AUTOMATIC CONTROL
KTH

Nonlinear Control, EL2620 / 2E1262

Exam 14.00–19.00 December 17, 2007

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text). Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2008-01-15 at STEX, Studerandeexpeditionen, Osquldasv. 10. **If you want your result emailed, please state this and include your email address.**

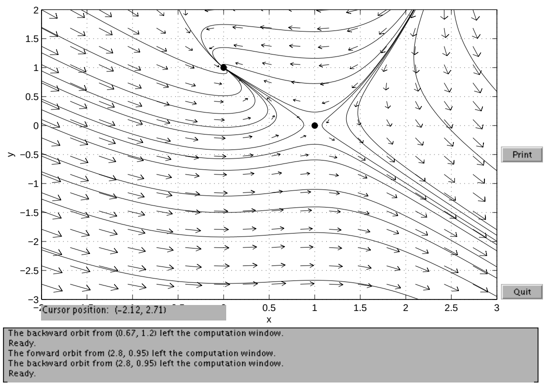
Responsible: Elling W. Jacobsen 0703 722 244, Björn Johansson 790 7424

Good Luck!

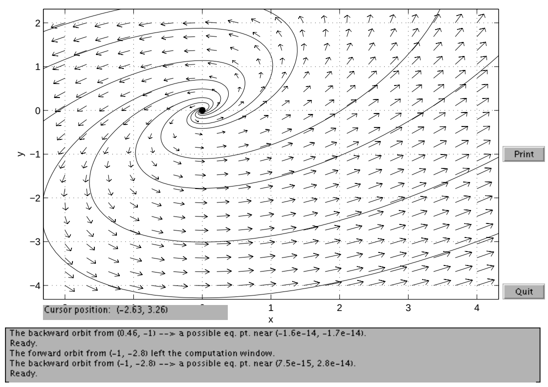
1. (a) The figure below shows phase portraits for four different systems. Pair each phase portrait with one of the systems below, and briefly motivate your answer. (4p)

$$
\begin{aligned}
(i)\quad & \dot{x}_1 = -x_2 + (1 - x_2^2)x_1 \\
& \dot{x}_2 = x_1 + x_2^2 x_1 \\
(ii)\quad & \dot{x}_1 = (1 - x_1)^2 - x_2 \\
& \dot{x}_2 = x_1(1 - x_1) \\
(iii)\quad & \dot{x}_1 = x_1 - x_2 \\
& \dot{x}_2 = x_1 \\
(iv)\quad & \dot{x}_1 = x_1 - x_2 + x_2^3 \\
& \dot{x}_2 = -x_2
\end{aligned}
$$

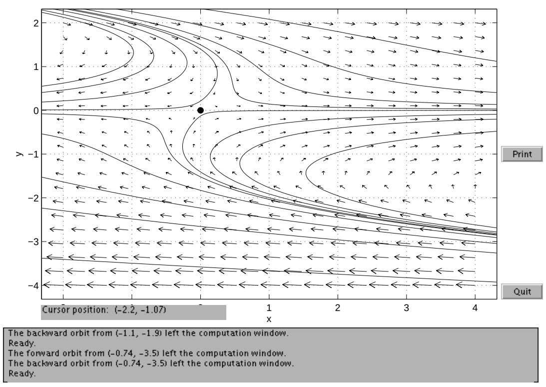
A



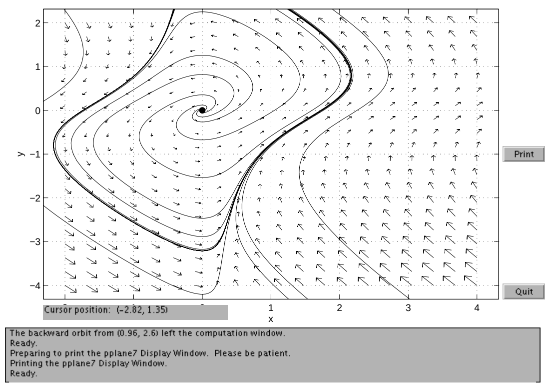
B



C



D



(b) Consider the nonlinear system

$$\dot{x} = -x^2 + sin(x) + u$$

Determine a feedback control $u(x, r)$ that makes the system linear from r to x (1p)

(c) Consider the nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_2 - x_1^2 \\ \dot{x}_2 &= -x_2 x_1^2 + u \\ y &= x_1\end{aligned}$$

(i) Determine a linearizing feedback control $u(x, r)$ such that the system becomes a (linear) double integrator from r to y, i.e.,

$$\ddot{y} = r$$

(4p)

(ii) Can there be problems with unstable zero dynamics when the controller derived in (i) is used? Motivate briefly. (1p)

2. (a) A biochemical system is described by

$$\dot{x}_1 = k_1 x_2 \left(1 - \frac{x_1}{1 + x_2^2}\right)$$
$$\dot{x}_2 = k_2 - x_2 - \frac{4x_1 x_2}{1 + x_2^2}$$

where x_1 and x_2 are concentrations of biochemical components and $k_1 > 0$ and $k_2 > 0$ are positive constants describing the reaction kinetics. We consider the case with

$$k_1 = 3, \quad k_2 = 10$$

The aim is to show that the system has a stable periodic solution, i.e., a stable limit cycle.

(i) Determine the equilibrium point and use Lyapunov's linearization method to show that the equilibrium is unstable. *To simplify calculations, recall that a 2nd order polynomial $s^2 + as + c = 0$ has all roots in the complex left half plane if and only if $a > 0$ and $c > 0$.* (2p)

(ii) Show that the region

$$K = \{(x_1, x_2) | x_1 \geq 0, x_2 \geq 0, x_1 \leq 1 + k_2^2, x_2 \leq k_2\}$$

is invariant. (4p)

(iii) What can you conclude from the results in (i) and (ii) concerning the existence of a limit cycle? Motivate briefly (1p)

(b) Consider the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2^3$$

(i) What can you conclude about the stability of the equilibrium using Lyapunov's linearization method? (1p)

(ii) Use the function $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ and LaSalle's theorem to show that the equilibrium is globally asymptotically stable. (4p)

3. (a) Consider the system

$$\dot{x}_1 = -x_1 x_2 + u$$
$$\dot{x}_2 = x_1^2 - x_2$$

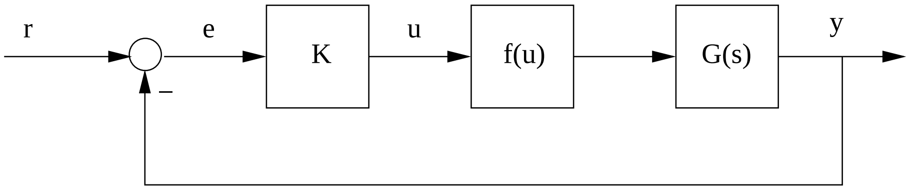
The aim is to make the equilibrium at the origin globally asymptotically stable. Use the control Lyapunov function $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ to find a linear control law that makes the origin globally asymptotically stable. (4p)

(b) Use control design based on back-stepping to determine a stabilizing controller for the origin $x = 0$ of the system

$$\dot{x}_1 = -x_1^2 + x_2$$
$$\dot{x}_2 = 1/(1 + x_1^2) - x_2 + u$$

(4p)

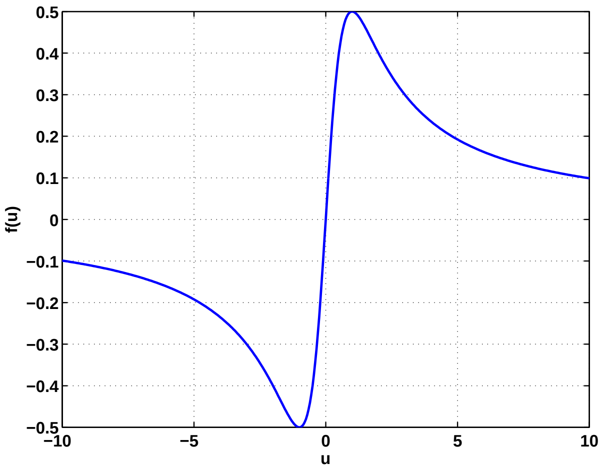
4. Consider the nonlinear feedback system in the figure below.

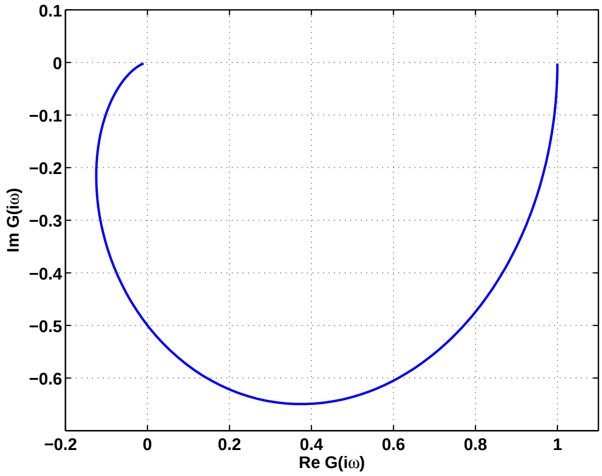


Here

$$G(s) = \frac{1}{(s+1)^2} \;; \quad f(u) = \frac{u}{1+u^2}$$

The static nonlinearity $f(u)$ and the frequency response $G(i\omega)$ are shown graphically below





We want to determine for which values of the controller gain K the closed loop system is stable

(i) Use the small gain theorem to determine values of the gain K that guarantees closed-loop stability. (3p)

(ii) Use the circle criterion to determine values of the gain K that guarantees closed-loop stability. (3p)

(iii) For what values of K will the describing function method predict sustained oscillations in the closed-loop system? *Hint: you do not need to compute the describing function for the nonlinearity to answer this question. It suffices to observe some characteristic properties of the nonlinearity and hence the describing function.* (3p)

(iv) Comment on the differences in the answers obtained with the three methods above. (1p)

5. We shall consider nonlinear control of a linear dynamic system

$$\dot{x}_1 = -x_1 + 2x_2$$
$$\dot{x}_2 = x_1 - x_2 + u$$
$$y = x_1 - x_2$$

The aim of the control is to drive the output y to zero in finite time. We consider using sliding mode control and define the sliding manifold as

$$\{x|y(x) = 0\}$$

(a) Determine a sliding mode controller that makes the sliding manifold globally attracting in finite time with Lyapunov function $V(y) = 0.5y^2$. (3p)

(b) Sketch the response $y(t)$ for an arbitrary initial condition $x_1(0) = a, x_2(0) = b$ when the controller in (a) is employed. (1p)

(c) Determine the equivalent control u_{eq} on the sliding manifold and show that the equivalent control corresponds to making the sliding manifold dynamics unobservable. Does the unobservability pose any problem in this case? (3p)

(d) Discuss the feasibility of alternative approaches to driving the output y to zero in finite time for the given system. (3p)