AUTOMATIC CONTROL

KTH

Nonlinear Control, EL2620 / 2E1262

Exam 14.00–19.00 December 16, 2009

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text) or equivalent basic control book if approved by the examiner beforehand. Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2010-01-15 at STEX, Studerandeexpeditionen, Osquldasv. 10.

Responsible: Elling W. Jacobsen 0703 722 244

Good Luck!

1. Consider an isolated island in which two species are introduced by the landowner. One species is a plant eater, the other species is a meat eater which prey on the first species. Denote the population of the plant eaters x_1 and the population of meat eaters x_2. A simple model of the evolution of the two species is then

$$\dot{x}_1 = x_1(1 - x_1 - x_2)$$

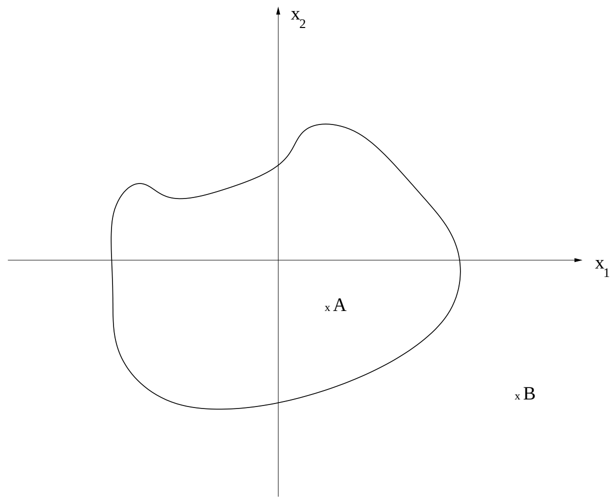
$$\dot{x}_2 = x_2(x_1 - x_2) + u$$

   (a) Assume $u = 0$. Find and classify all equilibria and determine if they are unstable, stable or asymptotically stable. Can you say something about the global stability of any of the equilibria? (6p)

   (b) Discuss what will happen with the populations on the island after some time when $u = 0$. (2p)

   (c) One day the landowner decides that he wants to get rid of the meat eaters by hunting them down. Propose a hunting strategy, corresponding to a linear feedback control law $u = Kx$, that will make the equilibrium $(1, 0)$ asymptotically stable. (2p)

2. (a) A second order time-invariant system has a limit cycle as indicated in the figure below.



(i) Assume that the limit cycle is stable and that the system is initiated in point A. Is it possible for the system trajectory to pass through point B? Motivate your answer! (2p)

(ii) Assume that the limit cycle is unstable. Is it then possible for the system to move from point A to point B? Motivate! (1p)

(b) Consider the nonlinear system

$$\begin{aligned}\dot{x}_1 &= -x_1(t)^2 + x_2(t)\\ \dot{x}_2 &= -x_1(t)x_2(t) + u(t)\\ y(t) &= x_2(t)\end{aligned}$$

(i) Determine a state transformation and a state feedback that results in a linear state space system. (3p)

(ii) Determine a state feedback that makes the system linear from the input u to the output y. Discuss whether there will be problems with unstable zero dynamics with this control. (4p)

3. (a) A mechanical system with two states is described by the model

$$\begin{aligned}\dot{x}_1 &= 5x_1(t)x_2(t)\\ \dot{x}_2 &= 2x_1(t)^5 + 3u(t)\end{aligned}$$

Use Lyapunov based methods to a find a feedback control law $u = c(x)$ such that the origin becomes globally asymptotically stable.

Hint: you may try $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ *as a Lyapunov function candidate.* (5p)

(b) Consider the system

$$\begin{aligned}\dot{x}_1 &= (x_1 - x_2)^2 + (x_1 - x_2)x_2 + u \qquad (= x_1^2 - x_1x_2 + u)\\ \dot{x}_2 &= -x_2 + u\end{aligned}$$

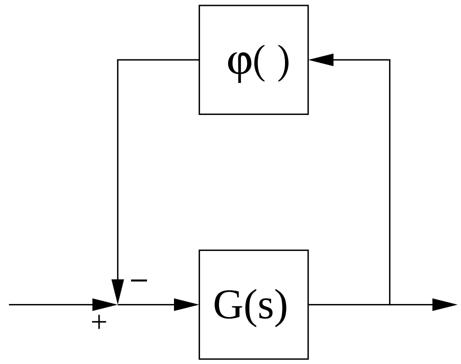
We want to use the principle of backstepping to design a stabilizing controller.

(i) Explain why the method of backstepping does not apply directly to the given system. (1p)

(ii) Introduce a state transformation so that backstepping can be applied, and design a stabilizing control based on backstepping for the transformed system.

Hint: note that $\dot{x}_1 - \dot{x}_2$ *will not contain* $u(t)$. (4p)

4. .



(a) Consider a feedback loop consisting of a stable linear system $G(s)$ and a static nonlinearity $\phi(\cdot)$ as shown in the block diagram above. The Nyquist diagram for G is shown on the next page.

(i) Assume that the static nonlinearity is sector bounded $-k < \phi(y)/y < k$. What is the largest k for which the small gain theorem guarantees closed-loop stability? (2p)

(ii) Assume now that the sector bound is $0 < \phi(y)/y < \alpha$. What is the largest α for which the circle criterion guarantees closed-loop stability? (2p)

(iii) Consider the case where the nonlinearity provides a negative gain, i.e., $\beta < \phi(y)/y < 0$. For which β will the circle criterion guarantee closed-loop stability? (2p)

(b) Consider again the feedback loop above but assume now that

$$G(s) = \frac{1}{(s+1)^4}$$

and that the nonlinear function $\phi(\cdot)$ has the describing function

$$N(A) = A + 3A^2$$

Will the describing function method predict a limit cycle? If so, determine the predicted amplitude, period and stability of the limit cycle. (4p)

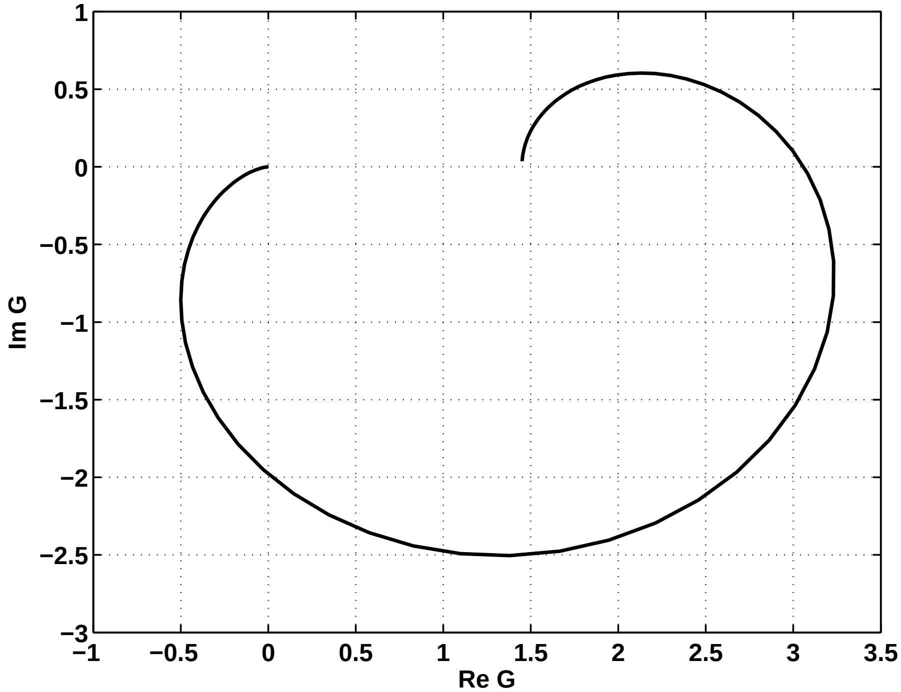


Figure 1: Nyquist diagram for problem 4a.

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5. We shall consider optimal heating of a bakers oven. The aim is to raise the temperature from $20°C$ to $220°C$ in 1 *hour* while keeping the total energy consumption as small as possible. The dynamics of the oven can be described by a first order model

$$c_P \frac{dT}{dt} = -\epsilon(T(t) - 20) + Q(t)$$

where c_P is the heat-capacity of the oven in $[kJ/°C]$, ϵ is the heat-loss coefficient in $kW/°C$ and Q is the heating in kW. The heat-capacity is assumed constant and is given by $c_P = 1\ kJ/°C$.

In order to reduce the total energy consumption we aim to minimize $\int Q^2 dt$

(a) Formulate the heating problem given above as an optimal control problem. (2p)

(b) What is the solution to the optimal control problem when there is no heat loss, i.e., $\epsilon = 0$. (2p)

(c) Assume the heat-loss coefficient $\epsilon = 0.01 kW/°C$. Solve the optimal control problem for this case. (5p)

(d) Consider that the maximum heating is $Q_{max} = 2kW$. Is the optimal control problem feasible then? Assume $\epsilon = 0.01 kW/°C$. (1p)