AUTOMATIC CONTROL

KTH

Nonlinear Control, EL2620 / 2E1262

Exam 14.00–19.00 January 10, 2012

Aid:

Lecture-notes from the nonlinear control course and textbook from the basic course in control (Glad, Ljung: Reglerteknik, or similar approved text) or equivalent basic control book if approved by the examiner beforehand. Mathematical handbook (*e.g.* Beta Mathematics Handbook). Other textbooks, exercises, solutions, calculators, etc. are **not** allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2012-02-01 at STEX, Studerandeexpeditionen, Osquldasv. 10.

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Good Luck!

1. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_2(1 - x_1^2 - 2x_2^2)\end{aligned}$$

(a) Determine all equilibrium points, determine their stability properties and sketch the corresponding local phase portraits. (3p)

(b) Show that the region defined by

$$0.5 \leq x_1^2 + x_2^2 \leq 1$$

is invariant for the system. *Hint: consider the function* $V = x_1^2 + x_2^2$. (5p)

(c) Based on the results in (a) and (b), what can you conclude about the system behavior? (2p)

2. (a) A phase-locked loop in communication networks can be described by

$$\ddot{y} + (a + b\cos y(t))\dot{y} + c\sin y(t) = 0$$

with the constant $c > 0$.

(i) Show that $\dot{y} = 0, y = 0$ is a stable equilibrium if

$$a \geq b \geq 0$$

You may consider the Lyapunov candidate $V = c(1 - \cos y) + 0.5\dot{y}^2$. (4p)

(ii) Show that $(0, 0)$ is asymptotically stable if $a > b \geq 0$. (3p)

(b) Consider the 2nd order system

$$\begin{aligned} \dot{x}_1 &= -x_1^2 + x_2 + u \\ \dot{x}_2 &= x_1 - 2x_2^3 \end{aligned}$$

Determine a state transformation combined with a state feedback such that the system becomes linear. (3p)

3. We shall consider control of a single linked robot arm described by the equation of motion

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau_u$$

Here q is the joint angle, M the inertia, C the centrifugal force, g the gravitational force and τ_u is the input torque.

You can assume that $M(q)$ is a positive scalar function for all $q \neq 0$ and that $\dot{M} - 2C = 0$.

(a) Assume that the gravitation term $g(q) = 0$ and show that the PD-controller

$$\tau_u = K(r - q) + K_d\dot{q}$$

provides asymptotic tracking for constant references r provided $K_d < 0$. *Hint: you can consider the Lyapunov candidate* $V = \frac{1}{2}\dot{q}^2 M(q) + \frac{1}{2}K(q - r)^2$. (5p)

(b) Consider now the case with gravitational effects, i.e., $g(q) \neq 0$. Can you conclude asymptotic tracking using the same Lyapunov candidate as in (a)? If not, propose a modification of the control law such that asymptotic tracking can be shown. (3p)

(c) Determine a control $\tau_u = c(q, \dot{q}, v)$ such that the relationship between the new control input v and the output q becomes linear. (2p)

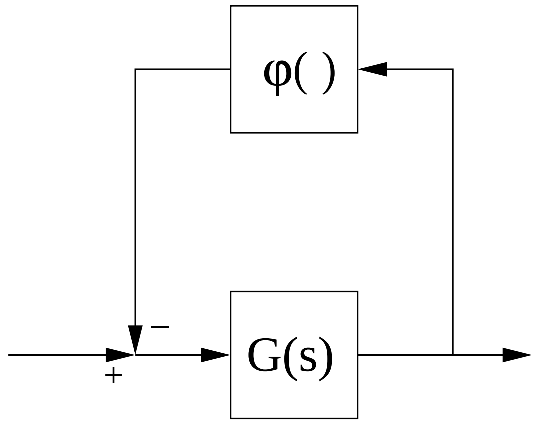

Figure 1: Block diagram for Problem 4.

4. Consider the system

$$\dddot{z} + \ddot{z} + \dot{z} = -\frac{1}{3}z^3$$

(a) Show that the system can be written as a feedback system composed of a linear dynamic system $G(s)$ and a static nonlinearity $\phi(\cdot)$ as shown in the figure above. (2p)

(b) Calculate the describing function for the static nonlinearity ϕ.
Hint: $\int_0^{2\pi} sin(x)^4 dx = \frac{3\pi}{4}$. (3p)

(c) Analyze the existence, stability, amplitude and period of possible limit cycles in the system. (5p)

5. We shall consider control of a bio reactor described by the model

$$\begin{aligned} \dot{x}_1 &= -x_1^2 + x_1 x_2 + u \\ \dot{x}_2 &= x_2(x_2^2 - 1) - x_1 \end{aligned}$$

The aim is to control the concentration of the metabolite corresponding to state x_1.

(a) Consider first designing a state feedback controller such that the closed-loop system is linear with the transfer-function

$$Y = \frac{1}{\tau s + 1} R$$

where $y = x_1$ and r is the reference for y. Discuss possible problems with this control strategy. (3p)

(b) To avoid the problems with the control strategy in (a), it is proposed to consider sliding mode control instead. The choice of the sliding manifold is then a critical step. Explain why the "standard" choice of a linear manifold

$$\sigma = x_1 + a x_2$$

is not a good choice in this case. Also propose a manifold which has desirable properties. (4p)

(c) For the manifold you proposed in (b), design a controller that takes the system from any initial state to the manifold in finite time. (3p)