Electricity Market Analysis,
EG2060
L5-L6

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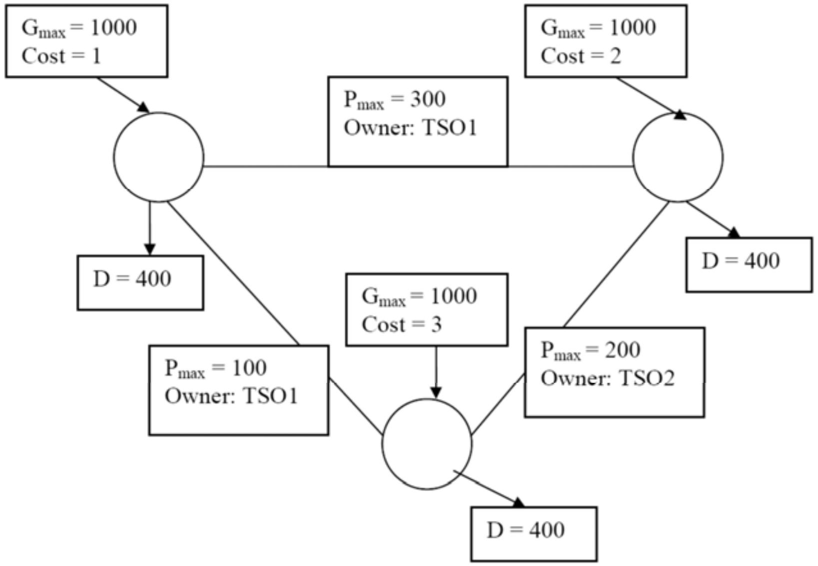
Transmission limits and congestion management

- Problem structure
- Transmission limits
- Transfer capacity
- Congestion Management Methods
- Examples – radial systems
- Example – meshed systems

Example 1



G_{max} = 1000
Cost = 1

P_{max} = 300
Owner: TSO1

G_{max} = 1000
Cost = 2

D = 400

D = 400

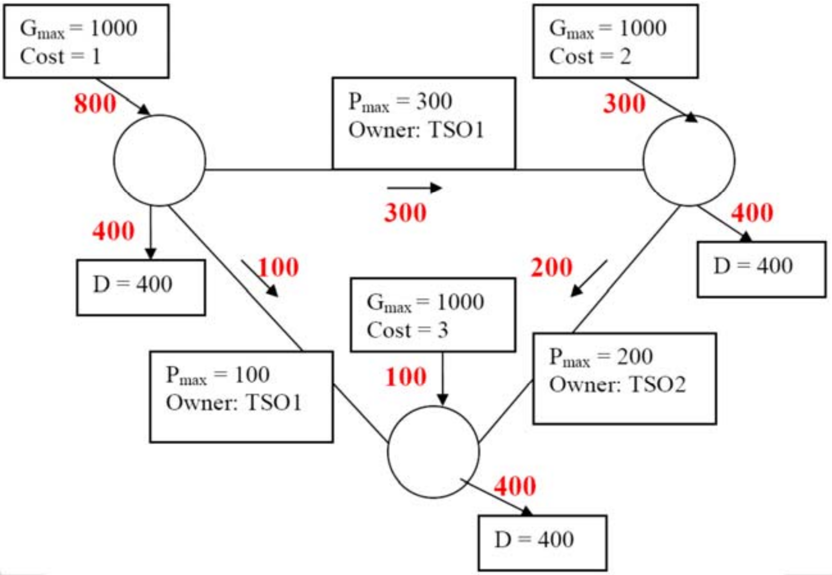
G_{max} = 1000
Cost = 3

P_{max} = 100
Owner: TSO1

P_{max} = 200
Owner: TSO2

D = 400

“Optimal” Solution



G_{max} = 1000
Cost = 1

800

P_{max} = 300
Owner: TSO1

G_{max} = 1000
Cost = 2

300

300

400

400

D = 400

100

200

D = 400

G_{max} = 1000
Cost = 3

P_{max} = 100
Owner: TSO1

100

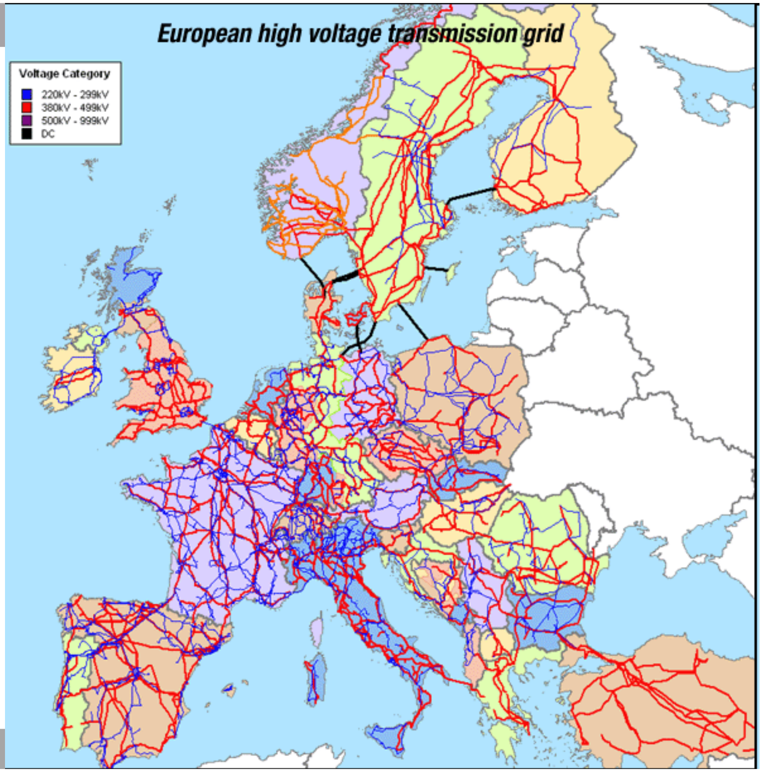
P_{max} = 200
Owner: TSO2

400

D = 400

Example 2

”Optimal solution=?”



Questions:

- Is this technically possible?
- What are the rules for “transmission limits”?
- Do we get the “optimal solution”?
- What are the prices in the different areas (same or different)?
- Are bilateral contracts possible/ allowed/ forbidden?
- How are the TSO:s, Transmission System Operators, affected economically by the congestions.

Transmission limits

Transmission limits can be caused by

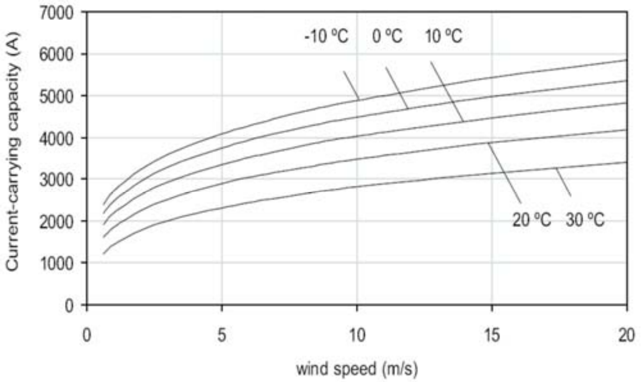
- Thermal limits
- Voltage stability
- Angle stability

Transmission line thermal limits - 1

- The thermal limit of an overhead transmission line is reached when the electric current flow heats the conductor material up to a temperature above which the conductor material gradually loses mechanical strength and sags due to conductor expansion, thus clearance to ground is decreased.
- The maximum allowed continuous conductor temperature varies from 50 °C to 100 °C

Transmission line thermal limits - 2

- The thermal limit of an overhead transmission line is dependent on wind speed and temperature



Transmission line thermal limits - 3

- The maximum allowable current is bounded to maximum allowable active power transfer by the following expression:

$$P_{max} = I_{max} \cdot U_{min} \cdot \cos \varphi_{min}$$

Transmission line thermal limits - 4

Concerning the maximum allowable power transmission:

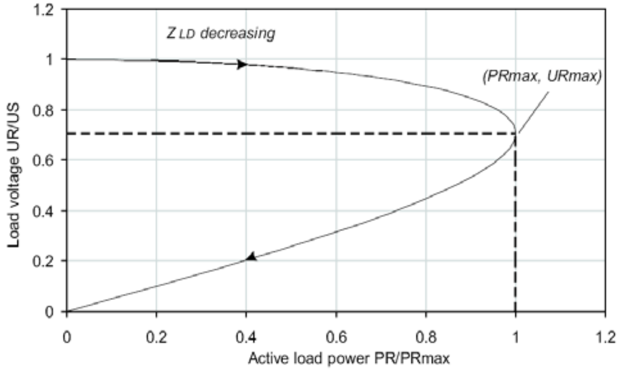
- There is an upper limit for each single line.
- For several components in series, the most sensitive component is the limiting one.
- The limit is depending on
  - Wind speed
  - Temperature
  - Operation details concerning voltage and phase shift between voltage and current.
- Overloading is possible for several minutes.

Transmission line voltage stability limits - 1

- Voltage stability is the ability of the system to maintain steady acceptable voltages at all buses in the system under normal conditions and after being subjected to the disturbance.
- Instability occurs in the form of a progressive fall or rise of voltages in some busses.
- Voltage stability sets a limit for power transfer between different areas (not single lines)

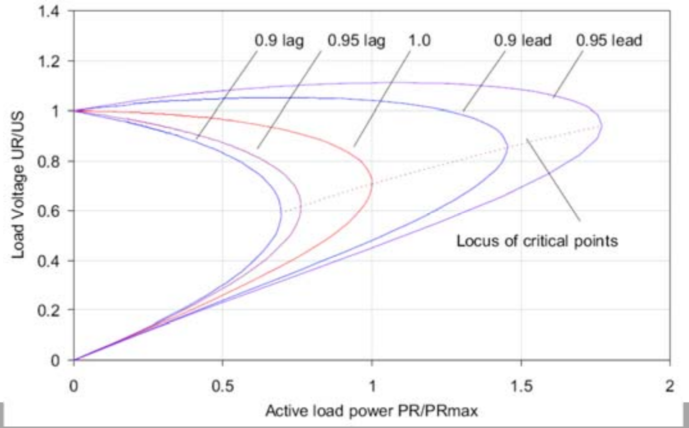
Transmission line voltage stability limits - 2

- The relationship between transferred power and the voltage can be illustrated by so called nose curve



Transmission line voltage stability limits - 3

- The phase angle between receiving end voltage and current has an impact of transfer capacity:



Transmission line voltage stability limits - 4

Concerning the maximum allowable power transmission

- There is an upper limit for total transmission between areas.
- The limit is depending on
  - Line type
  - Operating power plants in receiving end.
  - Load types in receiving end.
- This means different limits in different directions
- Overloading is often possible for several minutes.

Transmission line rotor angle stability limits - 1

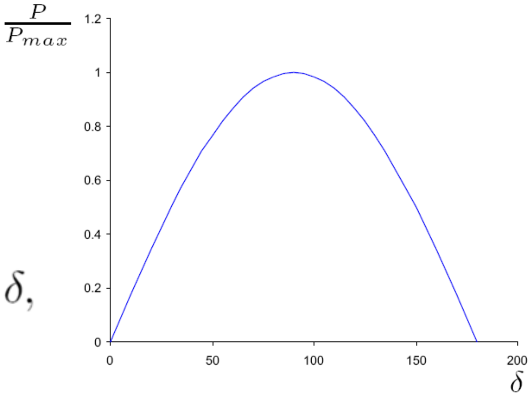
- Rotor angle stability is the ability of interconnected synchronous machines to remain in synchronism.
- Rotor angel instability can be viewed as two masses (=machines) on the same shaft (transmission line) that oscillates against each other.
- There is then a limit how large these oscillations can be until the shaft breaks.

Transmission line rotor angle stability limits - 2

Mathematical representation:

- U = end voltage
- X = reactance
- δ = volt. angle difference

$$P = \frac{U_G U_M}{X_T} \sin \delta,$$



Transmission line rotor angle stability limits - 3

Concerning the maximum allowable power transmission

- There is an upper limit for total transmission between areas.
- The limit is depending on
  - Line types (reactance)
  - Operating power plants in both ends.
  - Load types in receiving end.
- Overloading is **NOT** possible.

Transmission limit summary

	Thermal limit	Voltage stability	Rotor angle stability
Limit valid for	Single line	Between areas	Between areas
Overload-ability	Yes	Yes	No
Direction dependent	No	Yes	Can be

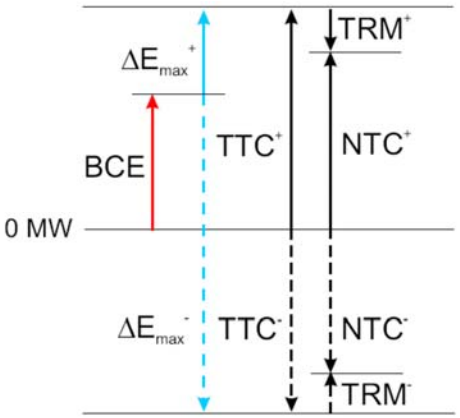
Transmission capacity – TC - 1

- Transmission capacity is determined by transmission system operators (TSO).
- Available TC depends not only on technical properties of the respective system, but also on the assumptions, economic and security considerations of respective TSO

Transmission capacity – TC - 2

European approach:

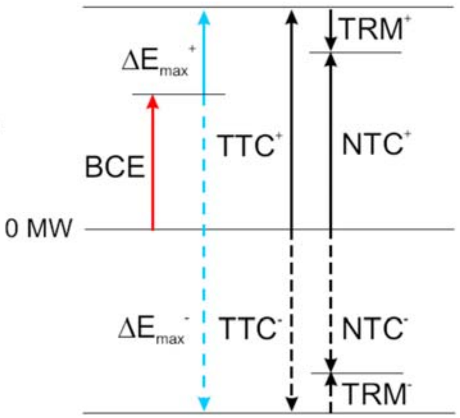
1. Start with Base Case Exchange, BCE
2. Add extra transfer ΔE until TSO security limits are violated
3. Common security limit = ”N-1”, i.e., outage of one component should not cause instability



Transmission capacity – TC - 3

European approach:

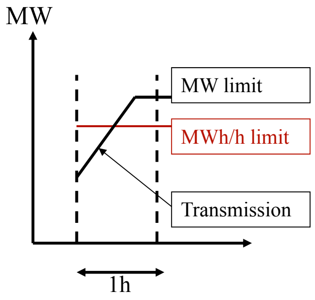
4. Total Transmission Capacity, TTC = BCE+ ΔE.
5. From TTC a Transmission Reserve Margin is subtracted, to consider uncertainties
6. The Net Transfer Capacity, NTC is then NTC = TTC - TRM



Transmission capacity – TC - 4

MW ≠ MWh/h!:

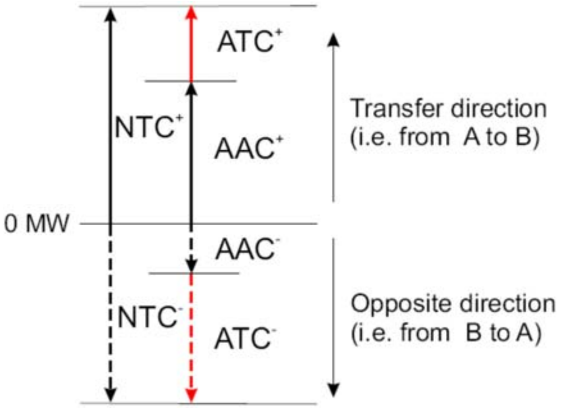
- A MW limit always (each second) has to be kept.
- The transmission can vary during an hour.
- This means that a MW-limit is not the same as a MWh/h limit if the transmission varies.



Transmission capacity – TC - 5

European approach:

6. Available Transmission Capacity, ATC, is the difference between Allocated Transmission Capacity, AAC and NTC,
7. ATC = NTC-AAC



Questions on transmission limits

1. With two parallel overhead lines where the limits are set by thermal limits, it is possible to overload one of them if the other one is below its rating.
2. If a transmission line has a limit set by transient stability, then overloading will directly lead to an interruption.
3. A voltage stability limit is never set for one single line, but only for corridors with many parallel lines.
4. ATC = Available Transmission Capacity from A to B can sometimes be larger than the physical limit.

Congestion Management

- The transmission system is built up together with power production to meet the expected electricity consumption.
- For economic reasons it is usually not over-dimensioned.
- This means that in some situations, local more expensive plants are used instead of more distant with lower operating costs.
- **Congestion Management** is the method used to handle these situations, i.e. when there is an economic interest to transfer more than technically possible.

Congestion Management methods

The questions include:

- Who has the right to use the limited capacity?
- The economic treatment of this

There are:

- **Market based CM methods**: The capacity is used by the ones that have the highest economic interest to use it.
- **Non-market based CM methods**: The capacity is allocated with other methods.

Congestion Management methods

Non-market based CM methods:

A. Type of contract
B. First come – first serve
C. Pro Rata
D. Channel

Congestion Management methods

Market based CM methods:

E. Implicit Auctioning
F. Explicit Auctioning
G. Counter trading
H. Re-dispatching

Aims of CM method:

- It should be economically efficient in the short run, i.e., the transmission line is used to maximize the total surplus (consumer + producer) in the whole system.
- It should give the long term incentives for
  - Producers to locate their new production units
  - Consumers to locate their consumption
  - Transmission operators to invest in increased capacity

A: Type of contract

- Can use the length of contract period as method of relieving congestion, e.g., contracts that is valid for a longer period get higher priority than a short term contract.

Possible drawback:

- Will not result in an economically efficient use of the system, since the flow can go from A => B even if marginal cost (= price?) is higher in A than in B.

B: First come – first serve

- Gives the capacity to the one who first submits a bid to use it.

Possible drawback:

- Will not result in an economically efficient use of the system, since the flow can go from A => B even if marginal cost (= price?) is higher in A than in B.

C: Pro Rata

- Gives the actors their share to total transmission capacity based on either their request to use it or, e.g., their installed capacity.

Possible drawback:

- Will not result in an economically efficient use of the system, since the flow can go from A => B even if marginal cost (= price?) is higher in A than in B.

D: Channels

- = Self owned transmission
- Gives an actors the right to use a transmission corridor with a guaranteed level of capacity.

Possible drawback:

- Will not result in an economically efficient use of the system, since the flow can go from A => B even if marginal cost (= price?) is higher in A than in B.

E: Implicit auctioning

- Transmission use and use of power plants scheduled simultaneously.
- Requires an organized exchange
- A fee is added to every bid, using the congested line until accepted bid volume is equal to ATC

Possible drawback:

- The fee is paid to the system operator.
- The smaller the capacity, the higher the fee
- => Negative incentive for TSO to reduce congestion

E: Implicit auctioning – market splitting

- Transmission and use of power plants scheduled simultaneously.
- Requires an organized exchange
- The prices in all areas are changed until accepted bid volumes correspond to inter-area transmission is ≤ ATC.

Possible drawback:

- The system operators benefit on price differences.
- The smaller the capacity, the higher the profit.
- => Negative incentives for TSO:s to reduce congestions
- “Small” congestion can cause high prices, which is a incentive to use market power

F: Explicit auctioning

- Transmission use and use of power plants **not** scheduled simultaneously.
- Common that use of congested sections are scheduled before generation is scheduled.
- The highest bids to use a certain section are accepted until amount of accepted bids = ATC.

Possible drawback:

- Can result in a not economically efficient use of the system, since the flow can go from A => B even if marginal cost (= price?) is higher in A than in B
- Perhaps accepted transmission bids are not used.

G: Counter trading

- A transmission ≥ ATC is first scheduled.
- The TSO is then "counter trading" the excess use, by buying power from producers on the receiving end and selling it on the sending end.

Possible drawback:

- Gives no incentives to producers to locate production in deficit areas.
- The cost for the TSO makes it possible for the TSO to "move bottlenecks" to reduce the cost.

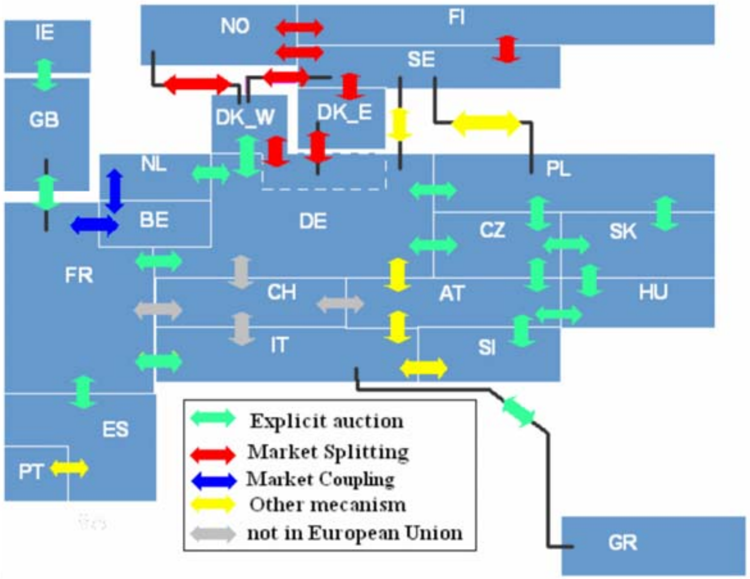
H: Re-dispatching

- ≈ Counter trading
- A transmission ≥ ATC is first scheduled.
- A system operator is then ordering re-dispatch, by buying power from producers on the receiving end and selling it on the sending end.

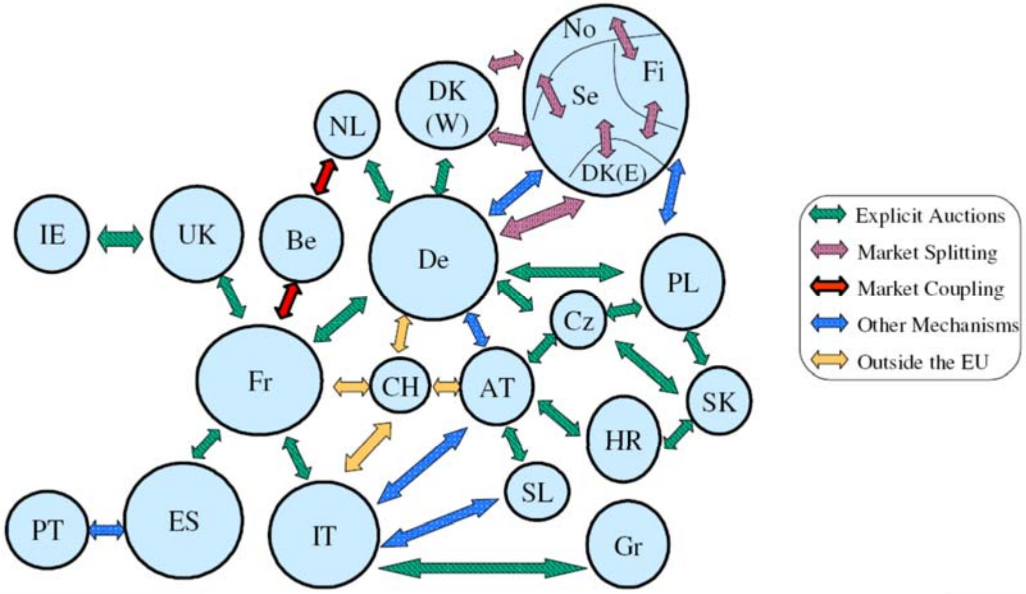
Possible drawback:

- Gives no incentives to producers to locate production in deficit areas.

Auctions in Europe



Auctions in Europe



Market splitting/coupling

Market splitting

- This method starts with one market and then this market is split into several markets with different prices if there are internal bottlenecks

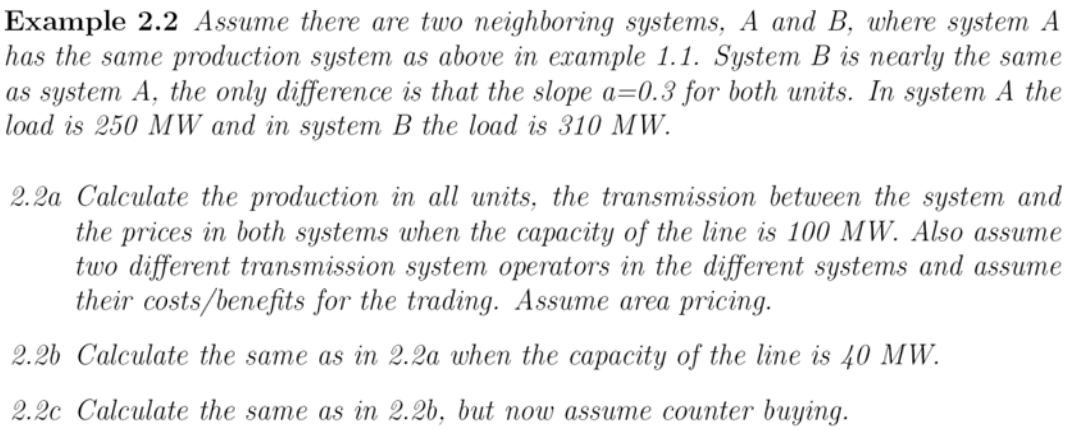
Market coupling

- This method starts with several individual markets, but if it is profitable to trade between the markets, then trading is allowed up to ATC

=>

- Same results concerning production-transmission utilization.

Transmission limits between 2 areas



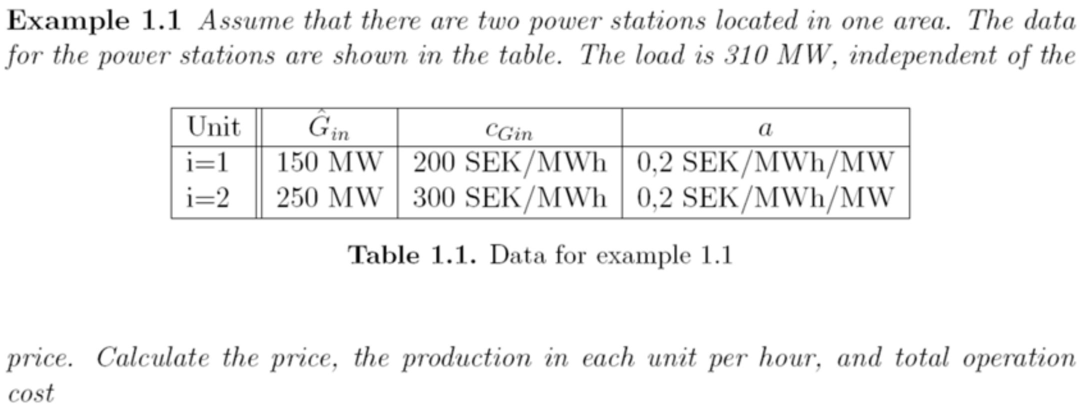
Example 2.2 *Assume there are two neighboring systems, A and B, where system A has the same production system as above in example 1.1. System B is nearly the same as system A, the only difference is that the slope a=0.3 for both units. In system A the load is 250 MW and in system B the load is 310 MW.*

2.2a *Calculate the production in all units, the transmission between the system and the prices in both systems when the capacity of the line is 100 MW. Also assume two different transmission system operators in the different systems and assume their costs/benefits for the trading. Assume area pricing.*

2.2b *Calculate the same as in 2.2a when the capacity of the line is 40 MW.*

2.2c *Calculate the same as in 2.2b, but now assume counter buying.*

Example 1.1

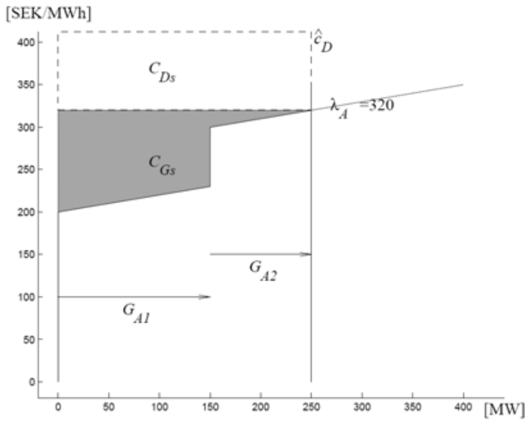


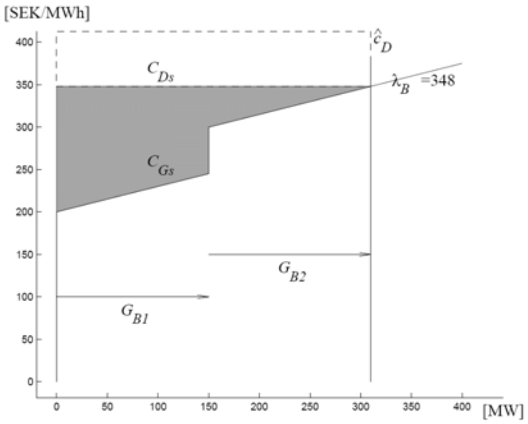
Example 1.1 *Assume that there are two power stations located in one area. The data for the power stations are shown in the table. The load is 310 MW, independent of the price. Calculate the price, the production in each unit per hour, and total operation cost*

Unit	$\hat{G}_{in}$	c_{Gin}	a
i=1	150 MW	200 SEK/MWh	0,2 SEK/MWh/MW
i=2	250 MW	300 SEK/MWh	0,2 SEK/MWh/MW

Table 1.1. Data for example 1.1

Graphic solution to example 2.2a: First assume that there is no transmission line between the two systems. Then do as in example 1.1, i.e., draw the supply curves, where the units are placed after each other according to increasing marginal operation cost. This is shown in figure 2.6. Then also draw the load curves as a vertical lines.



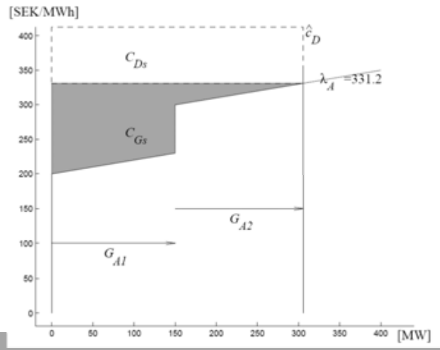


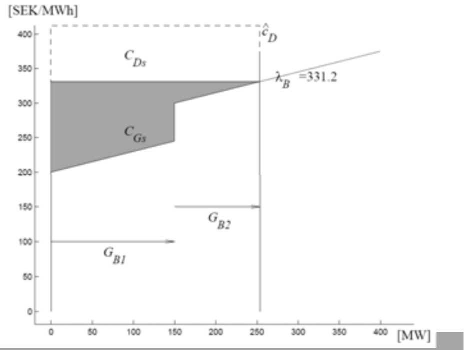
Price calculation with no transmission

$$\begin{aligned}\lambda_A &= c_A(G_2) = 300 + 0.2 \cdot 100 = 320 \quad \text{SEK/MWh}\\ \lambda_B &= c_B(G_2) = 300 + 0.3 \cdot 160 = 348 \quad \text{SEK/MWh}\end{aligned}$$

Price calculation with transmission

- Profitable to increase trade until the prices are equal. This happens when:
  $$\lambda_A = 300 + 0.2 \cdot (100 + P_{AB}) = 300 + 0.3 \cdot (160 - P_{AB}) = \lambda_B$$
- => P_{AB}= 56 MW $\lambda_A = \lambda_B$ =331.2 SEK/MWh.





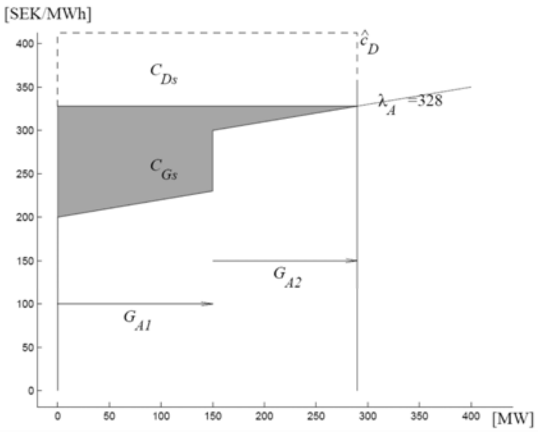
Example 2.2a with optimization

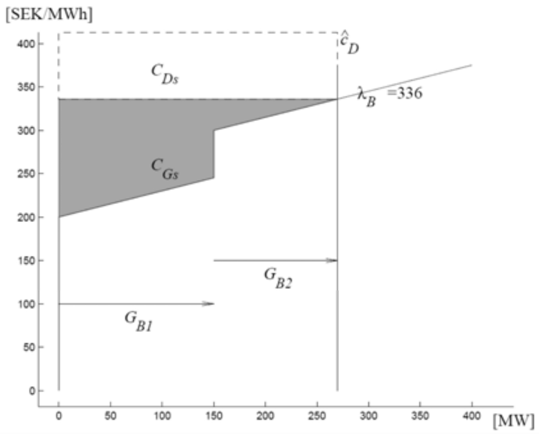
$$\max Z = C_D - \sum C_{Gi} \quad = \quad \hat{c}_D(D_1 + D_2) - \sum_{k=A}^{B}\sum_{i=1}^{2}\left(c_{Gi}G_i(k) + \frac{a(i,k)}{2}G_i(k)^2\right)$$

$$\begin{aligned} &\text{when} &\\ 0 \leq\ & G_1(k) \leq 150 &\\ 0 \leq\ & G_2(k) \leq 250 &\\ G_1(A) + G_2(A) &= D_A + P_{AB} &\\ G_1(B) + G_2(B) &= D_B - P_{AB} &\\ P_{AB} &= 0 &\end{aligned}$$

Example 2.2b, P_{AB} = 40 MW

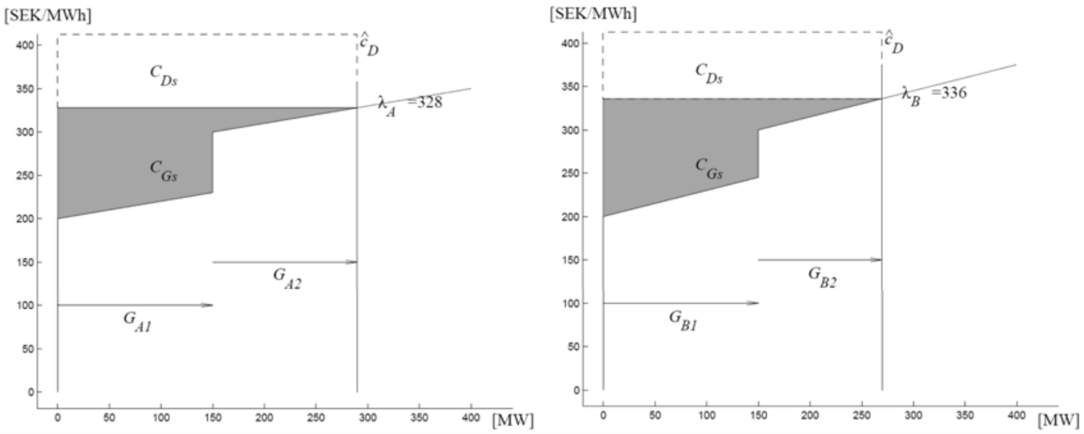
- Assume area pricing:
- 40 MW = Capacity < Need = 56 MW



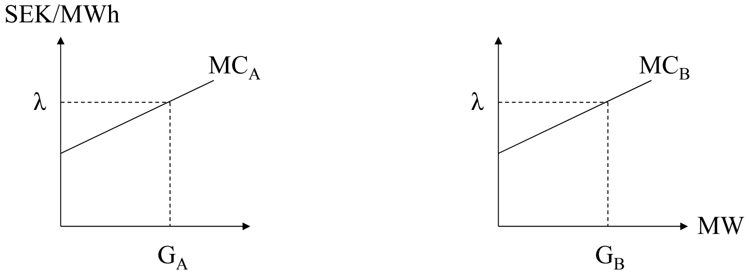


Example 2.2c, P_{AB} = 40 MW

- Assume counter buying (same price in both areas
- 40 MW = Capacity < Need = 56 MW
- Purchase cost = 16(336-328)=128 SEK/h

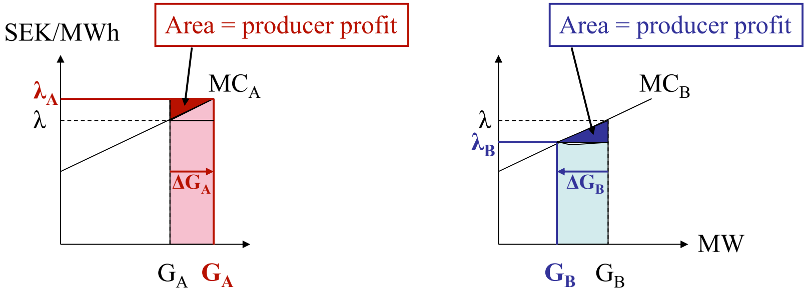


Counter trading - 1



- Assume a situation where prices are calculated without considering transmission congestions. This means that the prices will be the same everywhere: $\lambda_A = \lambda_B = \lambda$
- Assume now that too much power flows from B➔A. Then the TSO has to buy power in A and sell in B.

Counter trading - 2



- Assume now that too much power flows from B➔A. Then the TSO has to buy power **ΔG_A** in A and sell **ΔG_B** in B.
- **ΔG_A = ΔG_B**
- Area A: TSO pays **$\lambda_A \cdot \Delta G_A$** for buying of extra power
- Area B: TSO receives **$\lambda_B \cdot \Delta G_B$** for selling of excess power

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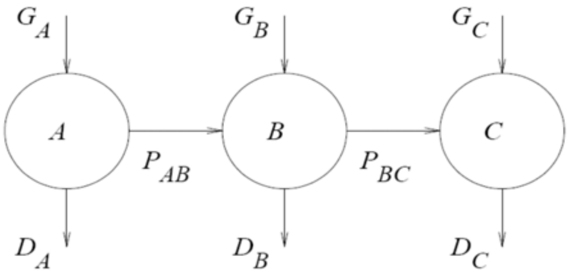
Example 2.3-1

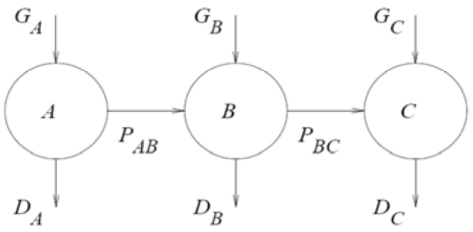
Example 2.3 *Assume there are three neighboring systems, A, B and C where there are connections between A-B and B-C but no connection between A and C. In each system there is one* $\hat{G}_A = \hat{G}_B = \hat{G}_C = 300$ *MW unit with* $c = 250$ *SEK/MWh and* $a = 0,3$ *SEK/MWh/MW. In system A the load is* $D_A = 70$ *MW, in system B* $D_B = 240$ *MW, and in system C it is* $D_C = 140$ *MW. The system is shown in figure 2.9*



Example 2.3-2

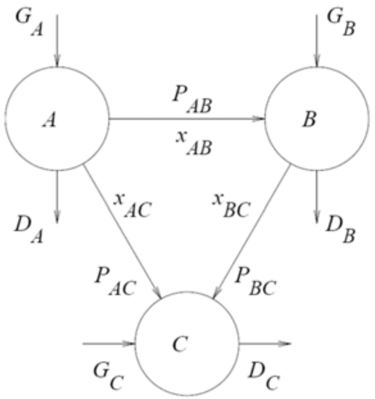
2.3d Assume line limits of 40 MW. Move 50 MW of load from B to C load so $D_B = D_C$=190 MW. Now assume that there is one TSO for area A+B and one for area C. Assume counter trading within each TSO area and price areas between them. Calculate area prices, total production cost and costs/benefits for the TSO:s for the congestion handling.

2.3e The same system as the previous, but now the TSO in area A+B limits the export to area C in order to minimize their costs for counter trading. Calculate area prices, total production cost and costs/benefits for the TSO:s for the congestion handling.



Transmission in meshed networks - 1

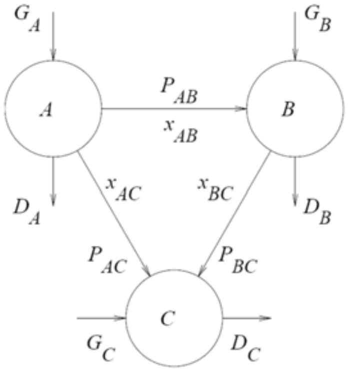
- "Meshed" means that there are possible loops in the system.



Transmission in meshed networks - 2

Two basic modeling approaches:

1. **AC-grid modeling**. This means that physical laws concerning flows in parallel lines are considered
2. **Pure area modeling**. This means that it is assumed that flows on parallel paths can be controlled individually.



Modeling of meshed grid - 1
DC load flow

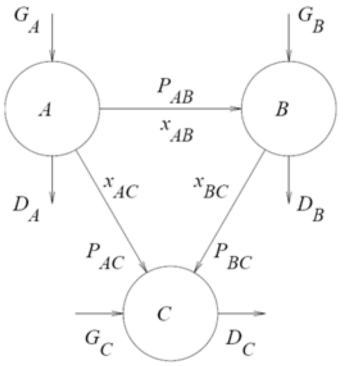
- Power flow on a transmission line:

$$P_{12} = \frac{U_1 U_2}{X_{12}} \sin(\delta_1 - \delta_2)$$

- Assumption of constant voltages (U=1) and small angle shifts (δ):

$$B_{12} = 1/X_{12}$$

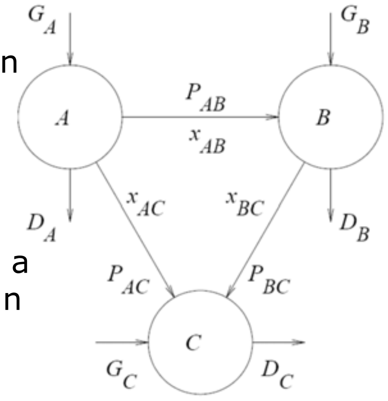
$$P_{12} = B_{12}(\delta_1 - \delta_2)$$



Modeling of meshed grid - 2
DC load flow

- Power flow division between parallel lines depends on relation between X.
- **Method:** Calculate transmission on all lines as a function of net production in each node.

$$\mathbf{P}_{\text{trans}} = \mathbf{M}\mathbf{P}_{\text{node}}$$



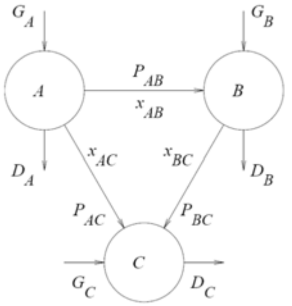
Modeling of meshed grid - 3
DC load flow

- Formulate system transmission for $P_{12} = B_{12}(\delta_1 - \delta_2)$

$$\mathbf{P}_{\text{trans}} = \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} B_{AB} & -B_{AB} & 0 \\ B_{AC} & 0 & -B_{AC} \\ 0 & B_{BC} & -B_{BC} \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_B \\ \delta_C \end{bmatrix} = \mathbf{B}'\mathbf{\Delta}'$$

- Set $\delta_A = 0$ =>

$$\mathbf{P}_{\text{trans}} = \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} -B_{AB} & 0 \\ 0 & -B_{AC} \\ B_{BC} & -B_{BC} \end{bmatrix} \begin{bmatrix} \delta_B \\ \delta_C \end{bmatrix} = \mathbf{B}\mathbf{\Delta}$$



Modeling of meshed grid - 4
DC load flow

- Net production is always = Net export in all nodes

$$\mathbf{P}'_{\text{node}} = \begin{bmatrix} G_A - D_A \\ G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} P_{AB} + P_{AC} \\ -P_{AB} + P_{BC} \\ -P_{AC} - P_{BC} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \mathbf{C}'\mathbf{P}_{\text{trans}}$$

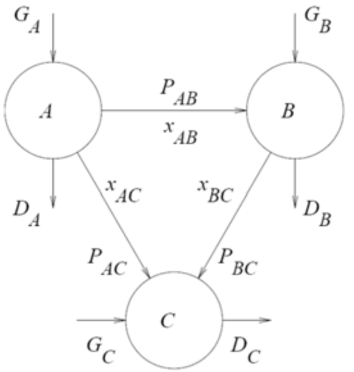
- Not full rang of matrix =>

$$\mathbf{P}_{\text{node}} = \begin{bmatrix} G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} -P_{AB} + P_{BC} \\ -P_{AC} - P_{BC} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \mathbf{C}\mathbf{P}_{\text{trans}}$$

Modeling of meshed grid - 5
DC load flow

- **Method:** Calculate transmission on all lines as a function of net production in each node.

$$\begin{aligned} \mathbf{P}_{\text{trans}} &= \mathbf{M}\mathbf{P}_{\text{node}} \\ \mathbf{P}_{\text{node}} &= \mathbf{C}\mathbf{B}\mathbf{\Delta} \\ &\Rightarrow \\ \mathbf{\Delta} &= (\mathbf{C}\mathbf{B})^{-1}\mathbf{P}_{\text{node}} \\ &\Rightarrow \\ \mathbf{M} &= \mathbf{B}(\mathbf{C}\mathbf{B})^{-1} \end{aligned}$$



Example 2.4

Example 2.4 *Assume there are three neighboring systems, A, B and C where there are connections between A-B, B-C and A-C. In each system there is one* $\hat{G}_A = \hat{G}_B = \hat{G}_C$*=300 MW unit with* $c = 250$ *SEK/MWh and* $a = 0,3$ *SEK/MWh/MW. In system A the load is* D_A*=70 MW, in system B* D_B*=240 MW, and in system C it is* D_C*=140 MW. The system is shown in figure 2.11. The impedances are* $x_{AB} = x_{BC} = 1$*, and* $x_{AC} = 2$ *(i.e. this line is double as long).*

2.4a Calculate the production in all units, the transmission between the systems and the prices in all systems when the capacity on the lines is 100 MW. Assume area pricing.

2.4b Calculate the same as in 2.4a, but now assume $x_{AB} = x_{BC} = 1$ *and* $x_{AC} = \infty$*. This means in reality the same case as studied in example 2.3*

2.4c Calculate the same as in 2.4a when the capacity of the line is 40 MW.

Example 2.4

- **Method:** Calculate transmission on all lines as a function of net production in each node.

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} -0.75 & -0.5 \\ -0.25 & -0.5 \\ 0.25 & -0.5 \end{bmatrix} \begin{bmatrix} G_B - D_B \\ G_C - D_C \end{bmatrix}$$

Example 2.4a

Solution to example 2.4a With an assumption of full transmission between all three areas, this is the same as only one area with three units and a total load of $D = D_A + D_B + D_C$= 450 MW. Since there are three units with the same data, then these three units will share the production equally, i.e.

$$\begin{aligned} G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\ &\Rightarrow \\ \lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \\ \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} &= \begin{bmatrix} -0.75 & -0.5 \\ -0.25 & -0.5 \\ 0.25 & -0.5 \end{bmatrix} \begin{bmatrix} 150 - 240 \\ 150 - 140 \end{bmatrix} = \begin{bmatrix} 62.5 \\ 17.5 \\ -27.5 \end{bmatrix} \text{ MW} \end{aligned}$$

Example 2.4a

use of optimization

$$\max Z = C_D - \sum C_{Gi} \quad = \quad \hat{c}_D(D_A + D_B + D_C) - \sum_{k=A}^{C} \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right)$$

when

$$\begin{aligned} 0 \leq \quad & G_k && \leq 250, \quad k \in [A, C] \\ P_{AB} + P_{AC} \quad & = && G_A - D_A \\ P_{AB} \quad & = && -0.75(G_B - D_B) - 0.5(G_C - D_C) \\ P_{AC} \quad & = && -0.25(G_B - D_B) - 0.5(G_C - D_C) \\ P_{BC} \quad & = && 0.25(G_B - D_B) - 0.5(G_C - D_C) \\ \underline{P}_{AB} \leq \quad & P_{AB} && \leq \hat{P}_{AB} \\ \underline{P}_{BC} \leq \quad & P_{BC} && \leq \hat{P}_{BC} \\ \underline{P}_{AC} \leq \quad & P_{AC} && \leq \hat{P}_{AC} \end{aligned}$$

Example 2.4b

Solution to example 2.4b As in the solution of example 2.4a, we first assume no limits on transmission between all three areas, i.e., one area with three units and a total load of $D = D_A + D_B + D_C$= 450 MW. Since there are three units with the same data, then these three units will share the production equally, i.e.

$$\begin{aligned} G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\ &\Rightarrow \\ \lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \end{aligned}$$

With $x_{AC} = \infty$, the **B**-matrix will change to

$$\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix} \Rightarrow \mathbf{B}(\mathbf{CB})^{-1} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow$$

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 150 - 240 \\ 150 - 140 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ -10 \end{bmatrix} \text{ MW}$$

Example 2.4c

Solution to example 2.4c In this example the restrictions are set to 40 MW. If there are no restrictions, then the transmission on the line A-B is $P_{AB} = 62.5$ MW as shown above in example 2.4a. The transmission on the other lines are below 40 MW in that example. This means that the flow on the line A-B now will be reduced to 40 MW. This means

$$\begin{aligned} P_{AB} &= -0.75(G_B - D_B) - 0.5(G_C - D_C) \\ 40 &= -0.75(G_B - 240) - 0.5(G_C - 140) \\ &\Rightarrow \\ G_C &= 420 - 1.5G_B \end{aligned}$$

Total production is equal to total consumption:

$$\begin{aligned} G_A + G_B + G_C &= D_A + D_B + D_C \\ G_A + G_B + 420 - 1.5G_B &= 70 + 240 + 140 \\ G_A &= 0.5G_B + 30 \end{aligned}$$

Example 2.4c

The total operation cost can now be calculated as a function of G_B

$$\begin{aligned} C_{Gtot} &= \sum_{k=A}^{C} \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right) \\ &= 250(G_A + G_B + G_C) + (0.5G_B + 30)^2 + G_B^2 + (420 - 1.5G_B)^2 \\ &= 250 \cdot 450 + 3.5G_B^2 - 1230G_B + 177300 \end{aligned}$$

Minimum cost is obtained when the derivative of total cost is zero

$$\begin{aligned} \frac{dC_{Gtot}}{dG_B} &= 0 = 7G_B - 1230 \\ &\Rightarrow \\ G_B &= 1230/7 = 175.71 \text{ MW} \\ G_C &= 420 - 1.5G_B = 156.43 \text{ MW} \\ G_A &= 0.5G_B + 30 = 117.86 \text{ MW} \\ P_{AC} &= -0.25(G_B - D_B) - 0.5(G_C - D_C) = 7.86 \text{ MW} \\ P_{BC} &= 0.25(G_B - D_B) - 0.5(G_C - D_C) = -24.29 \text{ MW} \end{aligned}$$

Example 2.4c

- Concerning prices:
- Prices equal to marginal cost =>

$$\lambda_A = c_{GA} + a_A G_A = 250 + 0.3 \cdot 175.71 = 302.71$$

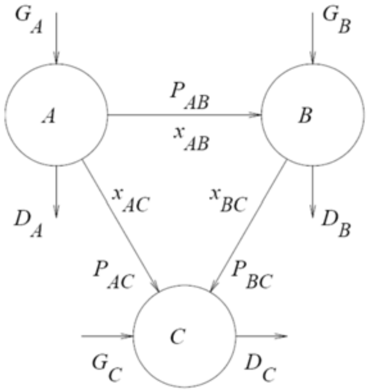
$$\lambda_B = c_{GB} + a_B G_B = 250 + 0.3 \cdot 156.43 = 296.93$$

$$\lambda_C = c_{GC} + a_C G_C = 250 + 0.3 \cdot 117.86 = 285.36$$

- Can be compared with prices without limits.
- Trading methods, market splitting/ counter buying, can be evaluated

Modeling of meshed grid - 2
Multi-area method

- It is assumed that power flows between the different areas are fully controllable, independent of each other

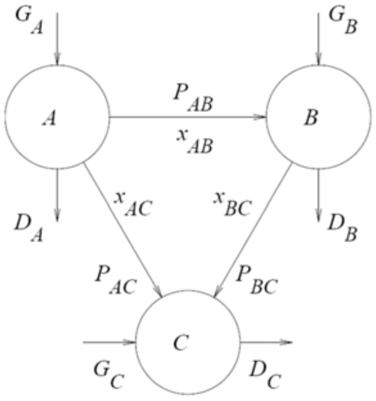


Modeling of meshed grid - 2
Multi-area method

- Power flow division between parallel lines depends **not** on relation between X.

- **Method:** Calculate transmission on all lines as a function of net production in each node.

$$\mathbf{P}_{\text{trans}} = \mathbf{M}\mathbf{P}_{\text{node}}$$



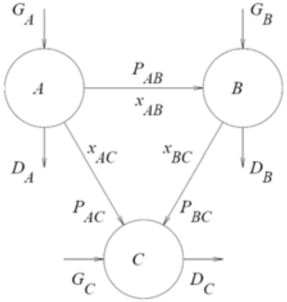
Modeling of meshed grid - 3
Multi-area method

- Formulate system transmission for

$$\begin{bmatrix} K \\ G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix}$$

- Inversion of the matrix =>

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} K \\ G_B - D_B \\ G_C - D_C \end{bmatrix}$$

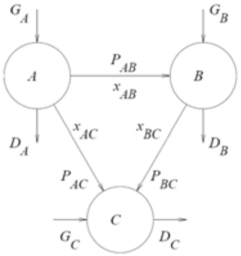


Example 2.5

Example 2.5 *This is the same example as 2.4, but here the multi-area modeling is used instead of the DC load flow. Assume there are three neighboring systems, A, B and C where there are connections between A-B, B-C and A-C. In each system there is one* $\hat{G}_A = \hat{G}_B = \hat{G}_C$*=300 MW unit with* c = 250 *SEK/MWh and* a = 0,3 *SEK/MWh/MW. In system A the load is* D_A*=70 MW, in system B* D_B*=240 MW, and in system C it is* D_C*=140 MW. The system is shown in figure 2.11.*

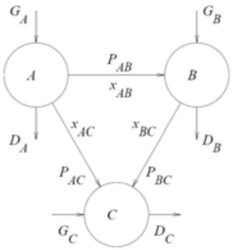
2.5a Calculate the production in all units, the transmission between the systems and the prices in all systems when the capacity on the lines is 100 MW. Assume area pricing.

2.5b Calculate the same as in 2.4a when the capacity of the line is 40 MW.



Example 2.5a solution

Solution to example 2.5a With an assumption of full transmission between all three areas, this is the same as only one area with three units and a total load of $D = D_A + D_B + D_C = 450$ MW. Since there are three units with the same data, then these three units will share the production equally, i.e.



$$\begin{aligned} G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\ &\Rightarrow \\ \lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \end{aligned}$$

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} K \\ 150 - 240 \\ 150 - 140 \end{bmatrix} = (K = 0) \begin{bmatrix} 10 \\ 70 \\ -80 \end{bmatrix}$$

This implies that the transmission is within the borders of the capacity for each line. There are also other possible solutions (all fulfilling area balances and transmission limits)

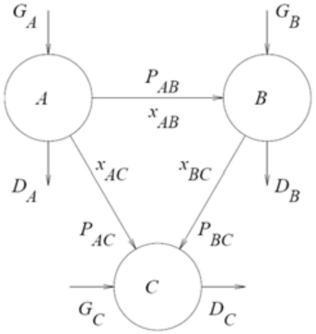
$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = (K = 10) \begin{bmatrix} 20 \\ 60 \\ -70 \end{bmatrix} \quad (K = 30) \begin{bmatrix} 40 \\ 40 \\ -50 \end{bmatrix} \quad (K = -20) \begin{bmatrix} -10 \\ 90 \\ -100 \end{bmatrix}$$

Example 2.5a
Optimization formulation

$$\max Z = C_D - \sum C_{Gi} = \hat{c}_D(D_A + D_B + D_C) - \sum_{k=A}^{C} \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right)$$

when

$$\begin{aligned} 0 \leq \quad & G_k \leq 250, \quad k \in [A, C] \\ G_A - D_A &= P_{AB} + P_{AC} \\ G_B - D_B &= -P_{AB} + P_{BC} \\ G_C - D_C &= -P_{AC} + P_{BC} \\ \underline{P}_{AB} \leq \quad & P_{AB} \leq \hat{P}_{AB} \\ \underline{P}_{BC} \leq \quad & P_{BC} \leq \hat{P}_{BC} \\ \underline{P}_{AC} \leq \quad & P_{AC} \leq \hat{P}_{AC} \end{aligned}$$



Example 2.5b – Max = 40 MW

- No restrictions =>

$$\begin{bmatrix} G_A - D_A \\ G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} 150 - 70 \\ 150 - 240 \\ 150 - 140 \end{bmatrix} = \begin{bmatrix} 80 \\ -90 \\ 10 \end{bmatrix}$$

- Maximum import to B = 80 MW =>

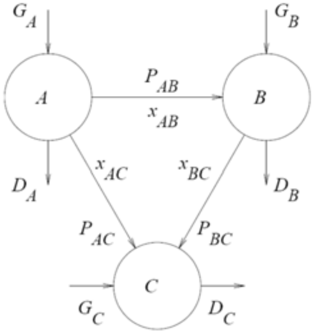
$G_B = 150 + 10 = 160$ MW.

- => decrease in A and C of 10 MW
- Same marginal cost in A and C => split decrease equally:

$$\begin{aligned} G_A &= 150 - 5 = 145 \text{ MW} \\ G_C &= 150 - 5 = 145 \text{ MW} \\ P_{AC} &= G_A - D_A - P_{AB} = 145 - 70 - 40 = 35 \text{ MW} \end{aligned}$$

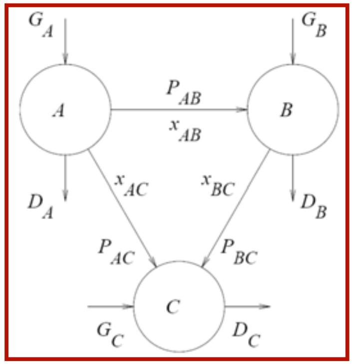
- =>

$$\begin{aligned} \lambda_B &= 250 + 0.3 \cdot 160 = 298 \text{ SEK/MWh} \\ \lambda_A = \lambda_C &= 250 + 0.3 \cdot 145 = 293.5 \text{ SEK/MWh} \end{aligned}$$



Method comparison

	DC load flow, 2.4c	Area model,2.5b
P_{AB}	40	40
P_{AC}	7.86	35
P_{BC}	-24.29	-40
λ_A	285.36	293.5
λ_B	302.71	298
λ_C	296.03	293.5
Tot cost	122885	122647



Pricing methods in deregulated markets

- Node pricing
  - – with consideration of losses
  - – without consideration of losses
- Area pricing

Assignment 3 (2011)

Part II: Transmission limits (max. 4 bonus points)

In this part of the assignment we introduce the handling of transmission limits, and the economy of system operators. Assume:

- consumers are not price sensitive,
- no emission costs,
- marginal pricing,
- perfect competition,
- operating cost of hydro power is equal to the water value,
- for hydro units: inflow = installed capacity.

Assignment 3 (2011)

Q8: (use Matlab) Calculate the transmission between the areas and the price in the four areas when the transmission system is modelled as an

a) AC system, i.e., consider DC load flow,

b) fully controllable transmission with transmission limitations, i.e. area pricing. Explain the difference in result.

c) Assume that you invest in the power system to make the transmission more controllable (go from a to b). How large is the change in economic value of this for consumers and producers?

Q9: (use Matlab) As in Q8b assume a fully controllable transmission system (area pricing) with transmission limitations.

a) Formulate the mathematical optimization problem which gives the area prices in Q8b.

b) Assume equal splitting of the profit/cost if two TSOs are involved in the trading. Calculate the profit for the two TSOs.

Assignment 3 (2011)

Q10 (use Matlab) Again, assume a fully controllable transmission system. Now, assume that an explicit auction is performed where 50 % of the capacity of all lines is already pre-scheduled. The direction is the same as in the solution of Q9.

a) It is first assumed that after the explicit auction is performed it is possible for the TSOs to trade between the different areas. Calculate the consumer prices in the four areas and motivate your answer.

b) What is the consumer prices in the four areas if the TSO:s are **not** allowed to trade after the explicit auction is performed?

Q11 (use Matlab) Again, assume a fully controllable transmission system. Assume that the counter trading method is used for treatment of all bottlenecks.

a) Calculate the consumer prices in the four areas.

b) Assume equal splitting of the profit/cost if two TSOs are involved in the trading. Calculate the extra costs/income for the system operators for the handling of the bottlenecks.

Q12 (by hand, no Matlab) Now, sum up the results: Comment and explain the equalities and differences between the three methods of treatment of bottlenecks. Calculate the difference in total surplus of the methods.

Questions on transmission limits

1. With two parallel overhead lines where the limits are set by thermal limits, it is possible to overload one of them if the other one is below its rating.
2. If a transmission line has a limit set by transient stability, then overloading will directly lead to an interruption.
3. A voltage stability limit is never set for one single line, but only for corridors with many parallel lines.
4. ATC = Available Transmission Capacity from A to B can sometimes be larger than the physical limit.

Questions on transmission modeling

1. “DC load flow” assumes that the transmission between the different areas/nodes can be fully controlled.
2. “Area modeling” can result in different prices even if there are no congestions.

Questions on congestion management

1. “DC load flow” assumes that the transmission between the different areas/nodes can be fully controlled.
2. “Area modeling” can result in different prices even if there are no congestions.
3. ”Counter trading” can lead to an income for the system operator
4. ”Explicit auction” leads to transmission from a low price area to a high price area
5. ”Implicit auction” leads to transmission from a high price area to a low price area