



Efternamn, förnamn

Personnummer

Program

Blad nr

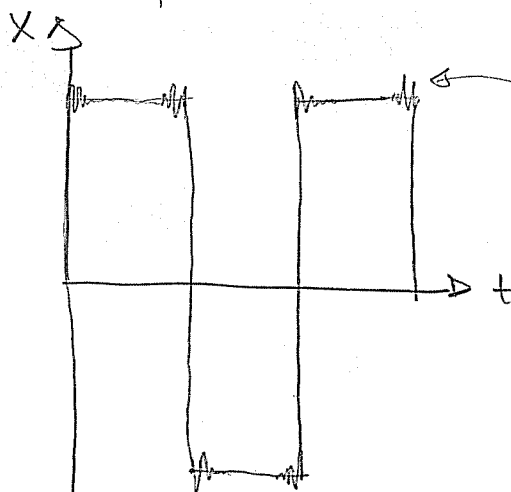
Uppgift nr

2011-11-07 ANSWERS by Karl Botin

P1. a) Transient & random

b) Transient & deterministic, not periodic
 $x(t) \neq x(t+T)$

P2. a) Gibbs phenomena



Non perfect description
of discontinuous function.

b) Gibbs phenomena occur due to the non-convergence of the Fourier series, that is, even if $N \rightarrow \infty$ a local overshoot at discontinuous points occur.



Efternamn, förnamn

Personnummer

Program

Blad nr

Uppgift nr

$$P3. \quad W(\omega) = F(\omega(t)) = \int_1^3 1 \cdot e^{-i\omega t} dt = -\frac{1}{i\omega} \left[e^{-i\omega t} \right]_1^3 =$$

$$= -\frac{1}{i\omega} \left(e^{-i3\omega} - e^{-i\omega} \right) = -\frac{1}{i\omega} e^{-i2\omega} \left(e^{-i\omega} - e^{i\omega} \right) =$$

$$= \left\{ \sin \omega = \frac{e^{i\omega} - e^{-i\omega}}{2i} \right\} = \boxed{2 \cdot \frac{e^{-i2\omega}}{\omega} \cdot \sin \omega}$$

$$W(\omega = \frac{\pi}{4}) = \frac{8}{\pi} e^{-i\frac{\pi}{2}} \cdot \sin \frac{\pi}{4}$$

$$\text{Amplitude: } |W(\frac{\pi}{4})| = \frac{8}{\pi\sqrt{2}} \approx 1,8$$

$$\text{Phase: } \arg(W(\frac{\pi}{4})) = -\frac{\pi}{2} \text{ rad} = -90^\circ$$



Efternamn, förnamn

Personnummer

Program

Blad nr

Uppgift nr

$$4. \quad f_{\max} = 1.6 \cdot 10^3 \text{ Hz}, \quad f_s = 2 f_{\max} \Rightarrow f_s = 3.2 \cdot 10^3 \text{ Hz}$$

alt

2.56 f_{\max} also ok

$$\Delta f = 10 \text{ Hz}$$

$$N = f_s / \Delta f = 3.2 \cdot 10^2$$

§ p 321 (Mathematical table):

$$\text{DFT} = N^2$$

$$\text{FFT} = N \log_2 N$$

N_s , number of spectra

$N_c = 10^5$, number of possible operations

$$N_{s_{\text{DFT}}} = \frac{N_c}{N^2} = \frac{10^5}{(3.2 \cdot 10^2)^2} \approx 0.98 \quad (\text{Not real time})$$

$$N_{s_{\text{FFT}}} = \frac{N_c}{N \log_2 N} \approx 38 \quad (\text{Real time possible})$$



Efternamn, förnamn

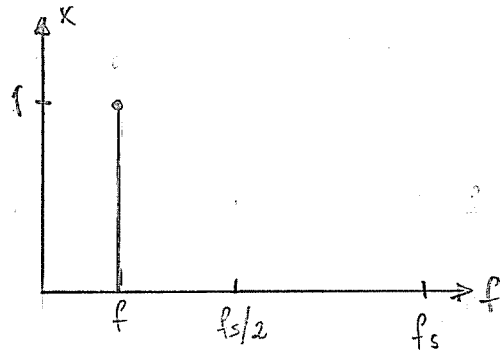
Personnummer

Program

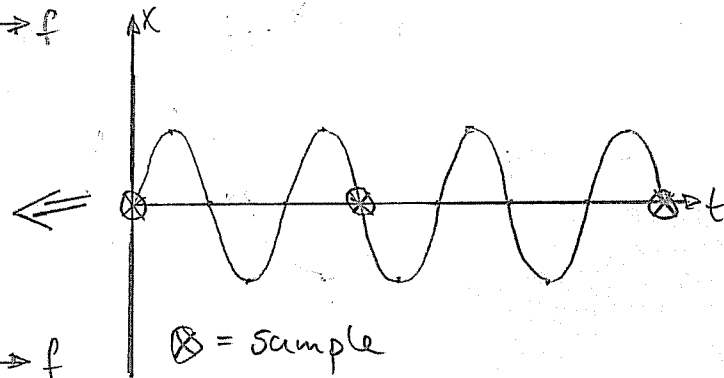
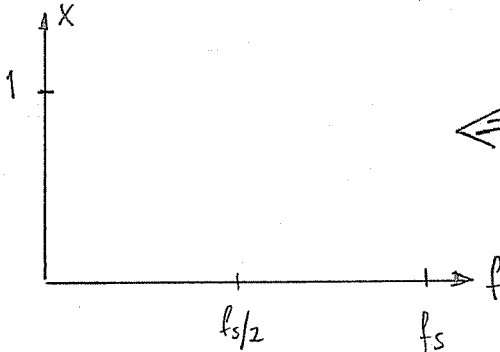
Blad nr

Uppgift nr

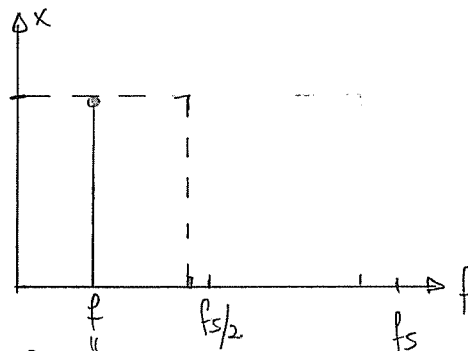
5. a) [1p]



b) [1p]



c) [1p]



$$f_s = 2.5f$$

$$f_s = f/2.5 \Rightarrow X_s(f) = 0$$

----- low-pass filter
(ideal, also ok
with non ideal)



Efternamn, förnamn

Personnummer

Program

Blad nr

Uppgift nr

P 6.
$$Y(\omega) = H(\omega) X(\omega) \Rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

↑

$$F(y(t)) = F(h(t)) F(x(t))$$

CONVOLUTION

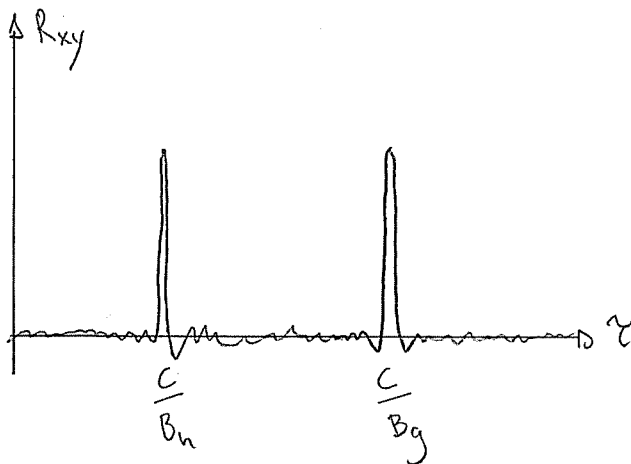
B p 313 F 13 :

$$F(\omega) G(\omega) \xrightarrow[\text{Fourier Transform}]{\text{Inverse}} f * g(t) = \int_{-\infty}^{\infty} f(t-\tau) g(\tau) d\tau$$

Inverse Fourier transform and convolution gives :

$$F^{-1}(Y(\omega)) = y(t) = F^{-1}(F(h(t)) F(x(t))) = h * g(t) = \int_{-\infty}^{\infty} f(t-\tau) g(\tau) d\tau$$

P 7.



Systems identical \Rightarrow constant product $C = vB$

$$\Rightarrow \tau_h = C/B_h < \tau_g = C/B_g$$

White noise \Rightarrow Peaks at time delay corresponding to propagation times τ_h and τ_g , otherwise $R_{xy} \sim 0$