AUTOMATIC CONTROL

Department of Signals, Sensors & Systems, KTH

Nonlinear Control, 2E1262

Exam 14.00–19.00 December 15, 2005

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text). Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number(*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five 10 credit problems.

Grading:

Grade 3: ≥ 23

Grade 4: ≥ 33

Grade 5: ≥ 43

Results:

The results will be posted within 2006-01-13 on the department's board, Osquldas väg 10, second floor. **If you want your result emailed, please state this and include your email address.**

Responsible: Bo Wahlberg 790 7242

Good Luck!

1.

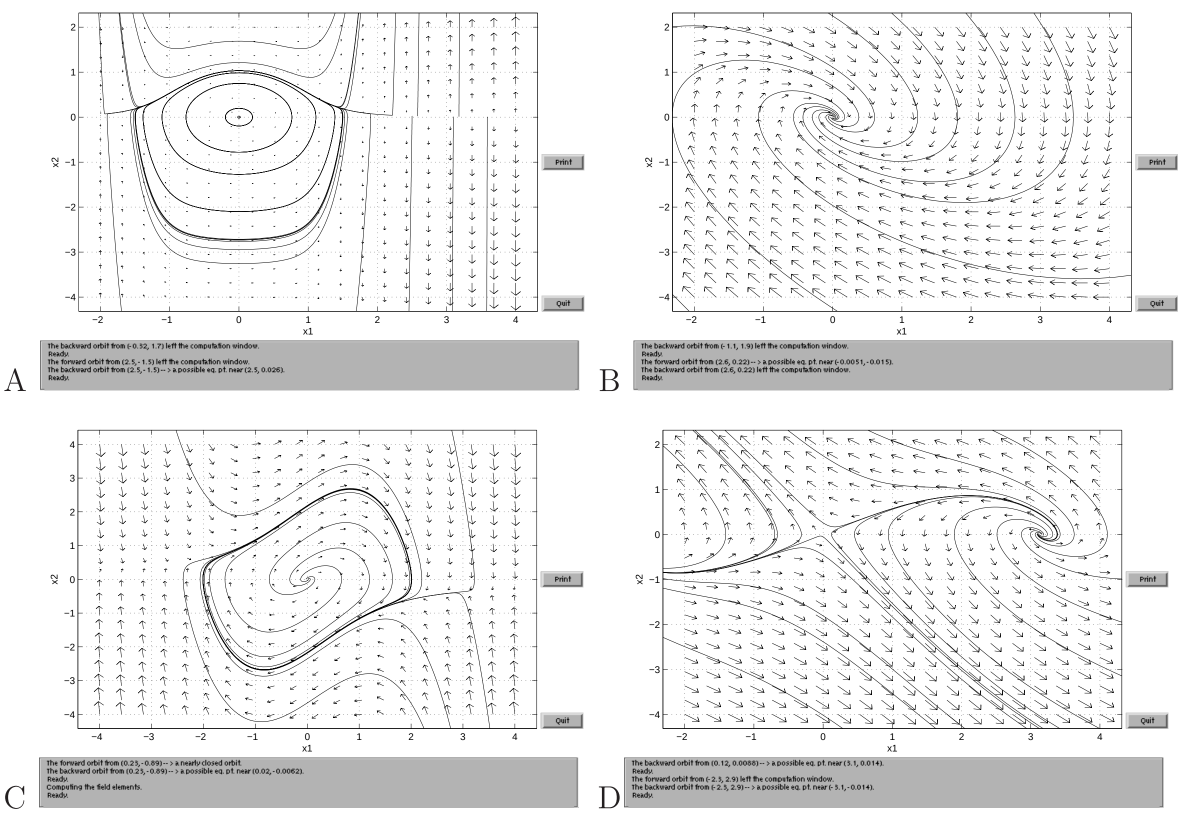
(a) [4p] Which phase portrait in Figure 1 belongs to what system? Briefly motivate your answers (no extensive calculations needed).

$$\text{(i)} \qquad \begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= x_2 - \sin(x_1) \end{aligned}$$

$$\text{(ii)} \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + (1 - x_1^2)x_2 \end{aligned}$$

$$\text{(iii)} \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_2 x_1^5 \end{aligned}$$

$$\text{(iv)} \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - x_2 \end{aligned}$$



Figur 1: The phase portraits in Problem 1(a)

(b) [4p] We want to study the stability properties of the nonlinear system defined by

$$\dot{x}_1 = x_1(x_1^2 + x_2^2 - 2) - 4x_1x_2^2$$
$$\dot{x}_2 = 4x_1^2x_2 + x_2(x_1^2 + x_2^2 - 2)$$

around its equilibrium point at the origin. Use the Lyapunov function candidate

$$V(x_1, x_2) = x_1^2 + x_2^2$$

to perform such an analysis.

(c) [2p] Give two examples of fundamental differences between linear and nonlinear dynamical systems.

2.

(a) [5p] Consider the system

$$\dot{x}_1 = -x_1 - x_2 + sign(-x_1 - 2x_2)$$
$$\dot{x}_2 = x_1$$

Find the sliding set and determine the sliding dynamics on/along the sliding set. (Hint: Use equivalent control and state in which region u_{eq} is valid!)

(b) [5p] Consider the system

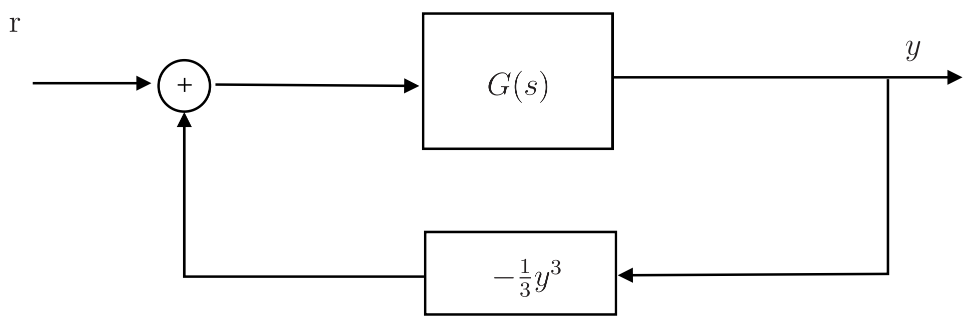
$$\dot{x}_1 = x_1^2 + x_2$$
$$\dot{x}_2 = u$$

Compute a controller using back-stepping to globally stabilize the origin.

3. Consider the nonlinear system

$$\frac{d}{dt}\left[\ddot{z} + \dot{z} + z\right] = -\frac{1}{3}z^3$$

(a) [2p] Show that the system can be separated into one linear and one nonlinear part as in the figure below. Determine the transfer function $G(s)$. Assume that $z(0) = \dot{z}(0) = 0$.



(b) [4p] Calculate the describing function of $f(y) = \frac{1}{3}y^3$.
Hint:

$$\int_0^{2\pi} \sin(x)^4 dx = \frac{3\pi}{4}$$

(c) [4p] Estimate the frequency and amplitude of possible limit cycles.

4. Consider the system

$$\begin{aligned}\dot{x}_1 &= -x_1 + 2x_2 + x_1 x_2 \\ \dot{x}_2 &= -2x_1 - x_1^2\end{aligned}$$

The task is to analyze the stability of this system using Lyapunov's Linearization Method.

(a) [2p] Show that the linear approximation around the origin is stable by using the function $V = x^T P x$, where

$$P = \begin{pmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{9}{8} \end{pmatrix}$$

(b) [8p] Consider the set $D = \{x_1, x_2, \quad |x_1| \leq \gamma\}$. Use $V(x)$ from a) to give conditions on γ so that the nonlinear system is locally asymptotically stable for $x \in D$.

5. A system

$$\dot{x} = f(x, u)$$
$$y = h(x, u)$$

is called output strictly passive if there exists a continuously differentiable positive semidefinite function $V(x)$ (called the storage function) such that

$$uy \geq \dot{V} + \delta y^2, \quad \delta > 0.$$

(a) [3p] Consider the system

$$\dot{x} = -x + u$$
$$y = x$$

Show that this system is output strictly passive.

(b) [7p] Show that an output strictly passive system is BIBO stable. To simplify the proof assume that $x(0) = 0$ and $V(0) = 0$.