EL2620 Nonlinear Control

Lecture 8

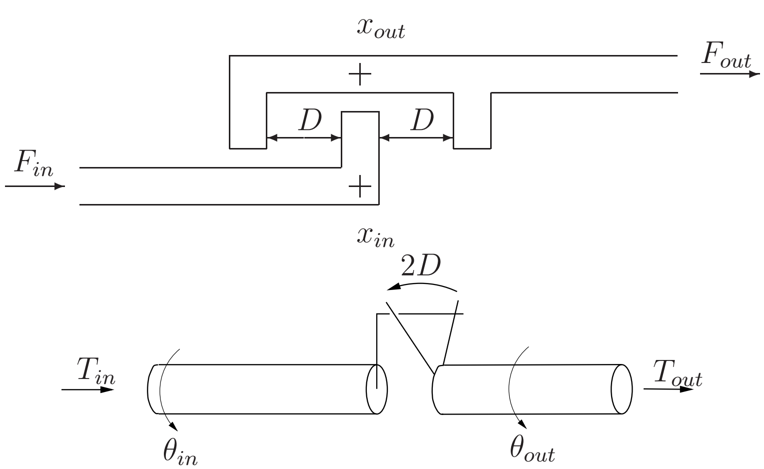
- Backlash
- Quantization

Today's Goal

You should be able to analyze and design for

- Backlash
- Quantization

Linear and Angular Backlash



Backlash

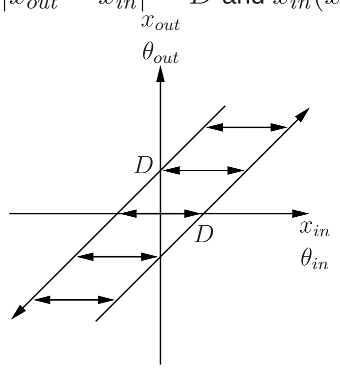
Backlash (*glapp*) is

- present in most mechanical and hydraulic systems
- increasing with wear
- necessary for a gearbox to work in high temperature
- bad for control performance
- sometimes inducing oscillations

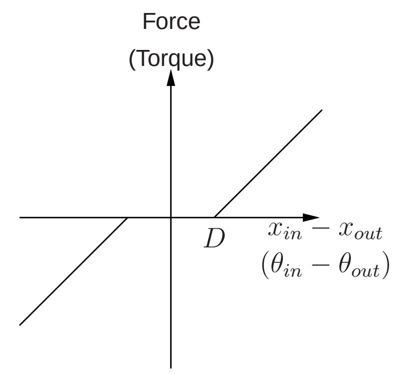
Backlash Model

$$\dot{x}_{out} = \begin{cases} \dot{x}_{in}, & \text{in contact} \\ 0, & \text{otherwise} \end{cases}$$

"in contact" denotes $|x_{out} - x_{in}| = D$ and $\dot{x}_{in}(x_{in} - x_{out}) > 0$.



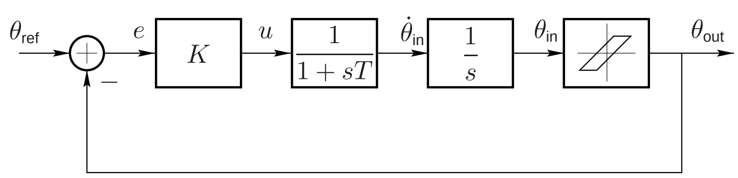
Alternative Model

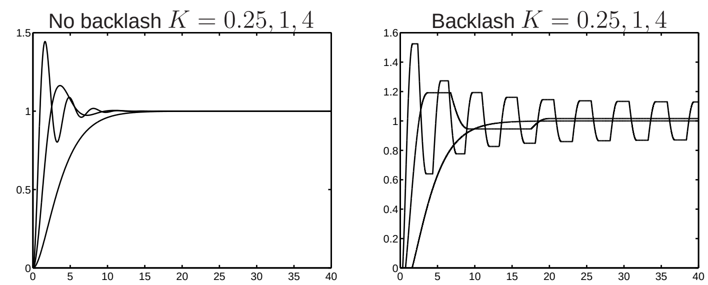


Not equivalent to "Backlash Model"

Effects of Backlash

P-control of motor angle with gearbox having backlash with $D = 0.2$





- Oscillations for $K = 4$ but not for $K = 0.25$ or $K = 1$. Why?
- Note that the amplitude will decrease with decreasing D, but never vanish

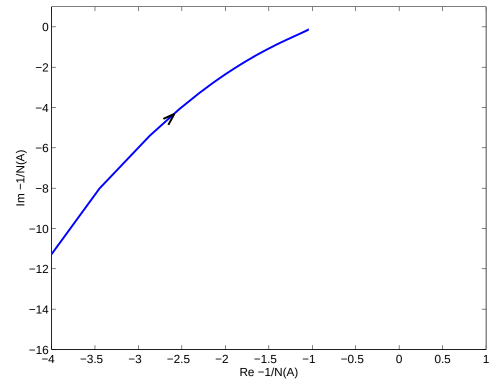
Describing Function for Backlash

If $A < D$ then $N(A) = 0$ else

$$\operatorname{Re} N(A) = \frac{1}{\pi}\left[\frac{\pi}{2} + \arcsin(1 - 2D/A) + 2(1 - 2D/A)\sqrt{\frac{D}{A}\left(1 - \frac{D}{A}\right)}\right]$$

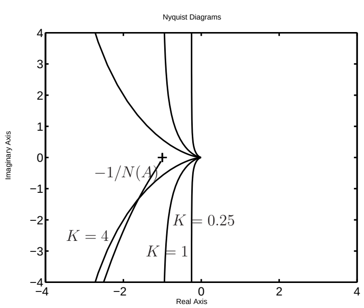
$$\operatorname{Im} N(A) = -\frac{4D}{\pi A}\left(1 - \frac{D}{A}\right)$$

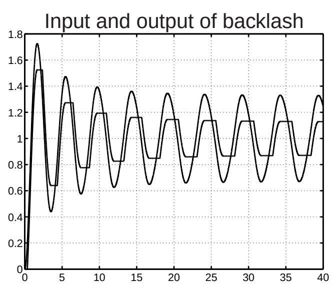
$-1/N(A)$ for $D = 0.2$:



Note that $-1/N(A) \to -1$ as $A \to \infty$ (physical interpretation?)

Describing Function Analysis





$K = 4, D = 0.2$:

DF analysis: Intersection at $A = 0.33, \omega = 1.24$

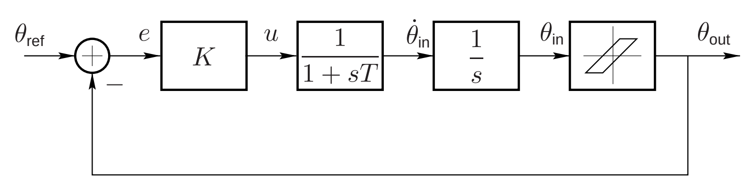
Simulation: $A = 0.33, \omega = 2\pi/5.0 = 1.26$

Describing function predicts oscillation well

Stability Proof for Backlash System

The describing function method is only approximate.

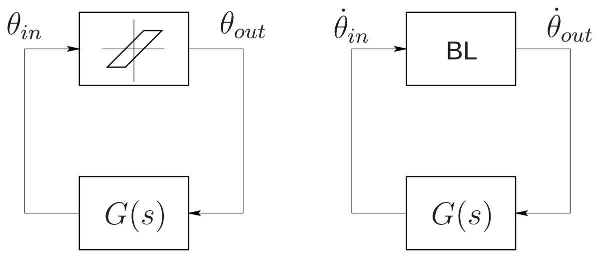
Do there exist conditions that **guarantee** stability?



Note that θ_{in} and θ_{out} will not converge to zero

Q: What about $\dot{\theta}_{in}$ and $\dot{\theta}_{out}$?

Rewrite the system:

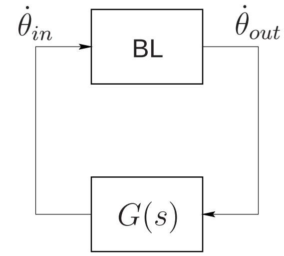


The block "BL" satisfies

$$\dot{\theta}_{out} = \begin{cases} \dot{\theta}_{in} & \text{in contact} \\ 0 & \text{otherwise} \end{cases}$$

Homework 2

Analyze this backlash system with input–output stability results:



Passivity Theorem BL is passive

Small Gain Theorem BL has gain $\gamma(BL) = 1$

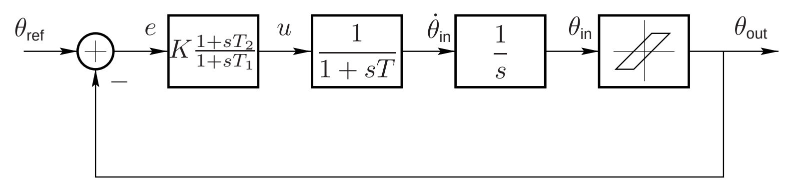
Circle Criterion BL contained in sector $[0, 1]$

Backlash Compensation

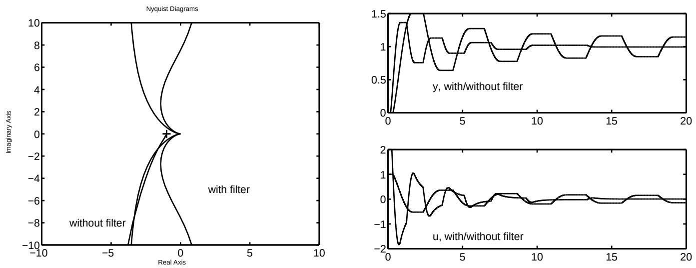
- Mechanical solutions
- Deadzone
- Linear controller design
- Backlash inverse

Linear Controller Design

Introduce phase lead compensation:



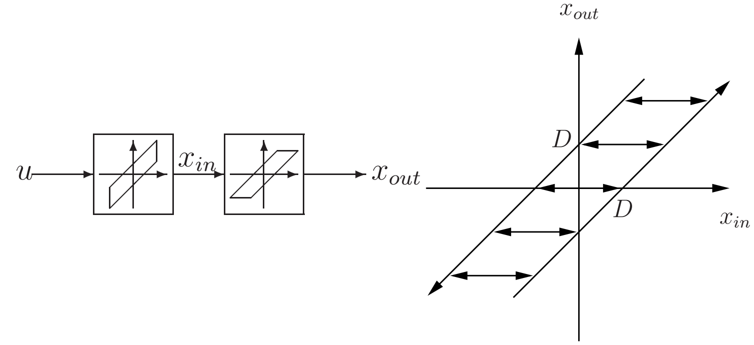
$F(s) = K\frac{1+sT_2}{1+sT_1}$ with $T_1 = 0.5, T_2 = 2.0$:

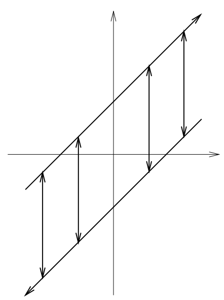


Oscillation removed!

Backlash Inverse

Idea: Let x_{in} jump $\pm 2D$ when $\dot{x}_{out}$ should change sign



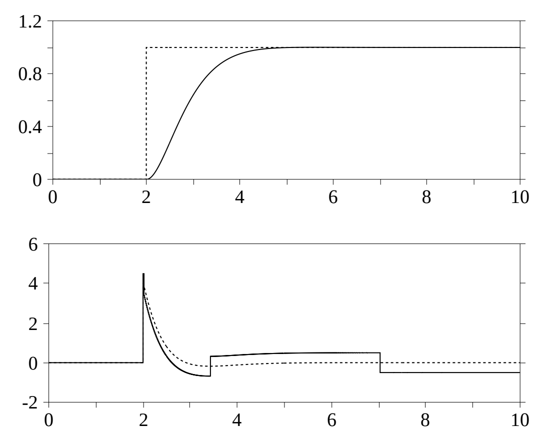


$$x_{in}(t) = \begin{cases} u + \widehat{D} & \text{if } u(t) > u(t-) \\ u - \widehat{D} & \text{if } u(t) < u(t-) \\ x_{in}(t-) & \text{otherwise} \end{cases}$$

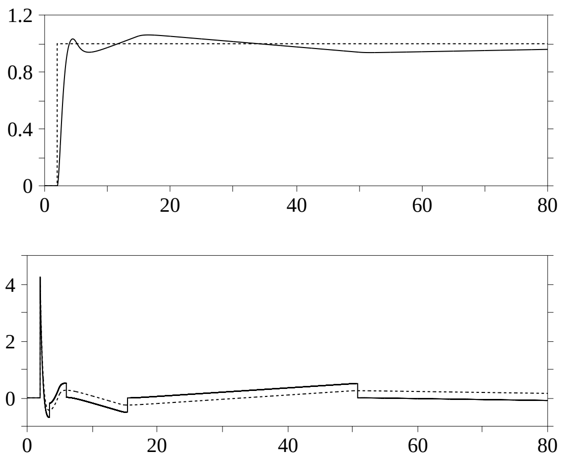
- If $\widehat{D} = D$ then perfect compensation ($x_{out} = u$)
- If $\widehat{D} < D$ then under-compensation (decreased backlash)
- If $\widehat{D} > D$ then over-compensation (may give oscillation)

Example—Perfect Compensation

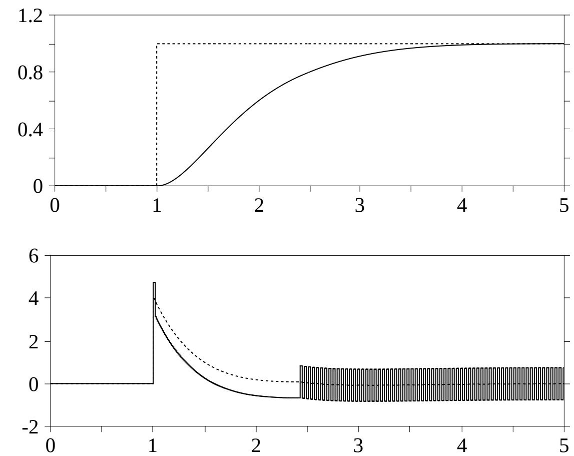
Motor with backlash on input in feedback with PD-controller



Example—Under-Compensation

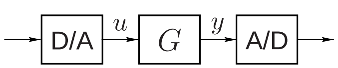


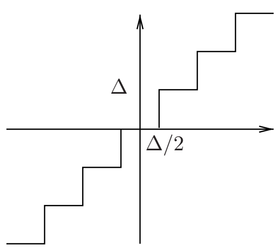
Example—Over-Compensation



Quantization

- What precision is needed in A/D and D/A converters? (8–14 bits?)
- What precision is needed in computations? (8–64 bits?)



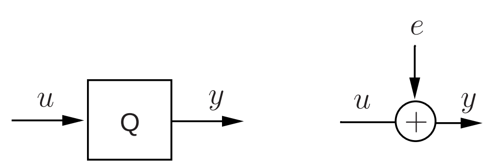


- Quantization in A/D and D/A converters
- Quantization of parameters
- Roundoff, overflow, underflow in computations

Linear Model of Quantization

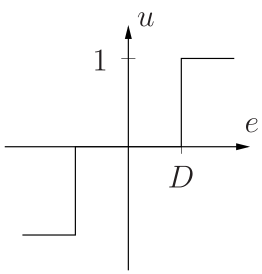
Model quantization error as a uniformly distributed stochastic signal e independent of u with

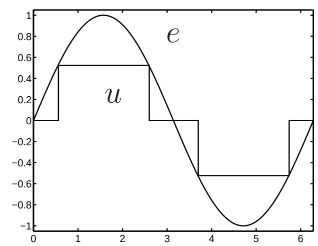
$$\mathrm{Var}(e) = \int_{-\infty}^{\infty} e^2 f_e \, de = \int_{-\Delta/2}^{\Delta/2} \frac{e^2}{\Delta} \, de = \frac{\Delta^2}{12}$$



May be reasonable if Δ is small compared to the variations in u

Describing Function for Deadzone Relay

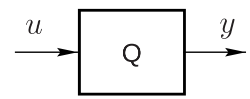


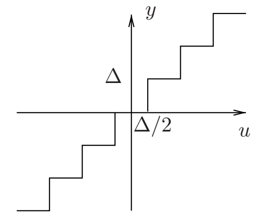


Lecture 6 $\Rightarrow$

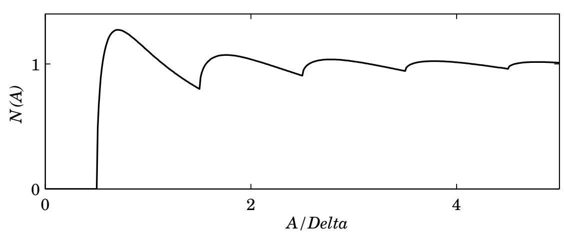
$$N(A) = \begin{cases} 0, & A < D \\ \frac{4}{\pi A}\sqrt{1 - D^2/A^2}, & A > D \end{cases}$$

Describing Function for Quantizer





$$N(A) = \begin{cases} 0, & A < \frac{\Delta}{2} \\ \frac{4\Delta}{\pi A}\sum_{i=1}^{n}\sqrt{1 - \left(\frac{2i-1}{2A}\Delta\right)^2}, & \frac{2n-1}{2}\Delta < A < \frac{2n+1}{2}\Delta \end{cases}$$



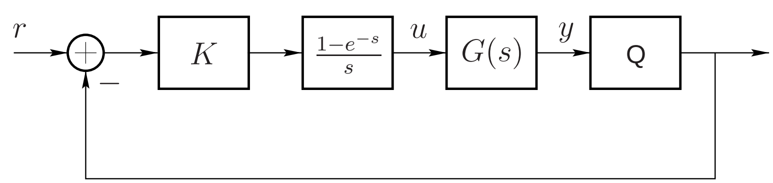
- The maximum value is $4/\pi \approx 1.27$ attained at $A \approx 0.71\Delta$.
- Predicts oscillation if Nyquist curve intersects negative real axis to the left of $-\pi/4 \approx -0.79$
- Controller with gain margin $> 1/0.79 = 1.27$ avoids oscillation
- Reducing Δ reduces only the oscillation amplitude

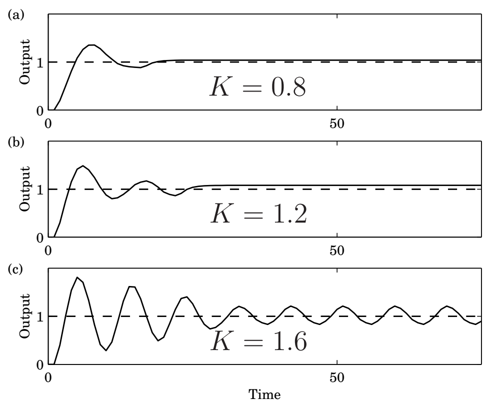
Example—Motor with P-controller.

Quantization of process output with $\Delta = 0.2$

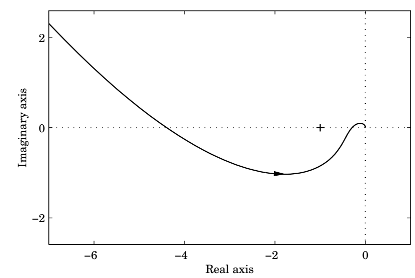
Nyquist of linear part (K & ZOH & $G(s)$) intersects at $-0.5K$: Stability for $K < 2$ without Q.

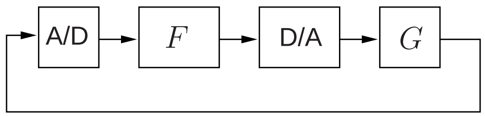
Stable oscillation predicted for $K > 2/1.27 = 1.57$ with Q.



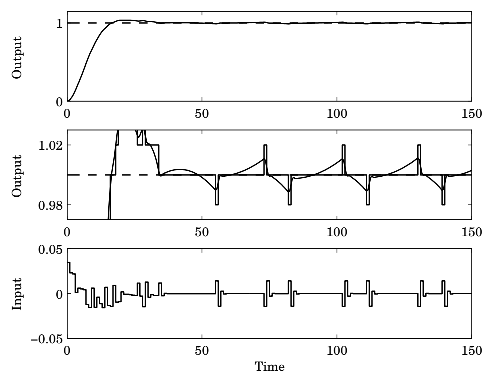


Example—$1/s^2$ & 2nd-Order Controller





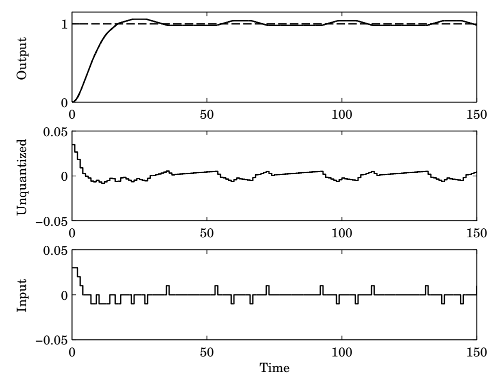
Quantization $\Delta = 0.02$ in A/D converter:



Describing function: $A_y = 0.01$ and $T = 39$

Simulation: $A_y = 0.01$ and $T = 28$

Quantization $\Delta = 0.01$ in D/A converter:

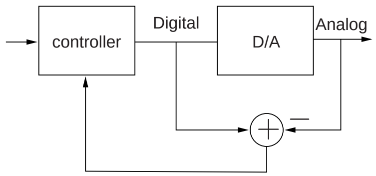


Describing function: $A_u = 0.005$ and $T = 39$

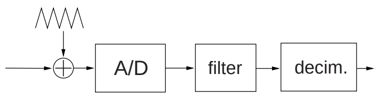
Simulation: $A_u = 0.005$ and $T = 39$

Quantization Compensation

- Improve accuracy (larger word length)
- Avoid unstable controller and gain margins < 1.3
- Use the tracking idea from anti-windup to improve D/A converter



- Use analog dither, oversampling, and digital lowpass filter to improve accuracy of A/D converter



Today's Goal

You should now be able to analyze and design for

- Backlash
- Quantization

Next Lecture

Nonlinear control design based on

- High-gain control
- Sliding-mode control