

- ÖVNING 1:
- STEGSVAR
  - "ÖVERFÖRINGSFUNKTIONER"
  - POLER OCH NOLLSTÄLLEN

UPPGIFTER: 2.10, 2.5, 2.11

## TEORI:

### DYNAMISKT SYSTEM



$u(t)$ : insignal

$y(t)$ : utsignal

- Ett dynamiskt system beror utsignalen också på tidigare insignaler. Systemet har "minne".
- Beskrivs av diffrentialekvation:

$$\frac{dy}{dt} = -a y(t) + u(t)$$

### LAPLACE TRANSFORM

$$\text{DEF: } \mathcal{L}[y(t)](s) = \int_0^\infty y(t) e^{-st} dt = Y(s)$$

## ÖVERFÖRINGSFUNKTION

$$\frac{dy}{dt} = -ay(t) + u(t) \quad \text{Laplace transform} \Rightarrow$$

$$sY(s) = -aY(s) + U(s) \Leftrightarrow (s+a)Y(s) = U(s)$$

$$Y(s) = \frac{1}{s+a} U(s) = g(s)U(s)$$

$g(s)$

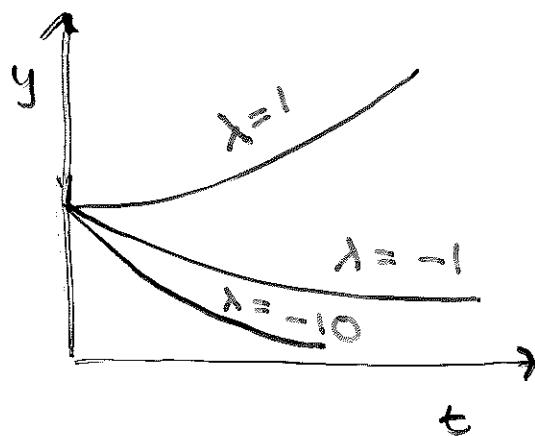
$g(s)$ : Överföringsfunktionen

## KOPPLING TILL DIFFERENTIALEKVATIONEN

$$\frac{dy}{dt} = -ay(t) + u(t)$$

$$\text{Låt } u(t) = 0 \Rightarrow y(t) = c_1 e^{-at}, \quad c_1: \text{konstant}$$

$\lambda = -a$  är systemets egenvärde



STABILITET:  $\lambda < 0 \Leftrightarrow y$  stabil  
 $\lambda > 0 \Leftrightarrow y$  instabil

SNABBHET:  $y \rightarrow 0$  snabbare för  $\lambda = -10$   
 än för  $\lambda = -1$

### I "LAPLACE VÄRLDEN"

- Tidsdomänens egenvärden motsvarar överföringsfunktionens poler.

POLER: Färs genom att sätta nämnaren i  $q(s) = 0$ .

STABILITET:  $\text{Pol} < 0 \Leftrightarrow$  stabilt system  
 $\text{Pol} > 0 \Leftrightarrow$  instabilt system

SNABBHET: Pol i  $-10$  ger snabbare system  
 än pol i  $-1$ .

IMAGINÄR POL: Lösningen till diff. ekvationen är en cos-funktion.

Ett oscillerande beteende fås.

NOLLSTÄLLEN: Färs genom att sätta  
täljaren i  $g(s) = 0$ .

Säger något om systemets transienta  
egenskaper.

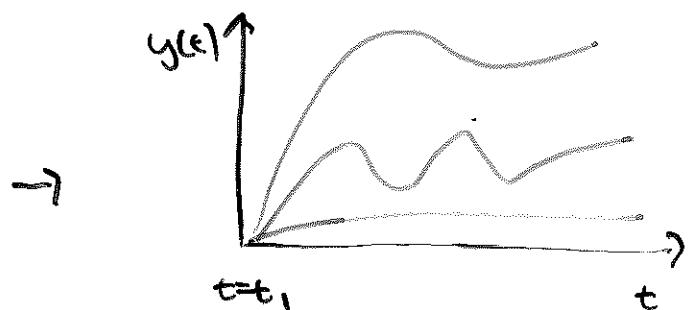
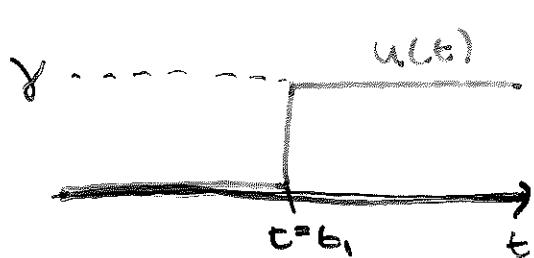
STATISKA FÖRSTÄRKNINGEN:

Systemets förstärkning vid konstant insignal.  
Ges av  $|g(0)|$ .

STEGSVAR:

STEGLI (INSIGNALEN  $u(t)$ ):

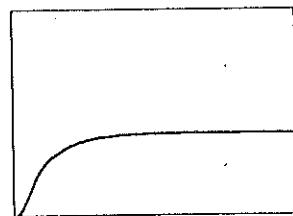
$$u(t) = \begin{cases} 0 & \text{om } t < t_1 \\ \gamma & \text{om } t \geq t_1 \end{cases}$$



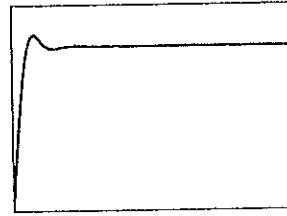
STEGSVAR

2.10 Figure 2.10a shows the step responses of four different systems. Combine each step response with a transfer function from the alternatives below.

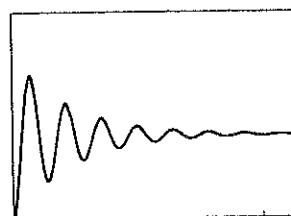
Transfer function	Poles	Zeros	$ G(0) $
$G_1(s) = \frac{100}{s^2+2s+100}$	$-1 \pm 10i$		1
$G_2(s) = \frac{1}{s+2}$	$-2$		$1/2$
$G_3(s) = \frac{10s^2+200s+2000}{(s+10)(s^2+10s+100)}$	$-10, -5 \pm 8.7i$	$-10 \pm 10i$	2
$G_4(s) = \frac{200}{(s^2+10s+100)(s+2)}$	$-2, -5 \pm 8.7i$		1
$G_5(s) = \frac{600}{(s^2+10s+100)(s+3)}$	$-3, -5 \pm 8.7i$		2
$G_6(s) = \frac{400}{(s^2-10s+100)(s+2)}$	$-2, 5 \pm 8.7i$		2



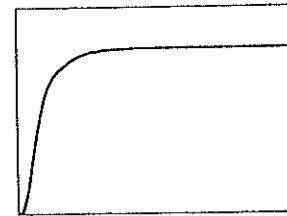
Step A



Step B



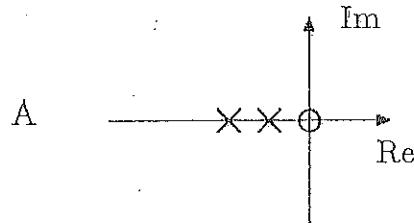
Step C



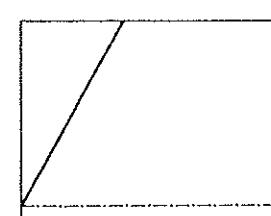
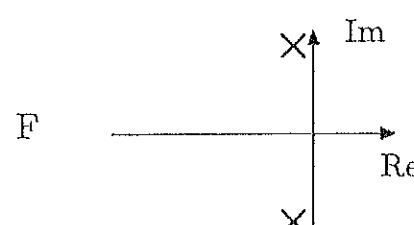
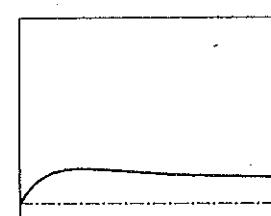
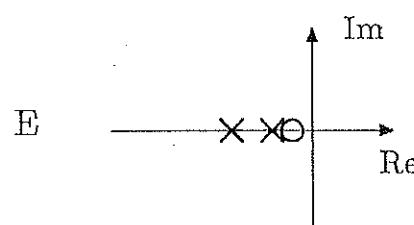
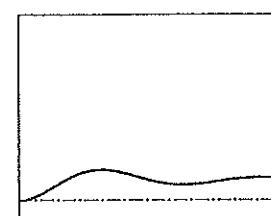
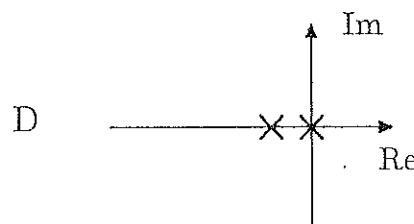
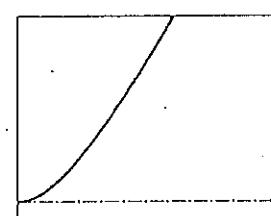
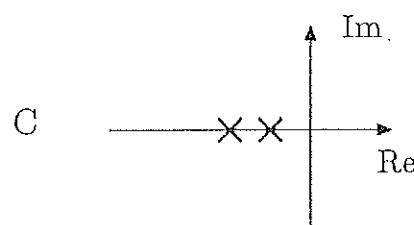
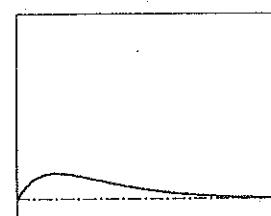
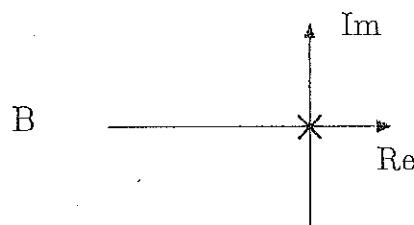
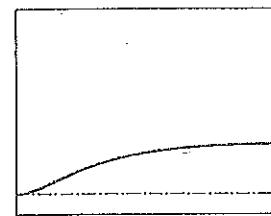
Step D

Figure 2.10a. All comparable axes have equal scaling.

Pole-zero map



Step response



Real part

Time

Figure 2.5a. All comparable diagrams have equal scaling. In the pole-zero maps, imaginary and real parts have equal scaling,  $\times$  marks poles, and  $\circ$  marks zeros.

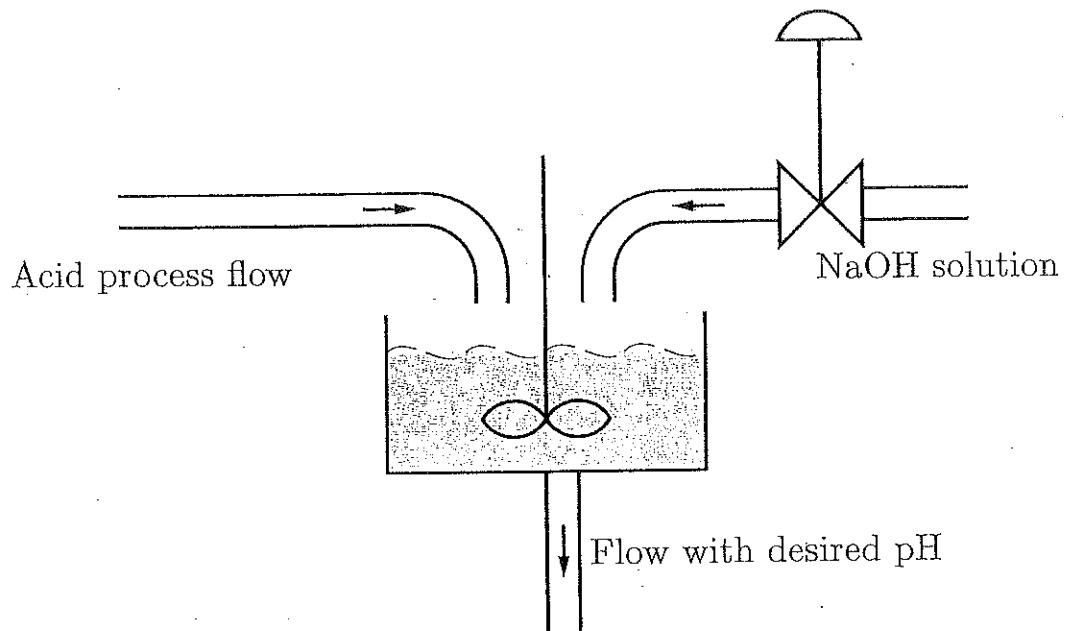


Figure 2.11a.

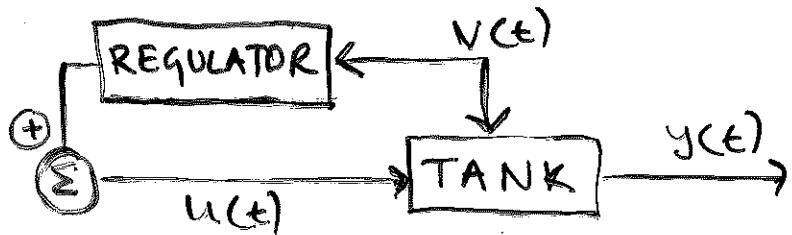
2.11 In the continuously stirred tank, see Figure 2.11a, an acid process flow is neutralized by adding a concentrated NaOH solution. The acid process flow has a tendency to vary its pH with time, which gives undesired variation of the pH in the outflow. In an effort to reduce these variations one has decided to use control.

- Classify the different signals as input, output, and disturbance signal.
- Draw a block diagram of the system with a control strategy.

2.11.b forts

(b)

## ALTERNATIV REGLERSTRATEGI: FRAMKOPPLING



- Kräver att man kan mäta störningen  $v(t)$  och kompensera direkt.

### FRAMKOPPLING

- + Korrigeras snabbare.
- Svårt att mäta alla störningar
- Känslig för modellfel.

### ÅTERKOPPLING

- + Inte så känslig för modell fel.
- + Behöver inte mäta störningar
- + Kan stabilisera instabilt system.
- Långsammare.
- Känslig mot brus i utsignal.