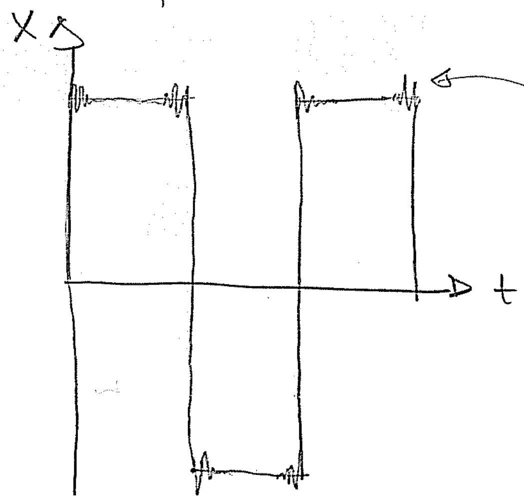
Efternamn, förnamn	Personnummer	Program	Blad nr	Uppgift nr

2011 - 11 - 07 ANSWERS by Karl Bolin

P1. a) Transient $\neq$ random

b) Transient = deterministic, <u>not</u> periodic $x(t) \neq x(t+T)$

P2. a) Gibbs phenomena



Non perfect description of discontinous function.

b) Gibbs phenomena occur due to the non-convergence of the fourier series, that is, even if $N \to \infty$ a local overshoot at discontinous points occur.

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P3. $W(\omega) = \mathcal{F}(w(t)) = \int_1^3 1 \cdot e^{-i\omega t}\, dt = -\frac{1}{i\omega}\left[e^{-i\omega t}\right]_1^3 =$

$$= -\frac{1}{i\omega}\left(e^{-i3\omega} - e^{-i\omega}\right) = -\frac{1}{i\omega} e^{-i2\omega}\left(e^{-i\omega} - e^{i\omega}\right) =$$

$$= \left\{ \sin \omega = \frac{e^{i\omega} - e^{-i\omega}}{2i} \right\} = \boxed{2 \cdot \frac{e^{-i2\omega}}{\omega} \cdot \sin \omega}$$

$$W\left(\omega = \frac{\pi}{4}\right) = \frac{8}{\pi} e^{-i\frac{\pi}{2}} \cdot \sin \frac{\pi}{4}$$

Amplitude: $\left| W\left(\frac{\pi}{4}\right) \right| = \frac{8\sqrt{2}}{\pi}$

Phase : $\arg\left(W\left(\frac{\pi}{4}\right)\right) = -\frac{\pi}{2}$

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4. $f_{max} = 1.6 \cdot 10^3$ Hz , $f_s = 2 f_{max} \Rightarrow f_s = 3.2 \cdot 10^3$ Hz

alt.

2.56 f_{max} also ok

$\Delta f = 10$ Hz

$N = f_s / \Delta f = 3.2 \cdot 10^2$

β p 321 (Mathematical table):

$DFT = N^2$

$FFT = N \log_2 N$

N_s, number of spectra

$N_C = 10^5$, number of possible operations

$$N_{s_{DFT}} = \frac{N_c}{N^2} = \frac{10^5}{(3.2 \cdot 10^2)^2} \simeq 0{,}98$$ (Not real time)

$$N_{s_{FFT}} = \frac{N_c}{N \log_2 N} \simeq 38$$ (Real time possible)

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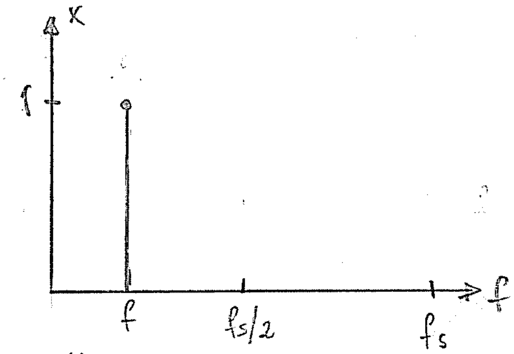
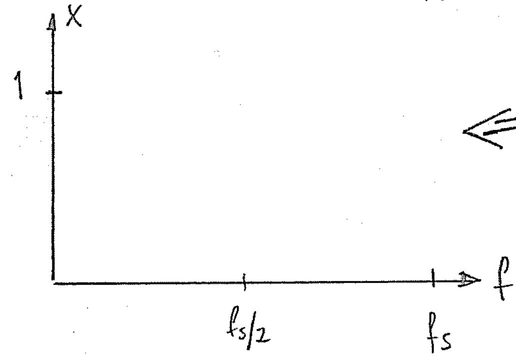
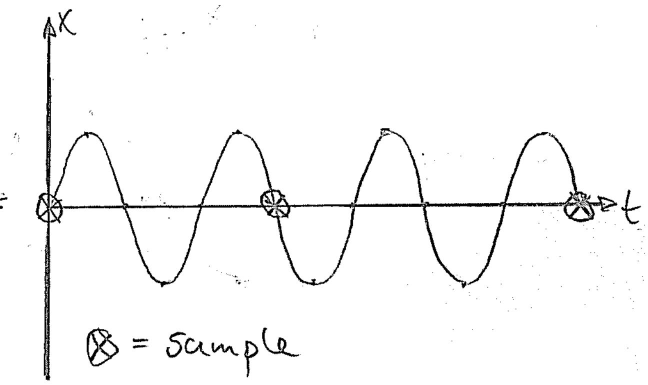
5. a) [1p]

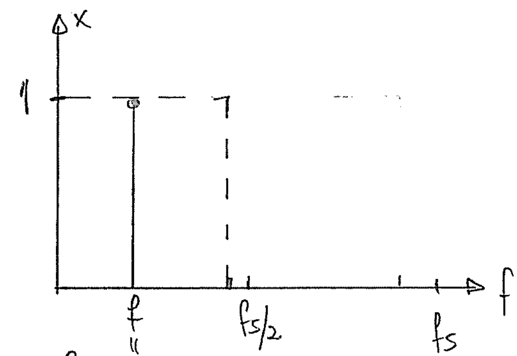


b) [1p]





c) [1p]



– – – low-pass filter (ideal, also ok with non ideal)

$f_s = 2.5 f$

$f_s = f/2.5 \Rightarrow X_s(f) = 0$

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P6.

$$Y(\omega) = H(\omega)\,X(\omega) \Rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$\Updownarrow$$

$$\mathcal{F}(y(t)) = \mathcal{F}(h(t))\,\mathcal{F}(x(t))$$

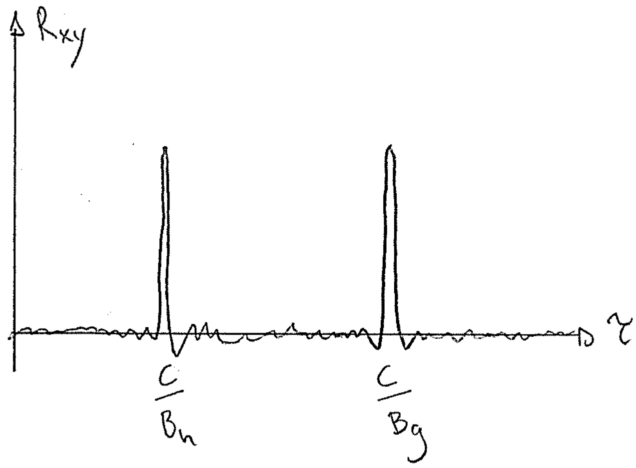
CONVOLUTION

$$\boxed{\begin{array}{l} \beta \quad p\ 313 \quad F\,13: \\ F(\omega)\,G(\omega) \xrightarrow[\text{Fourier Transform}]{\text{Inverse}} f * g(t) = \int_{-\infty}^{\infty} f(t-\tau)\,g(\tau)\,d\tau \end{array}}$$

Inverse fourier transform and convolution gives:

$$\mathcal{F}^{-1}(Y(\omega)) = y(t) = \mathcal{F}^{-1}\Big(\mathcal{F}(h(t))\,\mathcal{F}(x(t))\Big) = h * g(t) = \int_{-\infty}^{\infty} f(t-\tau)\,g(\tau)\,d\tau$$

P7.



Systems identical $\Rightarrow$ constant product $C = \tau B$

$$\Rightarrow \tau_h = C/B_h < \tau_g = C/B_g$$

White noise $\Rightarrow$ Peaks at time delay corresponding to propagation times τ_h and τ_g, otherwise $R_{xy} \sim 0$