# AUTOMATIC CONTROL
## KTH

## Nonlinear Control, 2E1262

Exam 14.00–19.00 December 18, 2006

**Aid:**

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text). Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

**Observandum:**

- Name and social security number(*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five 10 credit problems.

**Grading:**

Grade 3: $\geq 23$
Grade 4: $\geq 33$
Grade 5: $\geq 43$

**Results:**

The results will be posted within 2007-01-13 on the department's board, Osquldas vg 10, second floor. **If you want your result emailed, please state this and include your email address.**

**Responsible:** Bo Wahlberg 790 7242, Krister Jacobsson 790 74 27

*Good Luck!*

1.

(a) [4p] Consider the system

$$\dot{x}_1 = -x_2 + u$$
$$\dot{x}_2 = -x_2 + \sin(x_1) - \text{sat}(x_2)$$
$$y = x_2$$

where sat(·) denotes a saturation with slope one and saturation levels −1 and 1. Determine the equilibria for the system and their local stability property for $u = 0$ by sketching the corresponding phase plot.

(b) Consider the systems $\dot{x} = f(x)+u$, where the function $f(x)$ is plotted in Figure 1.

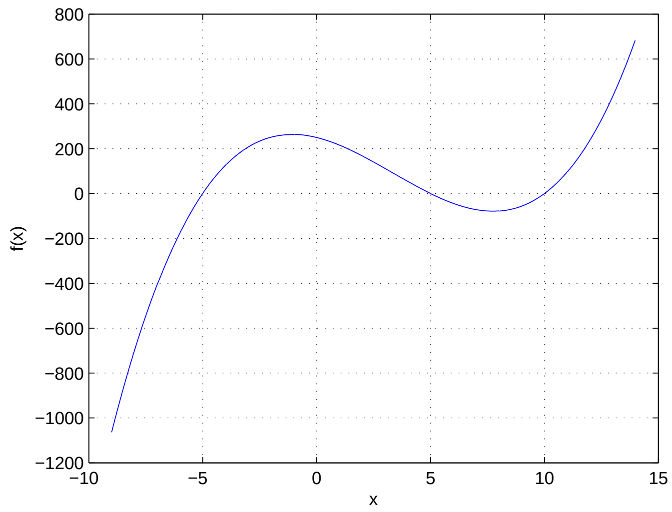


Figure 1: The the function $f(x)$ i Problem 1b.

(i) [2p] Assume $u = 0$ and determine for which initial conditions $x(0)$ the solutions will stay bounded.

(ii) [2p] Assume $u = 0$ and determine all equilibrium points and their stability properties.

(iii) [2p] Consider an unstable equilibrium point $x_0$ from ii). Introduce the feedback control $u = -K(x-x_0)$ and determine an approximate value of $K$ for which the feedback control law locally stabilize the equilibrium point $x_0$.

2.

(a) [1p] Consider the system

$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -2x_1 - x_2$$

Is the system asymptotically stable?

(b) [2p] Use the Lyapunov function candidate $V(x) = 3x_1^2 - 2x_1x_2 + x_2^2$ to investigate stability properties of the system in a).

(c) [3p] Use the Lyapunov function candidate $V(x) = x_1^2 + x_2^2$ to investigate stability properties of the system in a).

(d) [3p] Assume that the parameter $\lambda$ in the model

$$\dot{x}_1 = -\lambda x_1$$

is a function of time, $\lambda(t) = e^{-t}$. Introduce the extra state $x_2 = \lambda$ and rewrite the extended system in time invariant state space form.

3. Consider the system

$$\dot{x}_1 = x_1 x_2$$
$$\dot{x}_2 = x_1^5 + u$$

Use the Lyapunov function candidate

$$V(x) = x_1^2 + x_2^2$$

to find a control law using $u$ such that the origin can be shown to be globally asymptotically stable. [10p]

4.

(a) [7p] Consider the system

$$
\begin{aligned}
\dot{x}_1 &= -x_2 - 2x_1 + u \\
\dot{x}_2 &= x_1 \\
u &= -\text{sign}(x_1 + x_2)
\end{aligned}
$$

Determine the sliding set and the dynamics on the sliding set.

(b) [3p] Consider the nonlinear system

$$
\begin{aligned}
\dot{z}_1 &= -z_2^3 - 2z_1 + u \\
\dot{z}_2 &= z_1/(3z_2^2) \\
u &= -\text{sign}(z_2^3 + z_1)
\end{aligned}
$$

Show that this problem can be converted to the problem in (a) with a suitable choice of state transformation.

5. A system is called output strictly passive if there exists an $\epsilon > 0$ such that

$$\langle y, u \rangle_T \geq \epsilon |y|_T^2$$

with the definitions as in the lecture notes.

(a) [4p] Show that a stable linear system with transfer function $G(s)$ is output strictly passive if

$$\text{Re}G(i\omega) \geq \epsilon |G(i\omega)|^2$$

(b) [2p] Show that

$$G(s) = \frac{1}{s+1}$$

is output strictly passive.

(c) [2p] Consider the nonlinear static system $y = f(u)$. Show that this system is output strictly passive if

$$xf(x) \geq \epsilon f^2(x)$$

(d) [2p] Show that

$$f(x) = \frac{x}{1+|x|}$$

is output strictly passive.