AUTOMATIC CONTROL

KTH

Nonlinear Control, EL2620 / 2E1262

Exam 08.00–13.00 December 16, 2008

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text) or equivalent basic control book if approved by the examiner beforehand. Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2009-01-15 at STEX, Studerandeexpeditionen, Osquldasv. 10.

Responsible: Elling W. Jacobsen 0703 722 244

Good Luck!

1. Consider the nonlinear system

$$\dot{x}_1 = kx_1 - x_2$$
$$\dot{x}_2 = x_1 - x_2^3 + u$$

(a) Determine the local stability properties of all equilibrium points when $k = 1$ and $u = 0$. (4p)

(b) Find a linear control law $u = g(x)$ such that the origin becomes locally asymptotically stable when $k = 1$. (2p)

(c) Consider the case with $k = 0$ and $u = 0$. Show that the origin now is globally asymptotically stable. (4p)

2. (a) Consider the dynamic system

$$\begin{aligned}\dot{x} &= -y \\ \dot{y} &= x + y^3 - y\end{aligned}$$

(i) Show that

$$E = \{(x, y) | x^2 + y^2 \leq c\}$$

for some $c \in \mathbf{R}$ is an invariant region for the system and determine the values of c for which this holds. (4p)

(ii) What can you conclude from the result in (i)? (2p)

(b) A nonlinear system is described by

$$\begin{aligned}\dot{x}_1 &= 2x_1 - \frac{x_2}{1 + x_2^2} + u \\ \dot{x}_2 &= \frac{x_1}{1 + x_1^2}\end{aligned}$$

Use the control lyapunov function $V = x_1^2 + x_2^2$ to design a state feedback controller that makes the origin globally asymptotically stable. Show that the controller indeed is globally stabilizing! (4p)

3. (a) We shall consider linearizing control of the nonlinear system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 + x_2^2 + u \\ \dot{x}_2 &= x_2 - x_1^2 \\ y &= x_2 \end{aligned}$$

(i) Linearize the state-space by employing a suitable state transformation combined with a state feedback. Discuss possible limitations. (3p)

(ii) Linearize the input-output behavior by employing a state feedback. Discuss possible limitations and compare with the result in (i). (3p)

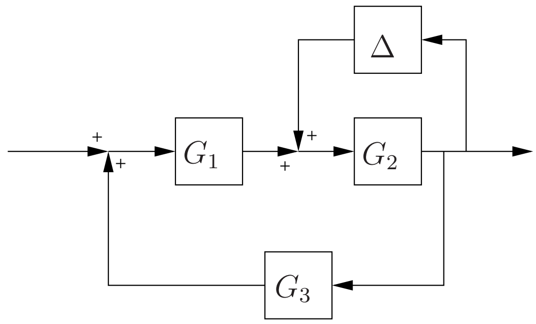
(iii) Will there be problems with unstable zero dynamics when the controller in (ii) is employed? Motivate your answer! (1p)

(b) A mechanical system with servo dynamics and no damping is described by the model

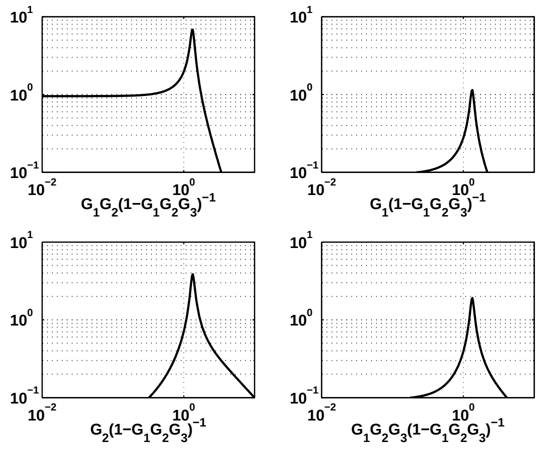
$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1^3 + z \\ \dot{z} &= -z + u \end{aligned}$$

Use backstepping to determine a control law that makes the origin locally asymptotically stable. (3p)

4. (a) Consider the system in the figure below

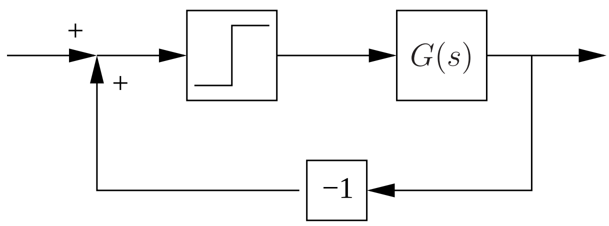


Here Δ denotes an unknown nonlinear system. Some amplitude plots for the closed-loop with $\Delta = 0$ are shown below.



Use the provided information to determine a bound K such that the system is guaranteed stable for all Δ with $\gamma(\Delta) < K$. State any additional assumptions you need to make! (4p)

(b) Consider the feedback system with a relay in the figure below



Here

$$G(s) = \frac{-s}{s^2 + 0.8s + 1}$$

(i) Do you expect the system to display sustained oscillations? If yes, provide an estimate of the amplitude and period of the oscillations. (4p)

(ii) A relay experiment can give information about the frequency for which $\arg(G(i\omega)) = -180^0$, and this can be used for autotuning of a controller. How would you modify the feedback term in the figure above to obtain information about the frequency where $\arg(G(i\omega)) = -90^o$? (2p)

5. We shall consider sliding mode control of the system

$$\dot{x}_1 = -2x_1 - \frac{x_2}{1+x_2^2} + u$$
$$\dot{x}_2 = \frac{x_1}{1+x_1^2}$$

(a) Someone has proposed the manifold

$$S = \{(x_1, x_2) | x_1 = x_2\}$$

as the sliding manifold. What will be the consequence of forcing the system to evolve on this manifold? Motivate your answer. (3p)

(b) Consider now the manifold

$$S = \{(x_1, x_2) | x_1 + a x_2 = 0\}$$

How will the choice of a affect the system dynamics on the sliding manifold? (4p)

(c) For the sliding manifold in (b), determine a controller that makes the sliding manifold globally asymptotically stable. Also give the equivalent control on the sliding manifold. (3p)