AUTOMATIC CONTROL
KTH

Nonlinear Control, EL2620 / 2E1262

Exam 14.00–19.00 May 30, 2008

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text). Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2008-06-19 at STEX, Studerandeexpeditionen, Osquldasv. 10.

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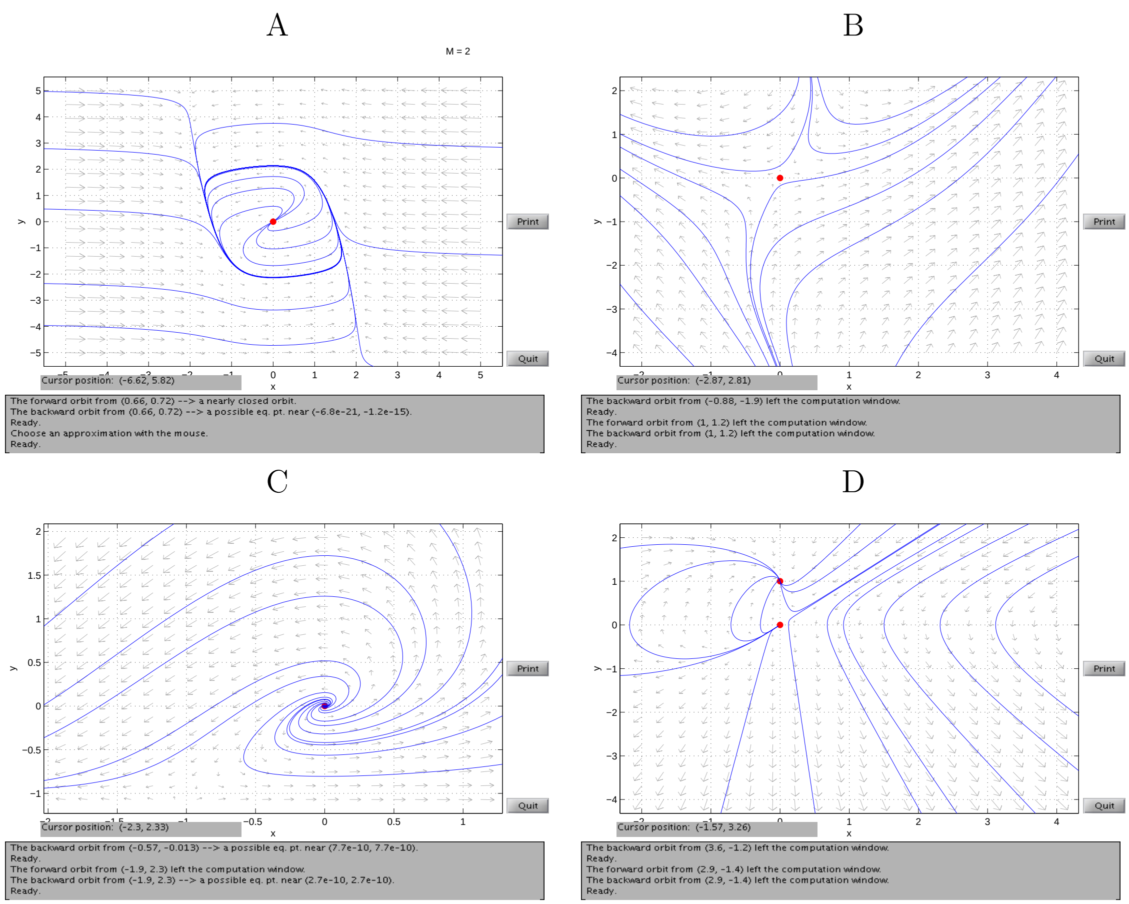
Good Luck!

1. (a) The figure below shows phase portraits for four different systems. Pair each phase portrait with one of the systems below, and briefly motivate your answer. (4p)

$$
\begin{aligned}
(i)\quad & \dot{x}_1 = 2x_1 - sin(x_2) \\
& \dot{x}_2 = x_1^2 - x_2 \\
(ii)\quad & \dot{x}_1 = 2x_1 - x_2 - x_1^3 \\
& \dot{x}_2 = x_1 \\
(iii)\quad & \dot{x}_1 = x_1 - x_2 \\
& \dot{x}_2 = x_1 + x_1 x_2 \\
(iv)\quad & \dot{x}_1 = -x_1 x_2 \\
& \dot{x}_2 = (1 - x_2)x_2 - x_1
\end{aligned}
$$



(b) Consider the nonlinear system

$$\dot{x} = -x^3 + \frac{1}{1 + x^4} + u$$

Determine a control law $u(x, r)$ that makes the system linear from r to x (1p)

(c) Consider the nonlinear system

$$
\begin{aligned}
\dot{x}_1 &= 2x_2 + x_1^3 + u \\
\dot{x}_2 &= -x_2 - u \\
y &= x_1
\end{aligned}
$$

(i) Determine a linearizing feedback control $u(x, v)$ such that the system becomes a (linear) integrator from r to y, i.e.,

$$\dot{y} = v$$

(4p)

(ii) The feedback linearized system in (i) is combined with a proportional feedback controller

$$v = -K_p y$$

Show that there is no value of K_p that will make the closed-loop system stable at the origin, and explain why this is so. (4p)

2. Consider the system

$$\dot{x}_1 = -x_1^3 + x_2$$
$$\dot{x}_2 = x_1^6 - x_2^3$$

(a) Determine all equilibria and analyze their stability properties. (2p)

(b) Show that the set

$$\Gamma = \{(x_1, x_2) \in \mathbf{R}^2 : 0 \leq x_1 \leq 1,\ x_2 \geq x_1^3,\ x_2 \leq x_1^2\}$$

is invariant (5p)

(c) Argue using Γ in (b) that the origin is an unstable equlibrium. (3p)

3. Consider the system

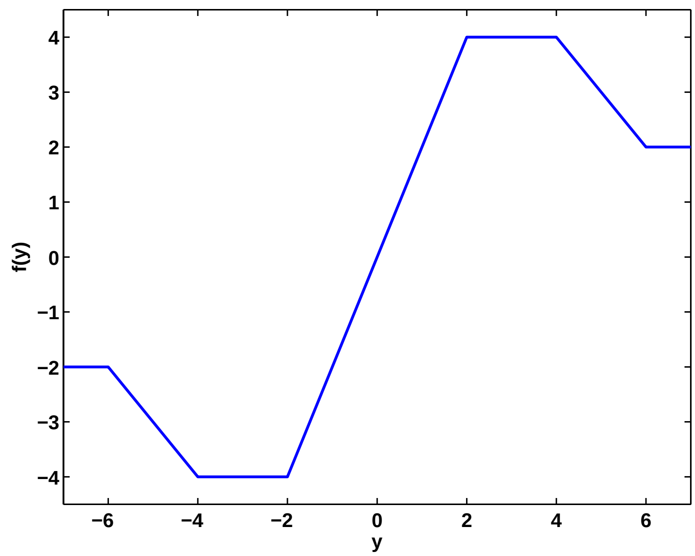
$$\dot{x}_1 = x_1^2 + x_1 x_2 + u$$
$$\dot{x}_2 = x_1 x_2^2 + x_1$$

Use the Lyapunov control function

$$V(x) = x_1^2 + x_2^2$$

to determine a control law $u(x)$ such that the origin is globally asymptotically stable. Show that the origin indeed becomes globally asymptotically stable. (10p)

4. Consider the static nonlinearity f below



(a) Sketch the describing function for f. You only need to draw an approximate sketch. (3p)

(b) Consider a feedback system that consists of a linear system $G(s)$ in negative feedback connection with f , that is,

$$y = Gu = -Gf(y)$$

Based on the describing function method, specify a transfer function $G(s)$ such that the closed-loop system is likely to have an oscillation. (2p)

(c) What is the gain of $f(y)$? What is the gain of the series connection $f(f(y))$? What is the gain of the parallell connection $f(y) + f(y)$? (2p)

(d) Consider a feedback system that consists of an integrator K/s in negative feedback connection with f, that is,

$$\dot{y} = -Kf(y)$$

For what values of $K > 0$ is the closed-loop system stable? Is it globally stable? (3p)

5. Consider the model of a nonlinear mechanical system

$$\dot{x}_1 = -x_1 + x_1 x_2 + u$$
$$\dot{x}_2 = -2x_2 + x_1 - 0.5u$$

The range of the control input is $u \in [-1, 1]$.

The owner of the system has asked you to design a controller that takes the system from any initial state to the manifold defined by

$$S = \{x | x_2 = 0\}$$

in finite time and then keeps it on the manifold S. That is, the controller should drive the state x_2 to zero in finite time and then keep $x_2 = 0$.

(a) Design a feedback controller that satisfies the specifications above. (3p)

(b) Show that the system with the controller from (a) is unstable on the manifold S and discuss the feasibility of choosing a control law that makes the system stable on the manifold (4p)