



KTH Machine Design

Dynamics and Motion control

Exercises part II
Modelling
of actuators and sensors



1. Modelling, simulation and analysis of DC-motor with load

The DC-motor with load depicted in figure 1 is the main modelling and control task of the exercises. It consist of a DC-motor with an incremental sensor mounted on the left hand side of the motor, the output of the motor is via a gearbox connected to a shaft which transfers the torque to the load on the right hand side. The load position is measured with a second incremental encoder. The first exercises will assume that the shaft can be modelled as a rigid body, where it later will be modelled as a flexible link between motor and load.

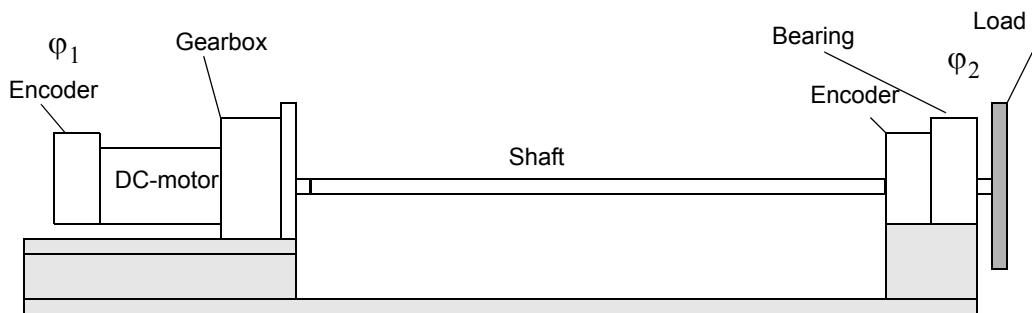


FIGURE 1. DC-motor with a gearbox two flywheels connected by a flexible shaft. Incremental encoders are mounted on both the motor (rotor position) and on the second flywheel.

Exercise 1.1 Model including inductance

Build a state space model using motor data from the data sheet in appendix 1 for the 12 V motor (012 S). The input to the motor is voltage, both position and velocity can be used as outputs. Assume that the shaft is stiff and without inertia.

Data: for load and gearbox and supply voltage and maximum current and friction in brushes.

Ratio of gearbox	$n = 5$
Load inertia	$J_l = 0.5e-6 \text{ kgm}^2$
Viscous friction in brushes	$d = 3.8e-6 \text{ Nms/rad}$
Supply voltage range	$V_{max} = 12 \text{ V}, V_{min} = -12 \text{ V}$
Maximum current	$I_{max} = 0.5 \text{ A}$

- Write a Matlab script that gives the following:
The state space model, poles & zeros, step response, frequency response.
- Build a Simulink model based on simple blocks, e.g., integrators, gains and summation blocks.
 - Simulate the state space block in the same Simulink model and verify that the result is the same as for the model based on simple blocks.
 - Check that a simulation in Simulink gives the same result as a step in Matlab

Exercise 1.2 Simplified model without inductance

Simplify the model by neglecting the inductance, show both position and velocity as outputs. Deliver the following:

- States space and transfer function models both in Matlab and Simulink and a block diagram model in Simulink
1. with voltage as input.
 2. with current as input.
 3. Compare the results with those from exercise 1.1.
 4. **Analysis:** poles & zeros, step response and frequency response

Exercise 1.3 Include static friction in the motor and gearbox

Include a static friction model in the model from exercise 1.2 in Simulink. Simulate both with and without static friction to show the effects of the static friction. See appendix B for a dataflow diagram for implementing the algorithm.

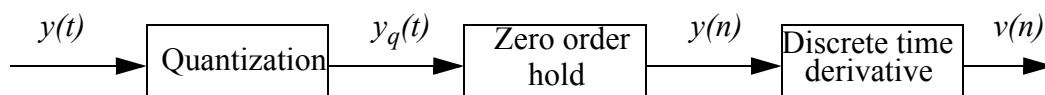
Total static friction for motor and gearbox, $F_c = 4e-4 Nm$

- step response
- sine wave input, $u = a \sin(\omega t)$, vary amplitude, a and frequency, ω and show when the static friction has most influence. For which voltage does the motor start?

Exercise 1.4 Sensor simulation

Include a model of the incremental encoder in Simulink, it should be simple to change the number of steps of the sensor. The sensor data sheet is in appendix B.

To simulate the behaviour of the real sensor and how the velocity is calculated on the embedded computer should the discrete time derivative of the sensor signal be included as shown in the figure below.



When you plot discrete time signals in Matlab you should use the command *stairs* instead of *plot*. *stairs* shows the signal level such as it is seen by the embedded computer whereas *plot* interpolates between samples.

- Simulate with two different sensor resolutions, 2048 and 8192 pulses per revolution and three different sampling intervals, 0.1 s, 10 ms, and 1 ms.
- Zoom in on the velocity and compare the simulated quantization level with the theoretical quantization level. **Tip**, you may have to do a simulation which gives low velocities to get the correct simulated quantization. Why?

Exercise 1.5 Gearbox and flexible shaft

When the stiffness of the shaft between load and motor is low with respect to the closed loop bandwidth of the controller it has to be modelled.

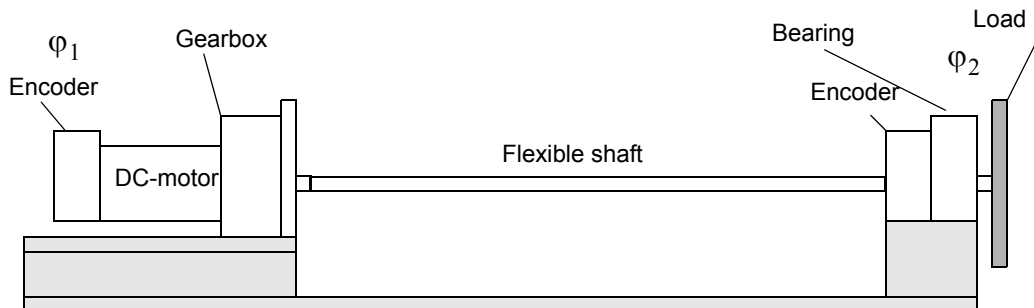


FIGURE 2. DC-motor with a gearbox two flywheels connected by a flexible shaft. Incremental encoders are mounted on both the motor (rotor position) and on the second flywheel.

Parameters:

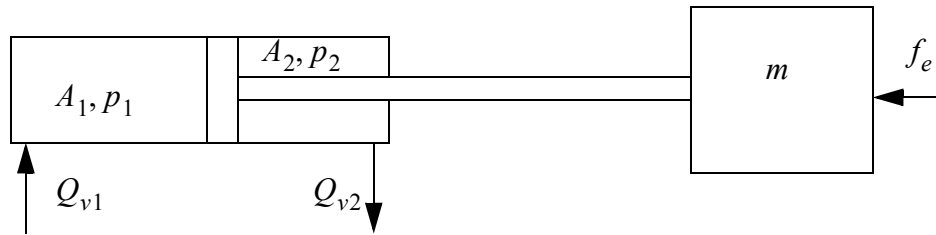
Gearbox ratio	$n = 5$
Shaft stiffness	$k_f = 0.01 \text{ Nm/rad}$
Viscous friction (damping) in shaft	0.00001 Nms/rad
Viscous friction at the bearings of second flywheel	0.0 Nms/rad

Deliver the following material:

1. The 4:th order state space model with voltage as input.
2. Poles and zeros of the model, the zeros should be for both motor position and load position. Why are they different?
3. Step and frequency response. (compare with ex. 1.1)
4. Divide the model into two parts one that is the model from 2.1 and the other part that corresponds to the resonance and anti-resonance frequencies. Verify that it is correct.

Exercise 1.6 Hydraulic actuator

Derive the nonlinear model for the non symmetric valve driven hydraulic cylinder shown in the figure. Build a Simulink model which can move the mass in both directions that is, so that either the first cylinder chamber is connected to the pump and the second to the tank or opposite. In the figure are the flow directions for extracting motion shown (to the right). The other direction with opposite flows as in the figure is called retraction (to the left).



Data: Cylinder:

Cylinder inner diameter	5.0 [cm].
Piston rod diameter	2.0 [cm].
Maximum piston stroke	0.7 [m]
Total moving mass	100 [kg]
Linear friction coefficient	200 [Ns/m]

Valve:

Flow constant	$R_v = 1e-4$
Maximum valve opening	1.0 [cm]

The valve dynamics can be neglected.

Pump:

Supply pressure	20 [MPa]
Tank return pressure	0.0 [MPa]

Simulate the nonlinear model for different constant valve openings for both positive and negative valve signals.

Apply a external force of $f_e = 10000$ [N] when the oscillations have stopped.

Plot both velocity and pressures, explain the difference in steady state pressures p_1 and p_2 .

Linearization:

Derive a linear model of the cylinder, set the crosssectional cylinder chamber areas equal. Take the mean value of A_1 and A_2 . Try to linearize around different operating points, i.e., different external force f_e and different valve openings x_v .

Compare the simulation results between the non linear and the linear models, look at both velocity and load pressure. Where are the poles and zeros located and what influences them the most?

The linear model is an approximation of the nonlinear model, what seems to make the difference between the models the largest?