AUTOMATIC CONTROL

KTH

Nonlinear Control, EL2620 / 2E1262

Exam 08.00–13.00   December 17, 2010

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text) or equivalent basic control book if approved by the examiner beforehand. Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2011-01-17 at STEX, Studerandeexpeditionen, Osquldasv. 10.

Responsible: Elling W. Jacobsen 0703 722 244, Torbjörn Nordling, 08 790 7324

Good Luck!

1. Consider the nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\sin x_1 + (x_2 - 1)x_2^3 + u\end{aligned}$$

(a) Consider first the case with $u = 0$. Determine all equilibrium points and classify the phase portraits of the linearizations about the equilibria. What can you deduce about stability from the linearizations? (4p)

(b) Show that the equilibrium at the origin, with $u = 0$, is locally asymptotically stable. You may use the Lyapunov function candidate

$$V(x) = (1 - \cos(x_1)) + \frac{1}{2}x_2^2$$

(4p)

(c) Determine a control law $u(t) = c(x)$ such that the origin becomes globally asymptotically stable. (2p)

2. A biochemical reaction system involving two components is described by the differential equations

$$\begin{aligned}\dot{x}_1 &= \frac{r_0}{K_0 + x_2^n} - K_1 x_1 + u_1 \\ \dot{x}_2 &= K_2 x_1 - K_3 x_2 + u_2\end{aligned}$$

where r_0, n and all K_i are positive constants and u_1 and u_2 represent addition of the respective biochemicals and serve as potential control inputs.

(a) Determine a state transformation and a state feedback that transforms the system into a linear state-space system when

(i) The input u_1 is used as a control variable (2p)

(ii) The input u_2 is used as a control variable (3p)

(b) Consider now that we want to control component x_2, i.e., $y = x_2$. Determine a state feedback that makes the input-output relationship linear when

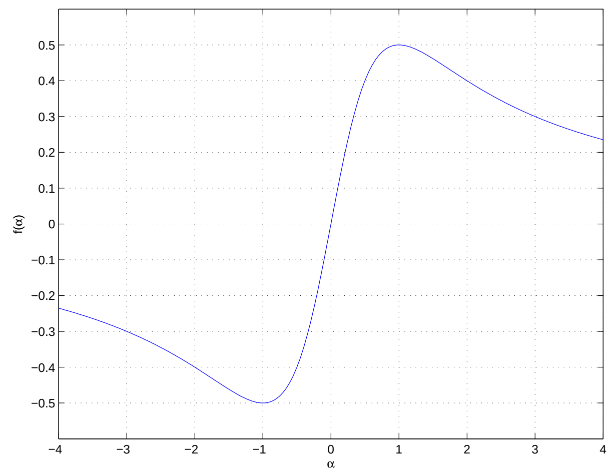
(i) The input u_1 is used as a control variable. Also determine if there will be problems with unstable zero dynamics with this control law. (2p)

(ii) The input u_2 is used as a control variable. Will there be unstable zero dynamics in this case? (3p)

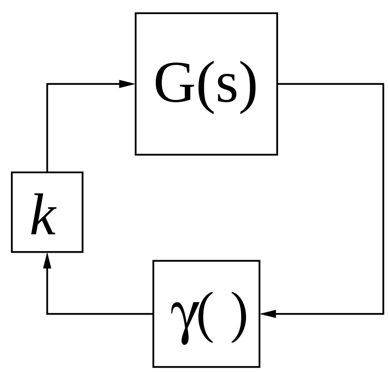
3. We shall consider stability of the system

$$\begin{aligned} \dot{x}_1 &= -x_1 + k\frac{x_3}{1+x_3^2} \\ \dot{x}_2 &= = x_1 \\ \dot{x}_3 &= = -x_3 + x_2 \end{aligned}$$

where k is a real constant.The nonlinearity $f(\alpha) = \frac{\alpha}{1+\alpha^2}$ is shown graphically in the figure below.



(a) Show that the system can be put on a feedback form as shown in the Figure below and determine the transfer-function $G(s)$ (you can assume zero initial conditions) and the nonlinear function $\gamma(\cdot)$. (2p)



(b) For what values of k can you guarantee stability of the system with the Small Gain Theorem? (2p)

(c) For what values of k can you guarantee stability of the system with the Circle Criterion? (3p)

(d) For what values of k will the describing function method predict sustained oscillations in the system? *You do not need to compute the describing function for this task, only sketch its main characteristics.* (3p)

4. A mechanical system with two states and one control input is described by the model

$$\begin{aligned}\dot{x}_1 &= x_1^2 + x_1 x_2 \\ \dot{x}_2 &= -2x_2^2 + u\end{aligned}$$

(a) Show that the system is on strict feedback form and use the method of back-stepping to design a control law $u = c(x)$ such that the origin becomes globally asymptotically stable. Also provide a corresponding Lyapunov function for the closed-loop system. (4p)

(b) Consider now using the concept of sliding mode control to stabilize the system.

(i) Show that the ”standard” choice of a linear manifold

$$S = \{(x_1, x_2) | \sigma(x) = ax_1 + x_2 = 0\}$$

is not viable in this case as it will not make all states converge to the origin when enforcing $\sigma = 0$. (1p)

(ii) Show that one by adding a quadratic term to the manifold equation $\sigma(x) = 0$ can make the origin globally attracting in S, i.e., all states converge to the origin when enforcing $\sigma = 0$. *Hint: Consider $x_2 = c(x_1)$ such that the differential equation for x_1 is stable.* (3p)

(iii) With the manfifold determined in (ii), determine a controller that makes any initial state move to the manifold in finite time. (2p)

5. Consider a linear multi-input-multi-output (MIMO) system

$$\dot{x} = Ax + B_1 w + B_2 u,$$

where w is an unknown disturbance and u is the control input. A so called worst-case design can be posed as a min-max optimization problem, where one first searches the worst case disturbance w with energy γ and then the control u that minimizes the effect of the disturbance. This can under certain conditions be formulated as a minimization problem with a linear quadratic objective

$$\min_{u,w} \frac{1}{2} \int_0^\infty (x^T Q x + u^T R u - \gamma^2 w^T w) dt,$$

where Q and R are constant symmetric positive definite weighting matrices.

(a) State the necessary conditions for this optimal control problem. Find the maximum disturbance, the optimal control law in terms of the states x and costates λ, and the equation system that gives x and λ. *Note that you do not need to solve the equations for x and λ here.* (8p)

(b) The optimal control law turns out to be a linear state feedback. Assume that the states and costates are related by a constant matrix P, i.e. $\lambda = Px$, and show that the solution of the equation system for x and λ then is given by the Riccati-like equation

$$A^T P + PA + Q + P(\frac{1}{\gamma^2} B_1 B_1^T - B_2 R^{-1} B_2^T) P = 0.$$

(2p)