AUTOMATIC CONTROL

Department of Signals, Sensors & Systems, KTH

Nonlinear Control, 2E1262

Exam 14.00–19.00 Dec 17, 2004

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text). Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number(*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five 10 credit problems.

Grading:

Grade 3: ≥ 23

Grade 4: ≥ 33

Grade 5: ≥ 43

Results:

The results will be posted within 2005-01-12 on the department's board, Osquldas väg 10, second floor. **If you want your result emailed, please state this and include your email address.**

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Good Luck and Merry Christmas!

1.

(a) [2p] A scalar nonlinear system is described by the differential equation

$$\dot{x}(t) = f(x(t)), \quad x \in \mathbb{R}$$

where the nonlinear function satisfies

$$\begin{aligned} f(0) &= 0 \\ f(x) &< 0, \quad x > 0 \\ f(x) &> 0, \quad x < 0 \end{aligned}$$

Show that $x = 0$ is a globally asymptotically stable equilibrium point.

(b) [2p] Consider the scalar differential equation

$$\dot{x}(t) = 1 + x^2(t), \quad x(0) = 0$$

Show that $x(t) = \tan(t)$ is a solution. What happens as $t \to \pi/2$?

(c) [6p] It is sometimes possible to do exact input/output feedback linearization. Consider the system

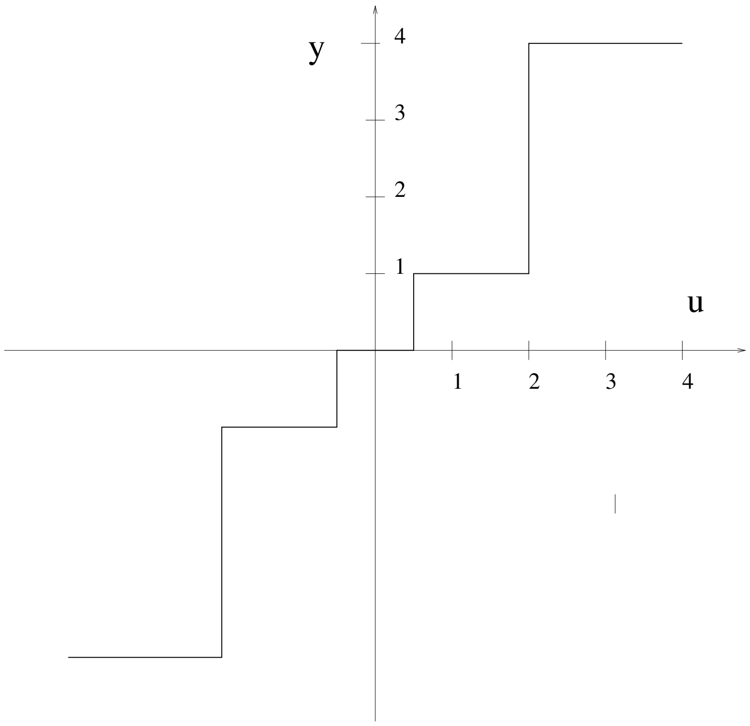
$$\begin{aligned} \dot{x}_1(t) &= u(t) \\ \dot{x}_2(t) &= 2 + \sin x_1(t) \\ y(t) &= x_2(t) \end{aligned}$$

Design a control law $u(t) = h(x(t), r(t))$ such that the relation between reference signal $r(t)$ and output signal $y(t)$ becomes

$$\ddot{y}(t) + 2\dot{y}(t) + y(t) = r(t)$$

(*Hint:* Calculate $\dot{y}(t)$ and $\ddot{y}(t)$ and then design $u(t)$ to give the specified reference to output differential equation.)

2. [10p] Consider a logarithmic quantizer with two levels:



This means that the output equals

$$y = \begin{cases} 0, & -0.5 \le u \le 0.5 \\ 1, & 0.5 < u \le 2 \\ -1, & -2 \le u < -0.5 \\ 4, & u > 2 \\ -4, & u < -2 \end{cases}$$

Calculate the describing function $N(A)$ for this quantizer.

3.

A plant can be described by

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t) - x_2(t) + x_3(t) + u(t)\\ \dot{x}_2(t) &= x_1(t)\\ \dot{x}_3(t) &= x_2(t)\end{aligned}$$

and let the control law be

$$u(t) = x_1(t) - x_3(t) - (2x_1(t) + x_3(t))^3$$

(a) [6p] Show that $x = 0$ is a global asymptotically stable equilibrium point for the closed loop system.
(*Hint:* Use

$$V(x) = x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2 + x_1x_3 \qquad)$$

(b) [4p] In order to evaluate the gain margin of the closed loop system, study the control law

$$u(t) = A_m(x_1(t) - x_3(t) - (2x_1(t) + x_3(t))^3)$$

where $A_m \geq 0$ is a gain. Decide for which A_m the closed loop system is stable or unstable.
(*Hint:* Study the stability properties of the linearized closed loop system.)

4.

Given the double integrator system

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= u(t),\end{aligned}$$

we have seen that the time optimal control is "bang-bang" type control. The task is now to show that this control can be written as

$$u(t) = -\text{sign}\{\sigma(x(t))\}$$

(a) [3p] Draw the phase plane diagram for the case $u = 1$, i.e. for the differential equation

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= 1\end{aligned}$$

Next, draw the phase plane diagram for the case $u = -1$, i.e. for the differential equation

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= -1\end{aligned}$$

Determine and mark the trajectories which goes through $x = 0$.

(b) [1p] From the optimal control theory we know that the time optimal control switches sign at most once. Combine the phase plane diagrams from (a) to sketch the control strategy which brings the state of the double integrator to the origin by switching control at most once.

(c) [3p] Show that the optimal control can be written in feedback form

$$u(t) = -\text{sign}\{\sigma(x(t))\}, \quad \sigma(x) = x_1 + \text{sign}\{x_2\}\frac{x_2^2}{2}$$

(d) [3p] Calculate the time it takes to bring the state from $x(0) = (2,\, -2)^T$ to zero using the optimal control strategy. Then calculate how long time it takes to bring it from $x(0) = (2,\, -2-\epsilon)^T$ to zero, where $\epsilon > 0$.

5. [10p] A nonlinear system is given by the state space model

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= -2x_2(t) - f(x_1(t))\end{aligned}$$

The function $f(\cdot)$ satisfies

$$\frac{R}{R+1} \leq \frac{f(y)}{y} \leq \frac{R}{R-1}, \qquad R > 1$$

Determine condition on $R > 1$, which guarantees that the system is stable.