AUTOMATIC CONTROL

KTH

Nonlinear Control, EL2620 / 2E1262

Exam 14.00–19.00 December 11, 2012

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text) or equivalent basic control book if approved by the examiner beforehand. Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2013-01-10 at STEX, Studerandeexpeditionen, Osquldasv. 10.

Responsible: Elling W. Jacobsen 070 372 22 44

Good Luck!

1. Consider the nonlinear dynamical system

$$\begin{aligned}\dot{x}_1 &= -\frac{6x_1(t)}{(1+x_1(t)^2)^2} + 3x_2(t) \\ \dot{x}_2 &= -3\frac{x_1(t)+x_2(t)}{(1+x_1(t)^2)^2}\end{aligned}$$

(a) Determine the equilibrium point(s) of the system (2p)

(b) Linearize the system around the equilibrium point(s) and determine the phase plane characteristics (2p)

(b) By using the candidate Lyapunov function

$$V(x_1, x_2) = x_1^2/(1+x_1^2) + x_2^2$$

can you prove that the equlibrium is stable? What about asymptotically stable? What about globally asymptotically stable? (6p)

2. (a) We shall consider feedback stabilization of the 2nd order system

$$\begin{aligned}\dot{x}_1 &= 2x_1x_2\\ \dot{x}_2 &= 2x_1^4 + 3u\end{aligned}$$

Employ Lyapunov based methods to design a controller which makes the origin $(x_1, x_2) = (0, 0)$ a globally asymptotically stable equilibrium.

Hint: you may start with the control Lyapunov function $V = x_1^2 + x_2^2$. (4p)

(b) A system is described by the model

$$\begin{aligned}\dot{x} &= y(t)\\ \dot{y} &= -x(t) + y(t) - y(t)x(t)^2 - 4y(t)^3\end{aligned}$$

Employ Lyapunov based methods to show that the system has a stable limit cycle. What can you say about the region of attraction of the limit cycle?

Hint: You may start with the function $V = x^2 + y^2$. (6p)

3. (a) A mechanical systems is described by the nonlinear model

$$\begin{aligned}\dot{x}_1 &= \cos(x_1(t)) - x_2(t)\\ \dot{x}_2 &= -x_1(t)x_2(t) + x_2(t)^3 + u(t)\end{aligned}$$

We shall consider use of feedback to linearize the input-output behavior

(i) Assume the output $y(t) = x_2(t)$. Determine a control law $u = c(x, v)$ which renders the system linear from $v(t)$ to the output $y(t)$. Discuss possible problems with zero dynamics. (3p)

(ii) Consider now $y(t) = x_1(t)$ and determine a control law $u = c(x, v)$ which renders the system linear from $v(t)$ to $y(t)$. Will there be problems with zero dynamics in this case? (2p)

(b) We want to obtain a globally stable equilibrium at $(0, 0)$ for the system

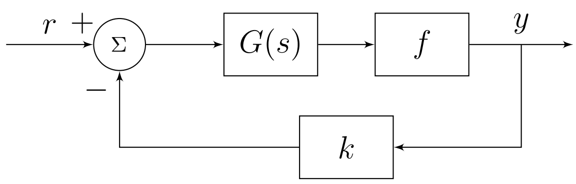
$$\begin{aligned}\dot{x}_1 &= x_1 + x_2\\ \dot{x}_2 &= x_1^2 + x_2 + u\end{aligned}$$

For this purpose we shall employ sliding mode control. Employ the sliding manifold

$$S = \{(x_1, x_2) | \sigma = x_1 + ax_2 = 0\}$$

and determine appropriate values of a, the controller that makes the sliding manifold globally attracting in finite time and the equivalent control that keeps the state on S. (5p)

4. Consider the nonlinear feedback system in the figure below



The linear time-invariant system is given by

$$G(s) = \frac{\sqrt{2}}{(s^2+s+1)(s+1)}$$

The frequency response $G(j\omega)$ is shown in Fig. 1.

We want do determine for which values of the controller gain k the closed loop is stable. The problem is that the odd static nonlinearity f is unknown.

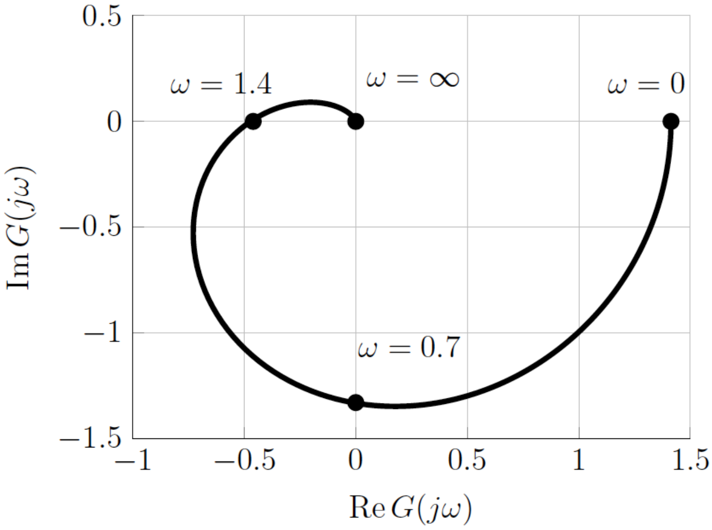


Figure 1: Frequency response $G(j\omega)$

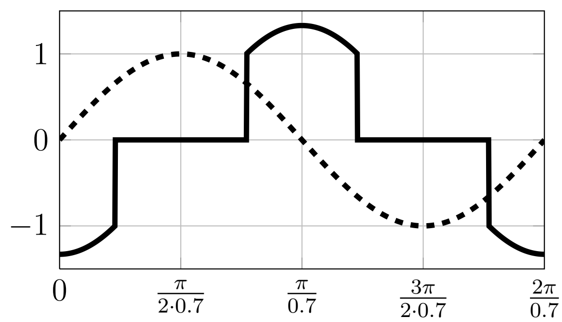


Figure 2: One period of the output y (——) and the input r (- - -) during the experiment.

(a) To determine the odd static nonlinearity f we set $k = 0$ (no feedback) and let $r(t)$ be a sinus wave with amplitude 1 and frequency $\omega = 0.7$ rad/s. After the transients have died out the output y is measured. The output y and the input r is shown during one period in Fig. 2. Sketch the static nonlinearity f. (3p)

(b) Use the small gain theorem to determine values of the gain k that guarantees closed-loop stability. (If you could not find the nonlinearity in (a), use the deadzone in Fig. 3.) (3p)

(c) Use the circle criterion to determine values of the gain k that guarantees closed-loop stability. (Hint: consider both positive and negative values of k.) (4p)

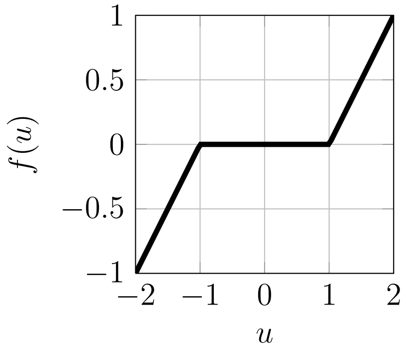


Figure 3: Deadzone nonlinearity

5. A supermarket has problems with long lines at the cashier and one problem appears to be that the conveyor belts are too slow. The manager first makes the obvious choice of increasing the belt speed, but the problem is that now bottles tip over and break and this makes things even worse. The shop manager therefore hires a student who has taken a nonlinear control course to optimize the conveyor belt speed. The conveyor belt is torque controlled implying that it can be accelerated directly by the control signal u according to

$$\ddot{x} = u$$

where x is the position. In order to avoid bottles falling over, experiments are performed and they show that the acceleration must be such that $|u| < 5$. In addition, the laser scanner can not read the EAN price tags at speeds above 0.2 m/s.

   (a) Formulate the optimization problem of moving a bottle from $x(0) = \dot{x}(0) = 0$ to $x(T) = 0.5m$, $\dot{x}(T) = 0.2m/s$ as fast as possible without tipping the bottle. (2p)

   (b) Start by using some heuristic reasoning: What would the control signal look like if we were to move the bottle as fast as possible without considering limitations on the end speed, and how long time would it take to move it $0.5m$? What if you want to move it as fast as possible with the constraint that it should stop at $0.5m$? (3p)

   (c) Now solve the optimization problem in (a) to determine the optimal u and corresponding minimum T. *Hint: consider the normal case with* $n_0 = 1$. (5p)