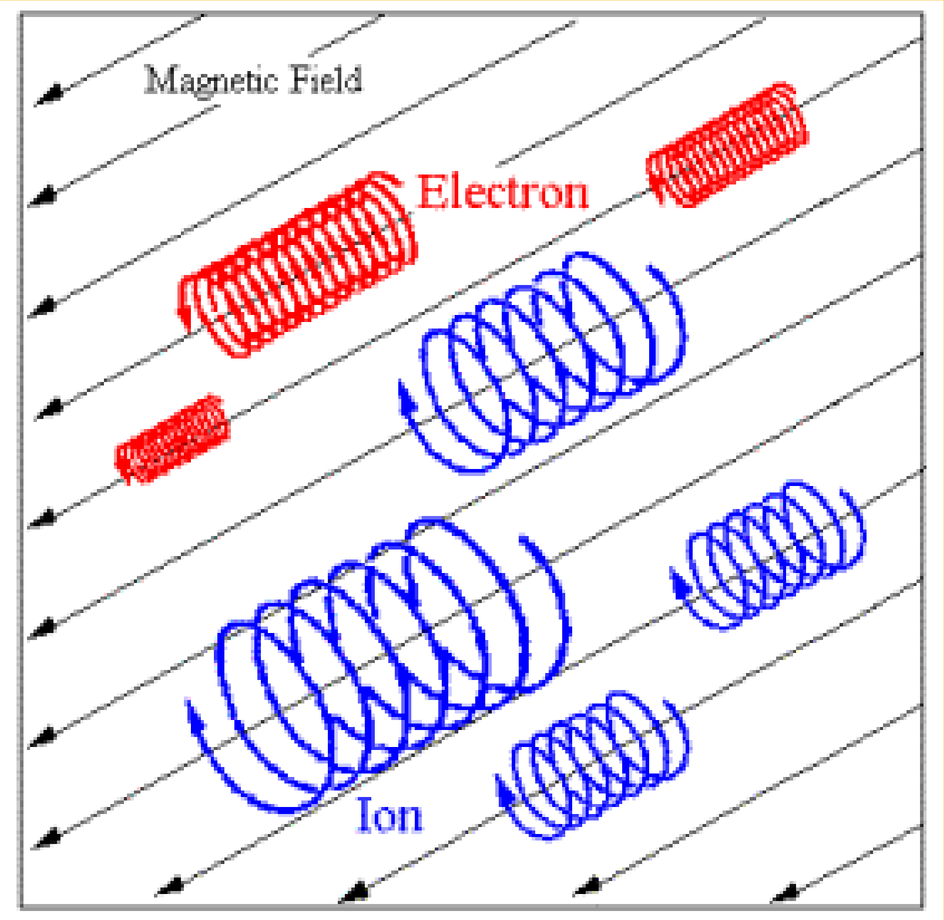
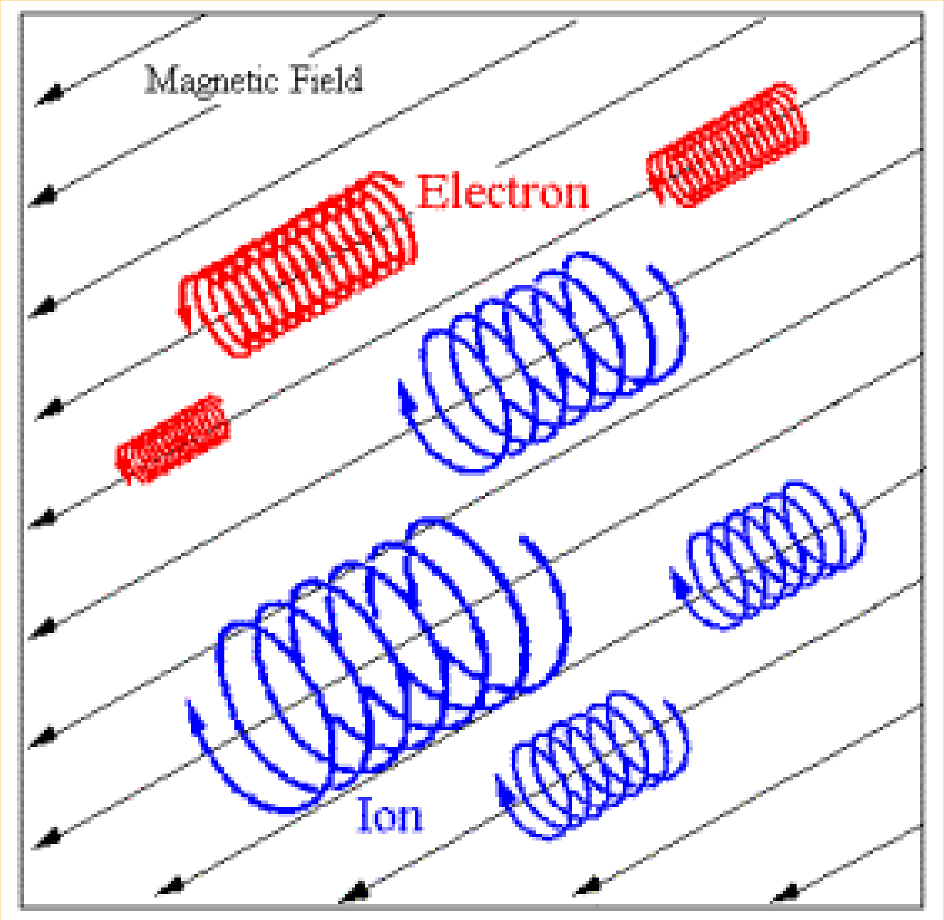


Figure 10: Gyration of charged particle along magnetic field lines.

Think about this:



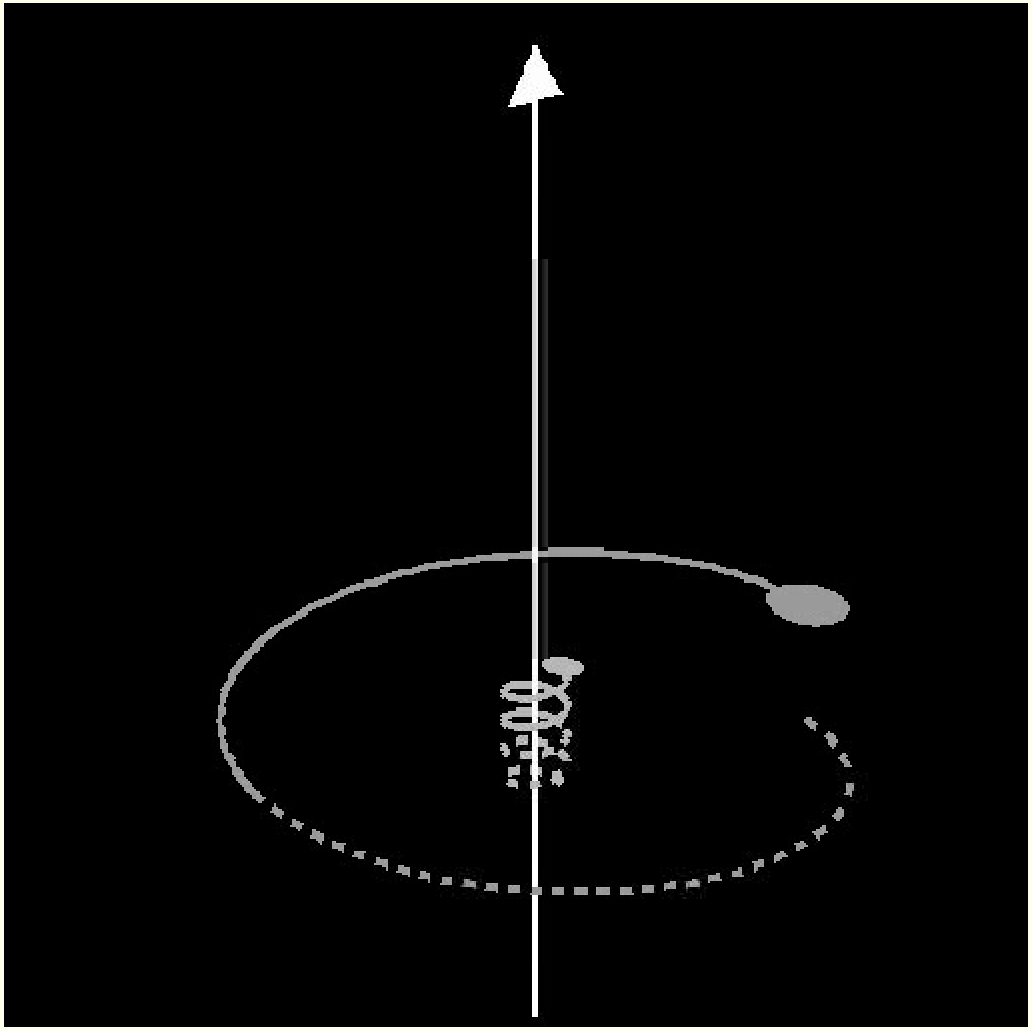
Can you think about a physical property of the plasma that varies with the direction?

(Such a property is called *anisotropic*.)

Coronal loops

Why does the plasma follow the magnetic field lines?

Gyro motion



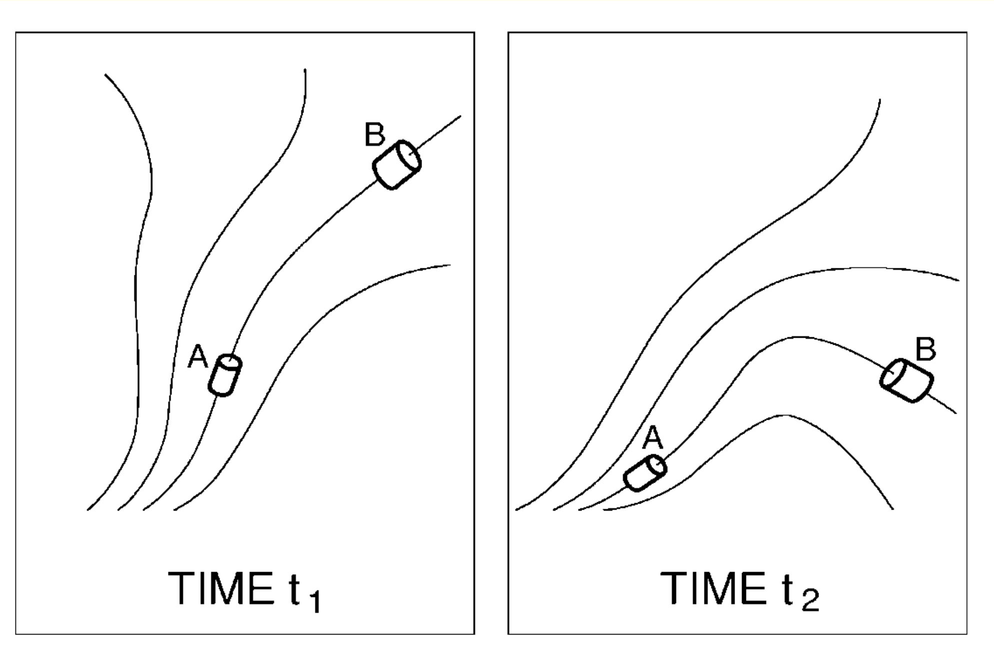
Equipartion principle

Statistically the kinetic energy is equally distributed along the three dimensions:

$$E_{\parallel} = \frac{1}{2} k_B T$$

$$E_{\perp} = \frac{2}{2} k_B T$$

Frozen in magnetic field lines



In fluid description of plasma two plasma elements that are connected by a common magnetic field line at time t_1 will be so at any other time t_2.

This applies if the magnetic Reynolds number is large:

$$R_m = \mu_0 \sigma l_c v_c >> 1$$

An example of the collective behaviour of plasmas.

Maxwell's equations

Gauss' law

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

No magnetic monopoles

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampére's law

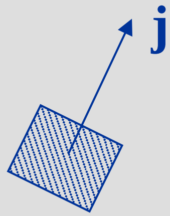
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Lorentz' force equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Ohm's law

$$\mathbf{j} = \sigma \mathbf{E}$$



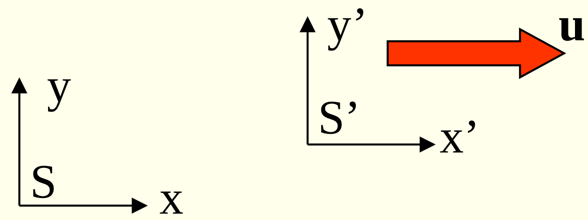
Energy density

$$W_B = \frac{B^2}{2\mu_0}, \quad W_E = \varepsilon_0 \frac{E^2}{2}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

Field transformations (relativistic)



Relativistic transformations
(perpendicular to the velocity $\mathbf{u}$*):*

$$\mathbf{E}' = \frac{\mathbf{E} + \mathbf{u} \times \mathbf{B}}{\sqrt{1 - u^2/c^2}}$$

$$\mathbf{B}' = \frac{\mathbf{B} - (\mathbf{u}/c^2) \times \mathbf{E}}{\sqrt{1 - u^2/c^2}}$$

For $u << c$*:*

$$\mathbf{E}' = \mathbf{E} + \mathbf{u} \times \mathbf{B}$$

$$\mathbf{E} = \mathbf{E}' - \mathbf{u} \times \mathbf{B}$$

induced electric field

$$\mathbf{B}' = \mathbf{B}$$

Frozen in magnetic flux *PROOF*

(1) $\mathbf{j} = \sigma \mathbf{E}' = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$ Ohm's law

(2) $\mu_0 \mathbf{j} = \nabla \times \mathbf{B} - \cancel{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}}$ Ampère's law

(3) $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$ Faraday's law

(1) $\Rightarrow$ $\mathbf{E} = \frac{\mathbf{j}}{\sigma} - \mathbf{v} \times \mathbf{B}$

(3+1) $\Rightarrow$ $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\frac{\mathbf{j}}{\sigma} - \mathbf{v} \times \mathbf{B}\right)$

(2) $\Rightarrow$ $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\frac{\nabla \times \mathbf{B}}{\mu_0 \sigma} - \mathbf{v} \times \mathbf{B}\right)$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B}\right) - \frac{1}{\mu_0 \sigma} \nabla \times \left(\nabla \times \mathbf{B}\right) =$$

$$\nabla \times \left(\mathbf{v} \times \mathbf{B}\right) - \frac{1}{\mu_0 \sigma}\left(\cancel{\nabla\left(\nabla \cdot \mathbf{B}\right)} - \nabla^2 \mathbf{B}\right)$$

$$\therefore \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B}\right) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Frozen in magnetic flux *PROOF II*

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_{A} + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}}_{B}$$

Order of magnitude estimate:

$$\frac{A}{B} = \frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}} \approx \frac{\frac{v \Delta B}{L}}{\frac{\Delta B}{\mu_0 \sigma L^2}} = v L \mu_0 \sigma \equiv R_m$$

Magnetic Reynolds number R_m:

$$R_m >> 1 \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Frozen-in fields!

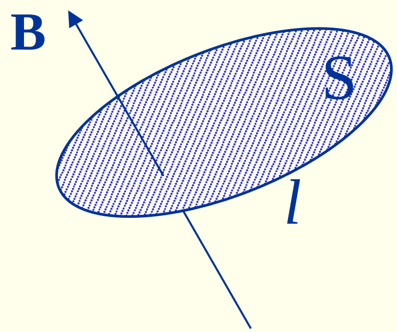
$$R_m << 1 \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Diffusion equation!

Frozen in magnetic flux *PROOF III*

$$R_m >> 1 \Rightarrow \boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})}$$

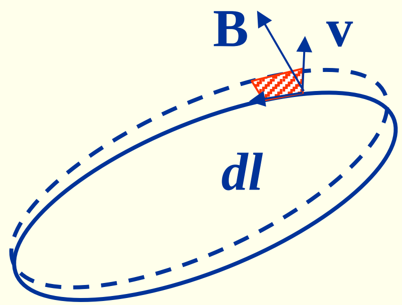
Consider the change of magnetic flux Φ through a surface S with contour l which follows plasma motion



$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \frac{d\Phi_c}{dt}$$

$\dfrac{d\Phi_c}{dt}$ This term is due to change in the surface S due to plasma motion

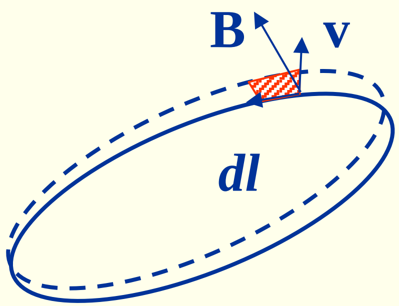


has an area of $(\mathbf{v} \cdot dt) \times d\mathbf{l}$

The flux through is $(\mathbf{v} \cdot dt) \times d\mathbf{l} \cdot \mathbf{B}$

$$\because \frac{d\Phi_c}{dt} = \int_l \mathbf{v} \times d\mathbf{l} \cdot \mathbf{B} =$$

Frozen in magnetic flux *PROOF IV*



$$\frac{d\Phi_c}{dt} = \int_l \mathbf{v} \times d\mathbf{l} \cdot \mathbf{B} =$$

$$-\int_l \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l} = -\int_S \nabla \times (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{S}$$

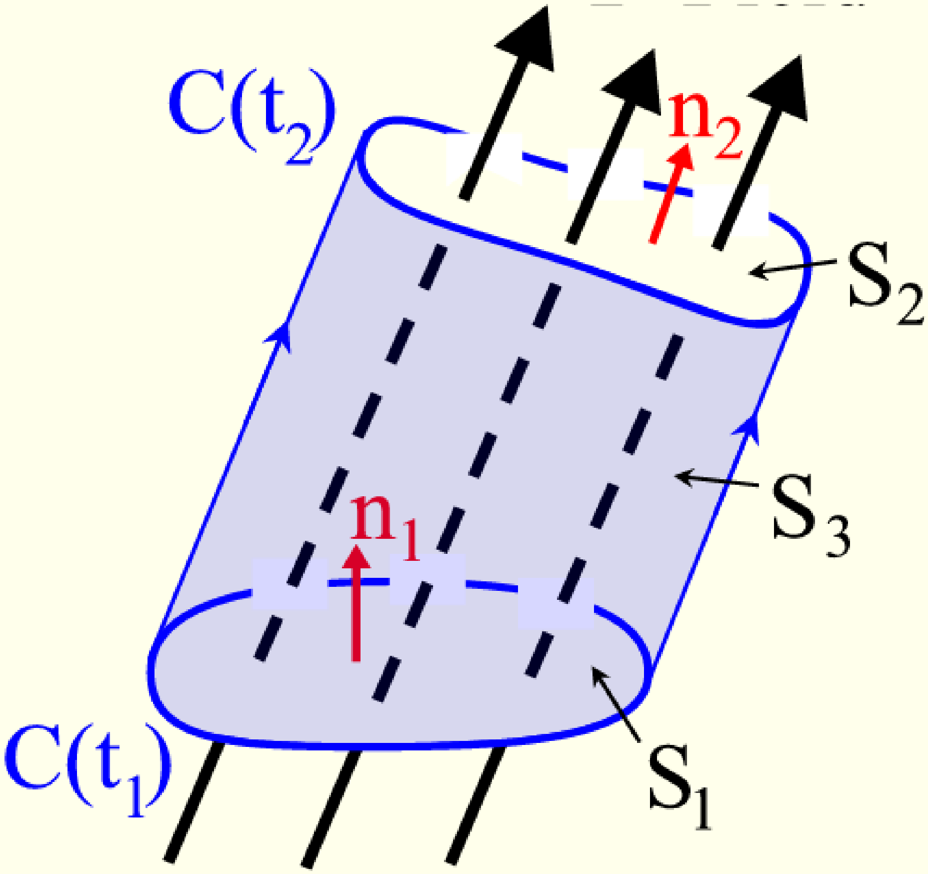
$$\because \frac{d\Phi}{dt} = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} - \int_S \nabla \times (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{S} =$$

$$\int_S \left[\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) \right] \cdot d\mathbf{S} = 0$$

★

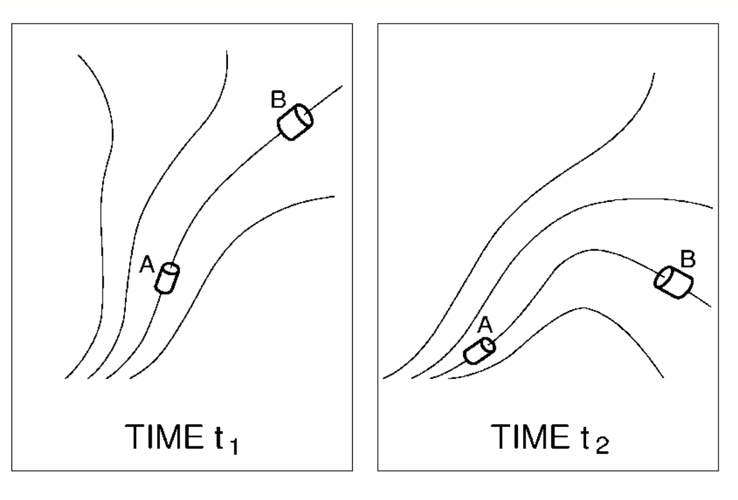
$$\because \frac{d\Phi}{dt} = 0$$

Frozen in magnetic field lines



A *flux tube* is defined by following **B** from the surface S. Due to the frozen-in theorem the flux tube keeps its identity and the plasma in a flux tube stays in it for ever.

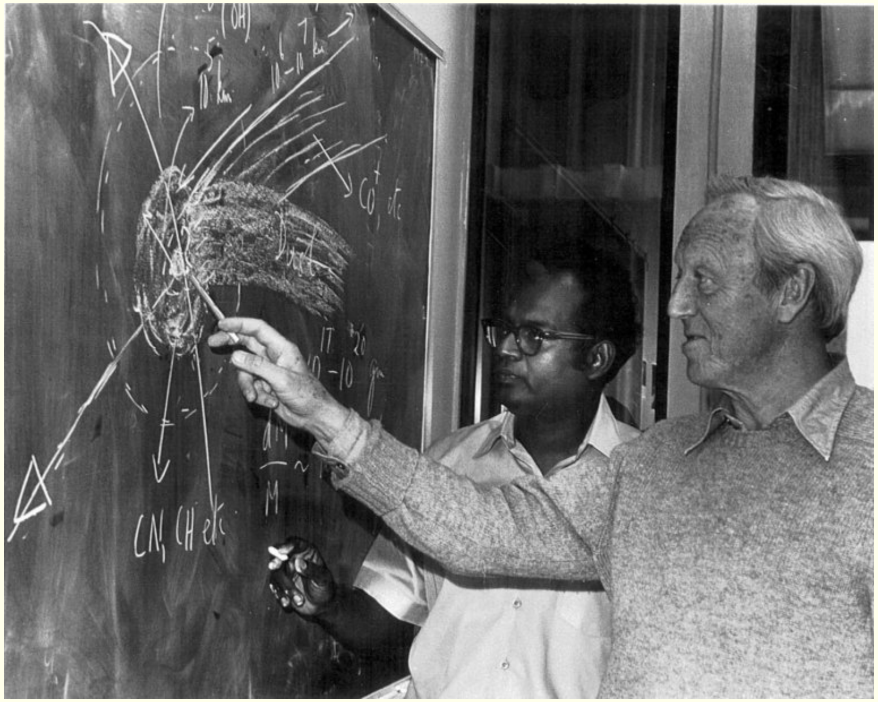
In particular if we let the tube become infinitely thin we have the theorem of frozen-in field lines.



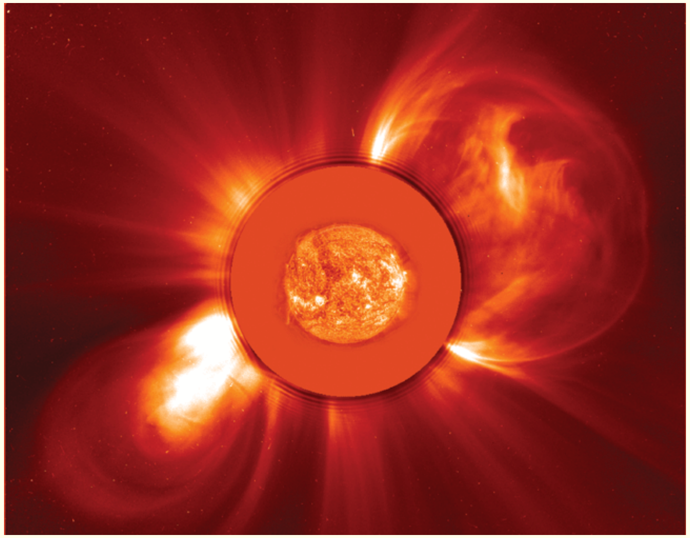
Frozen in magnetic field lines: some history

- Also known as *Alfvén's theorem*
- Hannes Alfvén (1908-1995), professor at KTH
- Alfvén received the Nobel prize in 1970

  '*for fundamental work and discoveries in magneto-hydrodynamics with fruitful applications in different parts of plasma physics*'



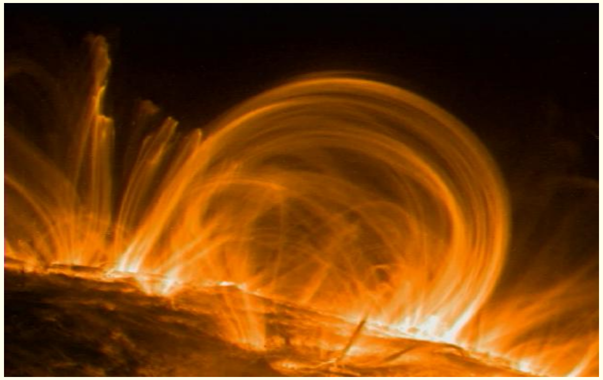
Magnetized plasma



Solar magnetic field

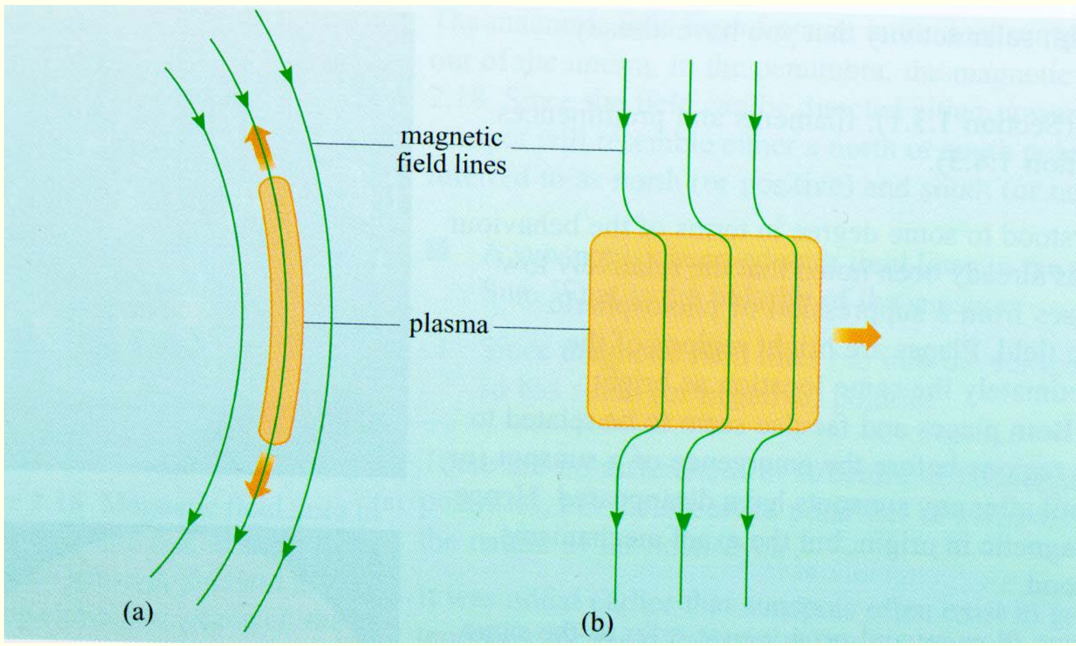
Northern lights (aurora)

Different *plasma populations* (plasmas with different temperature and density) keep to their own field line, and thus "paint out" the magnetic field lines.



Coronal loop

Does the plasma follow the magnetic field (a) or the other way around (b)?



Depends on relative energy density (pressure)

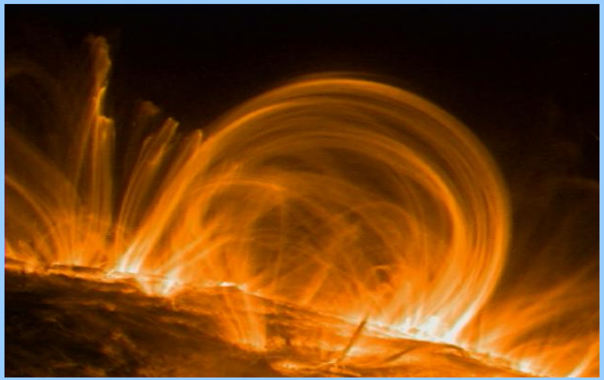
$$p_{pl} = nk_B T$$

$$p_B = \frac{B^2}{2\mu_0}$$

$$\beta = \frac{p_{pl}}{p_B}$$

$\beta << 1$ $\beta >> 1$

Plasma beta



Coronal loop

B = 0.2 T
n = 10^{23} m^{-3} (~1% of density at Earth surface)
T = 6000 K

Plasma (thermal) pressure/energy density: $p_{pl} = nk_bT$

Magnetic pressure/energy density: $p_b = B^2/2\mu_0$

$$\beta = \frac{p_{pl}}{p_B}$$

Green — $\beta >> 1$ The plasma dominates the magnetic field

Red — $\beta \sim 1$ Some complicated in-between behaviour

Blue — $\beta << 1$ The magnetic field dominates the plasma

Plasma beta

$B = 0.2$ T
$n = 10^{23}$ m^{-3} (~1% of density at Earth surface)
$T = 6000$ K

Plasma (thermal) pressure/energy density: $p_{pl} = nk_bT = 10^{23} \cdot 1.38 \cdot 10^{-23} \cdot 6000 \approx 8.3\,\text{kPa}$

Magnetic pressure/energy density: $p_b = B^2/2\mu_0 = \dfrac{0.2^2}{2 \cdot 4\pi \cdot 10^{-7}} \approx 16\,\text{kPa}$

Red

$\beta \sim 1$ Some complicated in-between behaviour

Last Minute!

Last Minute!

- What was the most important thing of today’s lecture? Why?
- What was the most unclear or difficult thing of today’s lecture, and why?
- Other comments