Exempel med Lösningar Skriftlig Tentamen
IE1204 Digital Design
2021-2022

Examiner/Examinator: Carl-Mikael Zetterling
Responsible teacher/Ansvarig lärare: Carl-Mikael Zetterling, 08-790 4344

Swedish/Svenska:
Tentamenstexten ska lämnas in när lösningarna lämnas in.
Inga tillåtna hjälpmedel utom linjal.
Examen består av två delar:
Del 1 har 16 uppgifter med max 1 poäng per uppgift som ska besvaras på "Answer Form".
Del 2 har 4 uppgifter med max 4 poäng per uppgift som ska besvaras på separat papper.
Lämna in båda delar samtidigt. Disponera tiden själv mellan delarna.
Uppgifterna är inte ordnade efter svårighetsgrad.
Om slutsumman av tentan har halvpoäng avrundas det uppåt.

$X = 1$ om studenten har minst 8 poäng på del 1
$Y = 1$ om studenten har minst 16 poäng totalt
$P = 1$ om studenter får godkänt på tentamen
$Fx = 1$ om studenten kan godkännas efter en extra uppgift

X	Y	P	Fx
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Totalt max 32 poäng på tentamen.
För godkänt krävs
(minst 8 poäng på del 1) OCH (minst 16 poäng totalt)
Fx om något villkor ej är uppfyllt med en poängs marginal.

Betygskalan för tentamen förutsatt att studenten har minst 8 poäng från del 1.

0-15	16-18	19-21	22-24	25-27	28-31	32
F	E	D	C	B	A	A+

Resultat meddelas inom tre veckor.

Extended Solutions Written Exam
IE1204 Digital Design
2021-2022

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English:
The exam text should be handed in after the exam.
No aids allowed except ruler.
The exam consists of two parts:
Part 1 has 16 exercises for max 1 point per exercise to be answered on the "Answer Form".
Part 2 has 4 exercises for max 4 points per exercise, to be answered on a separate paper.
Hand in both parts at the same time. Plan the time yourself between the parts.
The exercises are not in order of difficulty.
If the total sum of the exam has half points this will be rounded up.

X = 1 if student has at least 8 points from part 1
Y = 1 if student has at least 16 points in total
P = 1 if student passes exam
Fx = 1 if a student can pass after an extra task

X	Y	P	Fx
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Total max of 32 points on the exam.
To pass the exam requires
(at least 8 points from part 1) AND (at least 16 points in total)
Fx if any condition is not fulfilled by one point's margin.

Grades are given as follows provided the student has at least 8 points from part 1.

0-15	16-18	19-21	22-24	25-27	28-31	32
F	E	D	C	B	A	A+

The result will be announced within three weeks.

Del 1/Part 1, 1 point per exercise, fill in on "Answer Form"

1 Number Conversion

Swedish: Talet A är ett 16 bitars binärt tal.
Konvertera till ett hexadecimal tal.

English: The number A is an 16-bit binary number.
Convert to a hexadecimal number.

$A = 1010\ 1100\ 1101\ 1100_2$
Divide into groups of four bits, and convert each group to hexadecimal → ACDC

2 Addition

Swedish: Konvertera A och B från decimala till 8 bitars två-komplement kodade tal.
Beräkna A + B (binärt) och svara med ett 8 bitars två-komplement kodat tal.
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: Convert A and B from decimal to 8-bit binary (two's complement) numbers.
Calculate A + B (binary) and answer with an 8-bit binary (two's complement) number.
You can check your calculations using decimal numbers.

$A = 75_{10}$ 01001011
$B = 25_{10}$ + 00011001
$A + B$ = 01100100 = 100_{10}

3 Subtraction

Swedish: Konvertera A och B från decimala till 8 bitars två-komplement kodade tal.
Beräkna A - B (binärt) och svara med ett 8 bitars två-komplement kodat tal.
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: Convert A and B from decimal to 8-bit binary (two's complement) numbers.
Calculate A - B (binary) and answer with an 8-bit binary (two's complement) number.
You can check your calculations using decimal numbers.

$A = 35_{10}$ 00100011
$B = 77_{10}$ 01001101
$-B = -77_{10}$ + 10110011
$A - B$ = 11010110 = -42_{10}

4 CMOS

Swedish: Bestäm vilken/vilka av CMOS-grindnäten nedan som har den logiska funktionen Y. Det kan finnas mer än ett korrekt svar.

English: Determine which of the CMOS-circuits below have the logic function Y. There may be more than one correct answer.

$$Y = \overline{A} + \overline{B} + \overline{C} = \overline{A \cdot B \cdot C}$$ (De Morgans theorem), 3 input NAND

#1 is a 3 input NAND, OK
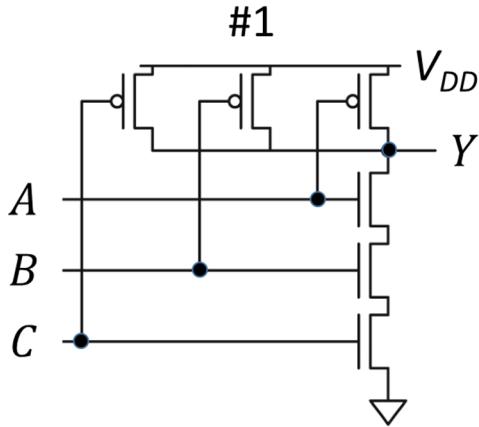
#2 $Y = \overline{\overline{A} + \overline{B} + \overline{C}} = A \cdot B \cdot C$ 3 input AND
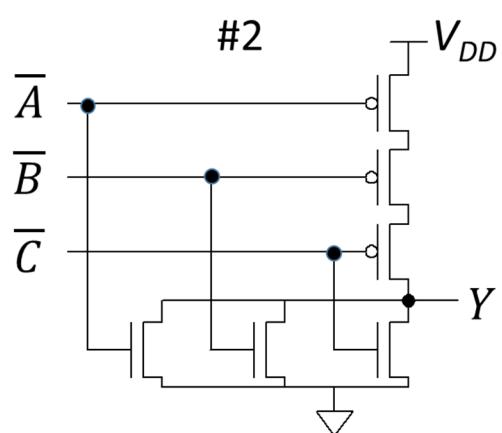
#3 $Y = \overline{\overline{\overline{A} + \overline{B}} \cdot C} = \overline{\overline{\overline{A} + \overline{B}}} + \overline{C} = \overline{A} + \overline{B} + \overline{C}$

OR $Y = \overline{\overline{\overline{A} + \overline{B}} \cdot C} = \overline{(\overline{\overline{A} \cdot \overline{B}}) \cdot C} = \overline{A \cdot B \cdot C}$ OK

#4 $Y = \overline{\overline{\overline{A} \cdot \overline{B}} \cdot C} = \overline{A} \cdot \overline{B} + \overline{C}$
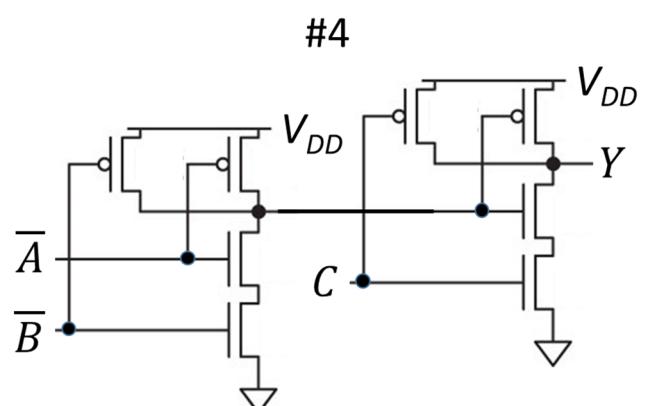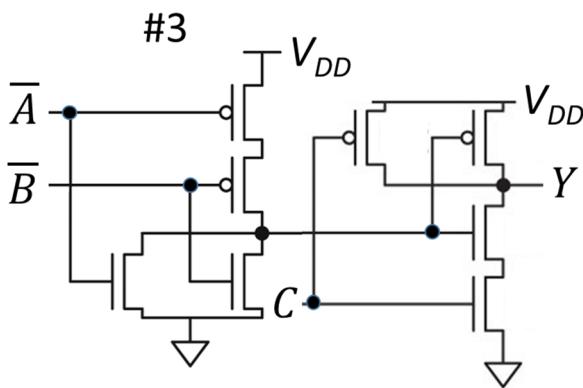
Answer: #1 and #3

5 Boolean Algebra

Swedish: Ta fram enklast möjliga booleska uttryck.

English: Derive the simplest possible Boolean expression.

$$Y = B + A \cdot D + \overline{(A + B + C)} \cdot D + A \cdot B \cdot C \cdot D =$$
$$B + A \cdot D(1 + C \cdot D) + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D = B + A \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D$$
$$= B + D(A + \overline{A} \cdot \overline{B} \cdot \overline{C}) = B + D(A + \overline{B} \cdot \overline{C})$$
$$= B(1 + D) + A \cdot D + D(\overline{B} \cdot \overline{C}) = B + A \cdot D + D(B + \overline{B} \cdot \overline{C})$$
$$= B + A \cdot D + D(B + \overline{C}) = B + B \cdot D + A \cdot D + \overline{C} \cdot D$$
$$= B(1 + D) + A \cdot D + \overline{C} \cdot D = B + A \cdot D + \overline{C} \cdot D$$

Can be verified with a K-Map:

$$Y = \boxed{B} + \boxed{A \cdot D} + \boxed{\overline{(A + B + C)} \cdot D} + \boxed{A \cdot B \cdot C \cdot D} = \boxed{B} + \boxed{A \cdot D} + \boxed{\overline{C} \cdot D}$$

Y	CD 00	CD 01	CD 11	CD 10
AB 00		1		
AB 01	1	1	1	1
AB 11	1	1	1	1
AB 10		1	1	

| Y	CD 00	CD 01	CD 11	CD 10
AB 00		1		
AB 01	1	1	1	1
AB 11	1	1	1	1
AB 10		1	1	

6 MUX to K-map

Swedish: Fyll i K-Map från MUX-kopplingen.
Ta fram enklast möjliga booleska uttryck för Y från K-map. Välj PoS eller SoP.

English: Fill in the K-Map from the MUX circuit.
Derive simplest possible Boolean expression from the K-map. Select PoS or SoP.

Y	CD 00	CD 01	CD 11	CD 10
AB 00	0	0	1	0
AB 01	0	1	1	0
AB 11	0	1	1	0
AB 10	0	0	1	0
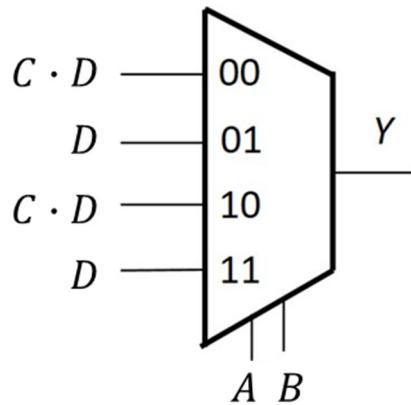
$$Y = B \cdot D + C \cdot D = D(B + C)$$

7 K-Map to MUX

Swedish: Använd en 4:1 MUX och valfria grindar eller 0 och 1 och gör en krets för K-map med CD som select-signaler.

English: Use a 4:1 MUX and any logic gates or 0 or 1 to draw a circuit for the K-map with CD as select signals.

| Y	CD 00	CD 01	CD 11	CD 10
AB 00	1	1	0	0
AB 01	0	1	1	1
AB 11	0	0	0	1
AB 10	0	0	1	1
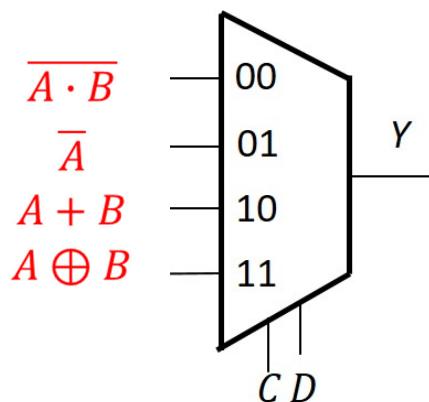
8 Timing diagram

Swedish: Rita tidsdiagram för D-latchen (D latch) i "Answer Form".

English: Draw the timing diagram for the D latch in the "Answer Form".
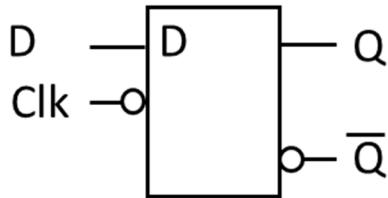

**NOTE: transparent when Clk = 0**
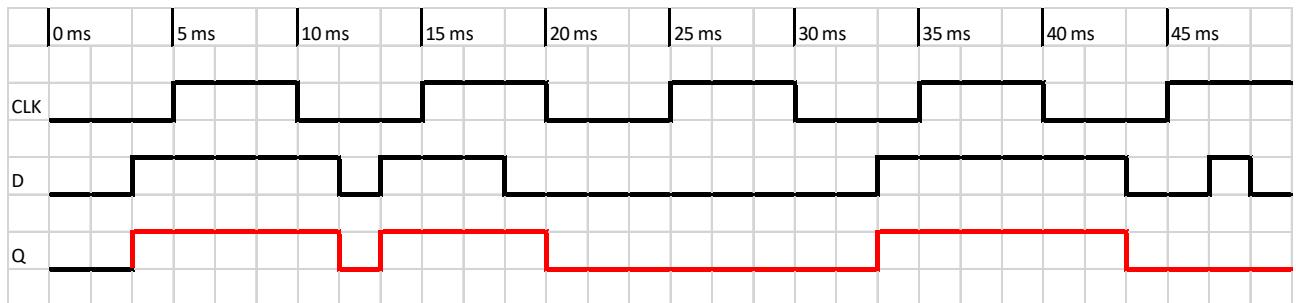
9 Timing diagram

Swedish: Rita tidsdiagram för T-vippan (T flip-flop) i "Answer Form".

English: Draw the timing diagram for the T flip-flop in the "Answer Form".
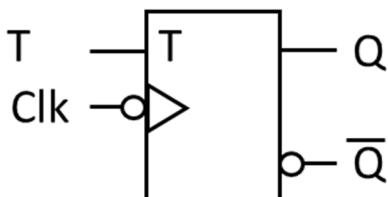
NOTE: negative edge; toggles if T = 1
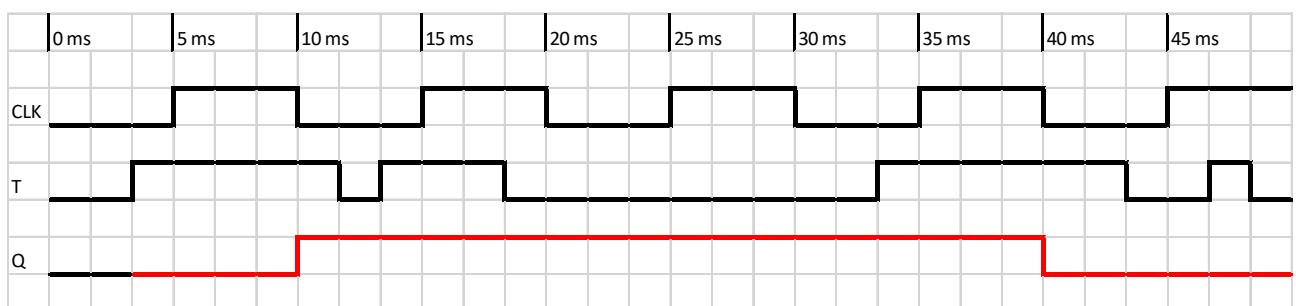
10 Timing calculation

Swedish: Beräkna maximal klockfrekvens för kretsen nedan?

English: Calculate the maximum clock frequency for this circuit?

Delay Per Register
t_{pcq} = 50 ps
t_{ccq} = 30 ps
t_{setup} = 60 ps
t_{hold} = 60 ps

Delay Per gate:
t_{pd} = 30 ps
t_{cd} = 20 ps
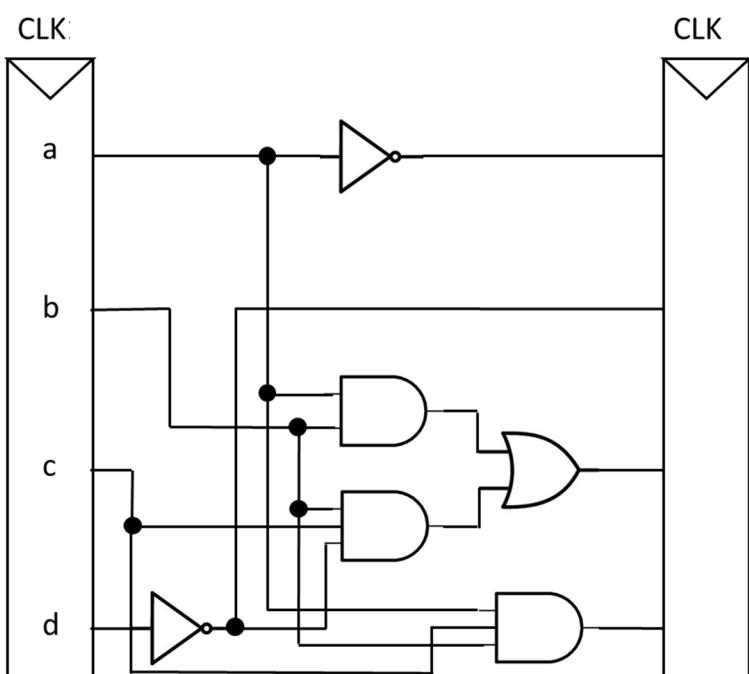
The longest path is three gates

$T_c \geq t_{pcq} + t_{pd} + t_{setup} = 50 + 3 \times 30 + 60 = 200$ ps

$f_c = 1/T_c = 5$ GHz

11 Counter

Swedish: Räknaren nedan har kommit till tillstånd $Q_D Q_C Q_B Q_A = 1110$
Vad blir nästa tillstånd?

English: The counter below has counted to state $Q_D Q_C Q_B Q_A = 1110$
What is the next state?
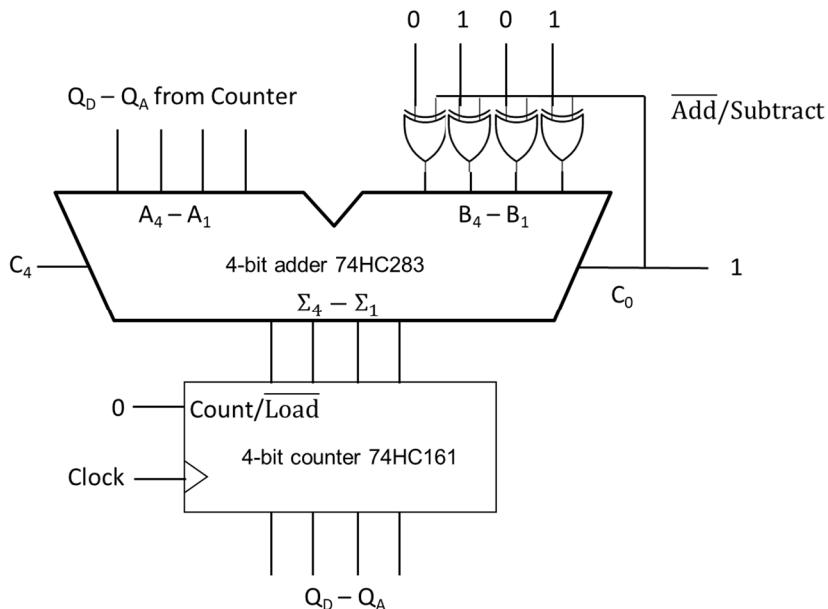
Subtract 5 for each clock, after 1110 (=14) comes 1001 (=9)

12 Multiplication

Swedish: A och B är 8 bitars två-komplement kodade tal.
Beräkna A x B (binärt) och svara med ett 16 bitars två-komplement kodat tal.
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: A and B are 8-bit binary (two's complement) numbers.
Calculate A x B (binary) and answer with a 16-bit binary (two's complement) number.
You can check your calculations using decimal numbers.

$A = 11111001_2$
$B = 01000111_2$ $= 71_{10}$

A is negative, convert to positive: $-A = 00000111_2 = 7_{10}$
Multiply $-A \times B = B \times (-A)$:

```
        01000111
    x   00000111
        01000111   1st multiplication
    +   01000111   2nd multiplication
       011010101   add intermediate result first two rows
    +   01000111   add 3rd multiplication
000000111110001   = 497₁₀ (511 - 14)

1111111000001111  Two's complement = A x B
```

13 Division

Swedish: A och B är 8 bitars två-komplement kodade tal.
Beräkna A / B (binärt) och svara med kvot och rest (8 bitars två-komplement kodade tal).
Tänk på att du kan kontrollera dina beräkningar med decimala tal.

English: A and B are 8-bit binary (two's complement) numbers.
Calculate A / B (binary) and answer with quotient and remainder (8-bit binary two's complement numbers).
You can check your calculations using decimal numbers.

$A = 01010110_2$ $=86_{10}$
$B = 00010001_2$ $=17_{10}$

(A and B are both positive, nothing to convert)

```
            101   Quotient = 5, check: 5 x 17 = 85
10001/01010110
    -  10001
          10010
       -  10001
              1   Remainder
```

14 Fixed point

Swedish: Ett format för fixed point tal med tecken är att använda två-komplementtal
med 4 bitar för talet och 4 bitar för bråkdelen. Omvandla A till ett decimalt tal.

English: One format for fixed point signed numbers is to use two's complement numbers
with 4 bits for the integer and 4 bits for the fraction. Convert A to a decimal number.

$A = 0101.1010_2$
$A = 4 + 1 + \frac{1}{2} + 1/8 = 5.5 + 0.125 = 5.625$

15 Full Adder

Swedish: Vad blir resultatet från heladderarkretsen nedan?
Svara med 5 bitar (S4 S3 S2 S1 S0)

English: What is the result for the full adder circuit below?
Answer with 5 bits (S4 S3 S2 S1 S0)

$A = 0101$ $B = 1100$ $\overline{Add}/Sub\ 1 = 0$ $\overline{Add}/Sub\ 2 = 1$
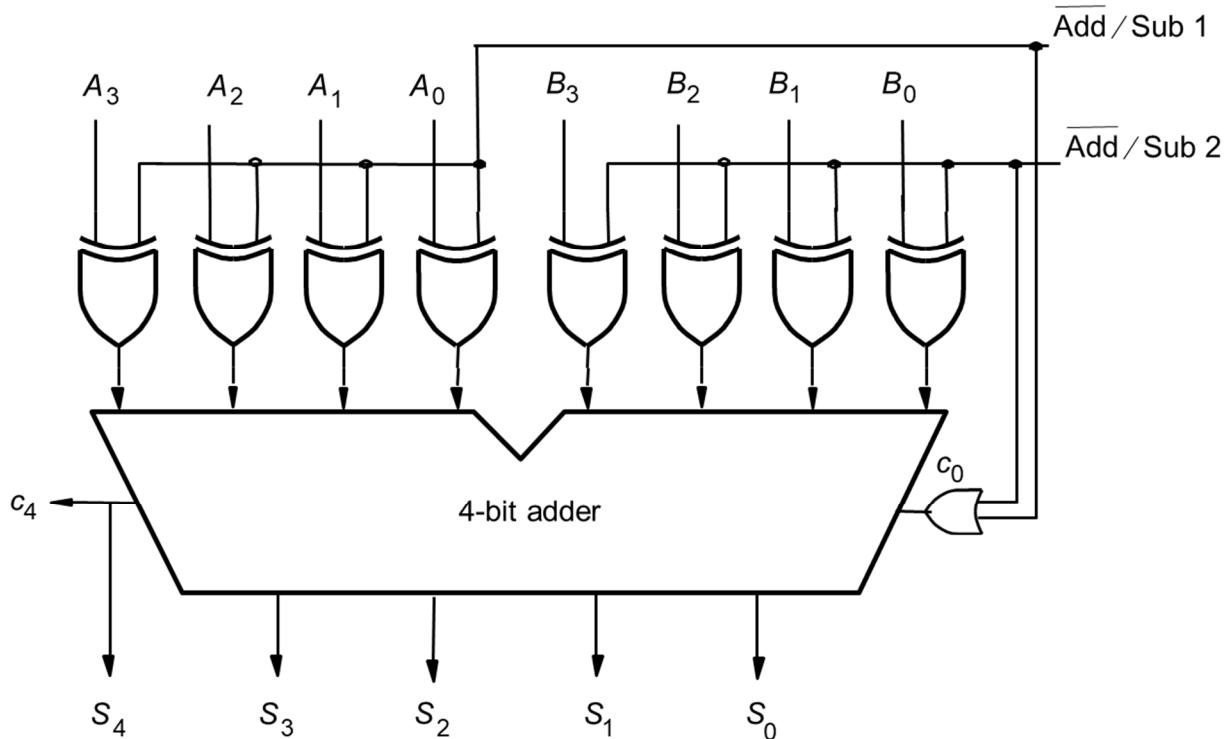
A is not affected since $\overline{Add}/Sub\ 1 = 0$

B will be subtracted since $\overline{Add}/Sub\ 2 = 1$

$S = A - B = 0101 - 1100 = 0101 + 0011 + 1 = 01001$ (no carry out)

Check: $A = 5$, $B = -4$, $S = 5 - (-4) = 9$

16 ALU

Swedish: Med A och B enligt nedan blev flaggorna i ALU som visas.
Vilken instruktion genomfördes? Svara med 2 bitar för ALUControl.

English: Using A and B below, the flags gave the result shown.
Which instruction was executed? Answer with 2 bits for ALUControl.

A = 1000   B = 0110   →   V C N Z = 0001
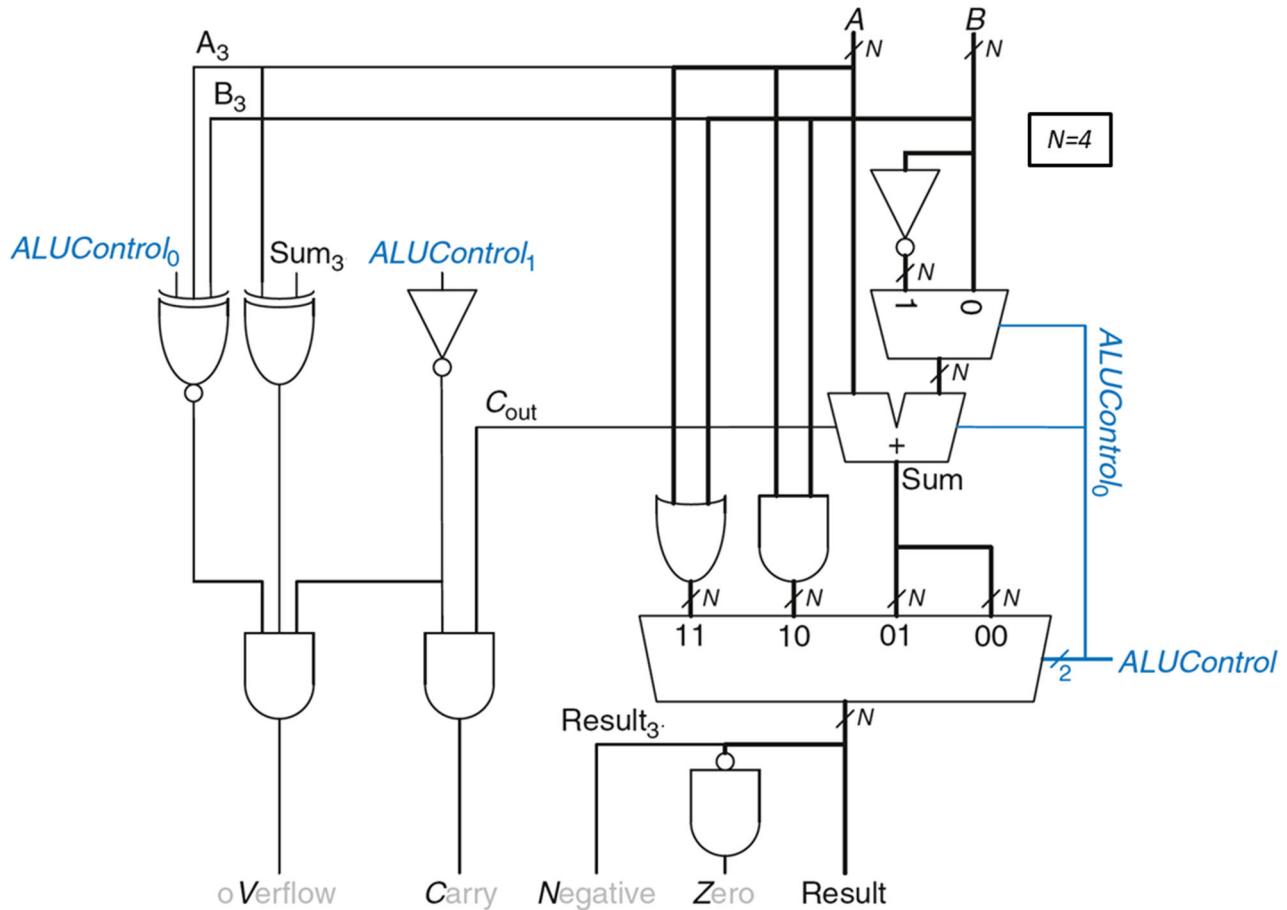
ALUControl$_{1:0}$	Function
0 0	Add
0 1	Subtract
1 0	AND
1 1	OR

Since Z = 1 the result is 0000

Adding or subtracting the A and B will not have a 0000 result

A AND B = 0000,   A OR B = 1110

# Del 2/Part 2, 4 points per exercise, answer on separate paper

17 Analysis of Combinational Circuit

Swedish:
1. Ta fram booleskt uttryck för kretsen nedan.
2. Rita K-map för kretsen med variabelordning som i figuren.
3. Förenkla uttrycket med hjälp av K-map.
4. Rita ny krets med enbart 2- och 3-ingångars NOR-grindar.

English:
1. Derive the Boolean expression for the circuit below.
2. Draw a K-map for the circuit with variables as in the figure.
3. Simplify the expression using the K-map.
4. Draw a new circuit using only 2 and 3 input NOR gates.
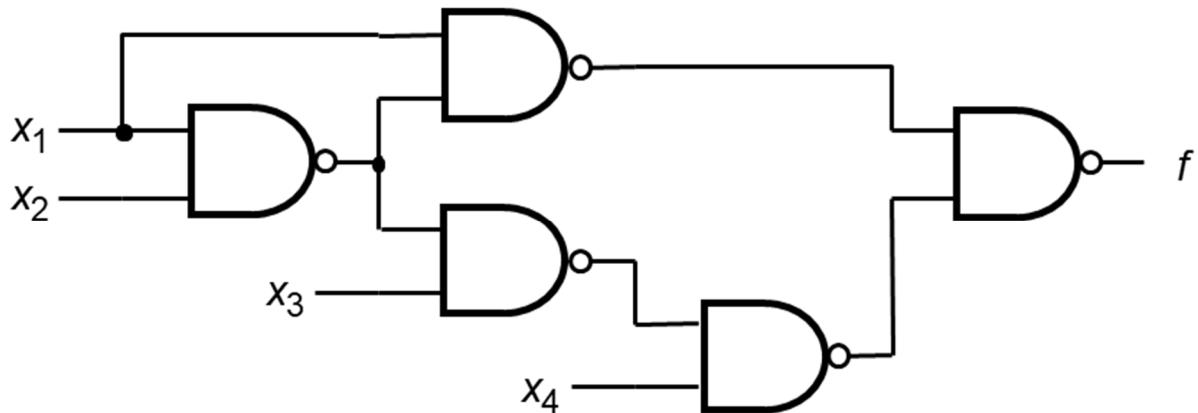
1. Boolean expression:

$$f = \overline{x1\overline{(\overline{x1 \cdot x2})} \cdot x4 \cdot x3\overline{(\overline{x1 \cdot x2})}} = x1\overline{(\overline{x1 \cdot x2})} + x4 \cdot x3\overline{(\overline{x1 \cdot x2})}$$
$$= x1\overline{(\overline{x1} + \overline{x2})} + x4\overline{(\overline{x3} + x1 \cdot x2)} = x1 \cdot \overline{x2} + \overline{x3} \cdot x4 + x1 \cdot x2 \cdot x4$$
$$= x1\overline{(x2} + x2 \cdot x4) + \overline{x3} \cdot x4 = x1 \cdot \overline{x2} + x1 \cdot x4 + \overline{x3} \cdot x4$$

Output: F

$$\overline{\overline{x1 \; \overline{x1 \; x2} \; x4 \; x3 \; \overline{x1 \; x2}}}$$

~(~(x1 ~(x1 x2)) ~(x4 ~(x3 ~(x1 x2))))

2. K-Map with 3. Simplified expression

Format:	Sum of products ∨

x2, x1

		00	01	11	10
x4, x3	00	0	1	0	0
	01	0	1	0	0
	11	0	1	1	0
	10	1	1	1	1

$\overline{x2}\,x1 + x4\,\overline{x3} + x4\,x1$

Format:	Product of sums ∨

x2, x1

		00	01	11	10
x4, x3	00	0	1	0	0
	01	0	1	0	0
	11	0	1	1	0
	10	1	1	1	1
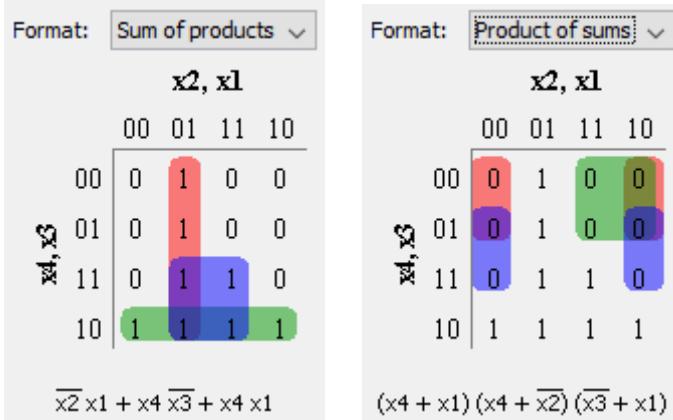
$(x4 + x1)\,(x4 + \overline{x2})\,(\overline{x3} + x1)$

Use POS for NOR only (inverters are ok if you note that they can be made with a NOR)
No deductions if not simplest possible.

$$f = \overline{\overline{(x4 + x1)(x4 + \overline{x2})(\overline{x3} + x1)}} = \overline{\overline{x4 + x1} + \overline{x4 + \overline{x2}} + \overline{\overline{x3} + x1}}$$
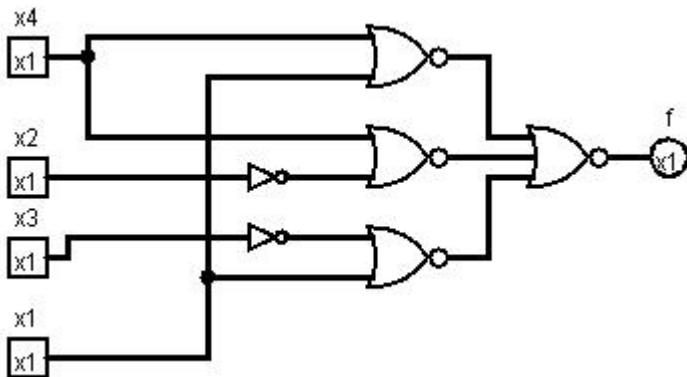
For SoP you need double double inversion, and an extra NOR/Inverter at the output

$$f = \overline{\overline{\overline{x2} \cdot x1}} + \overline{\overline{x4 \cdot \overline{x3}}} + \overline{\overline{\overline{x4} \cdot \overline{x1}}} = \overline{\overline{x2 + \overline{x1}}} + \overline{\overline{\overline{x4} + x3}} + \overline{\overline{x4 + \overline{x1}}}$$

$$= \overline{\overline{\overline{x2 + \overline{x1}}} + \overline{\overline{\overline{x4} + x3}} + \overline{\overline{x4 + \overline{x1}}}}$$
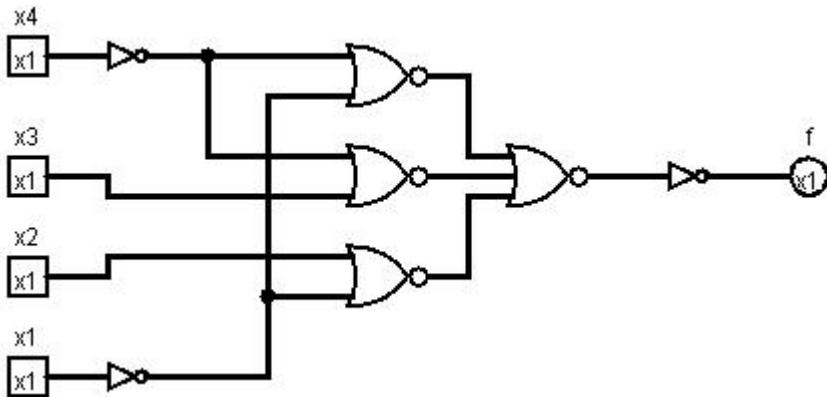
18 Design of Combinational Circuit

Swedish:

Designa en kombinatorisk krets för Y=f(Q3, Q2, Q1, Q0) där

    Y = 1 för ensiffriga primtal, dvs 2, 3, 5, och 7

    Y = 0 för alla övriga ensiffriga tal

    Y = x (don't care) för tvåsiffriga tal

1. Rita sanningstabellen.
2. Rita K-map för sanningstabellen med variabelordning som i figuren.
3. Uttnyttja x = don't care. Ta fram enklast möjliga booleska uttryck från K-map.
4. Rita en krets för uttrycket med enbart 2- och 3-ingångars NAND-grindar.

English:

Design a combinational circuit for Y=f(Q3, Q2, Q1, Q0) where

    Y = 1 for single digit prime numbers, i. e. 2, 3, 5, and 7

    Y = 0 for all other single digit numbers

    Y = x (don't care) for double digit numbers

1. Draw the truth table.
2. Draw a K-map for the truth table with variables as in the figure.
3. Use x = don't care. Derive simplest possible Boolean expression from the K-map.
4. Draw a circuit for the expression using only 2 and 3 input NAND-gates.

$Q_3Q_2 =$ \ $Q_1Q_0 =$	00	01	11	10
00				
01				
11				
10				

> Rita om K-map i dina inlämnade svar.
>
> Redraw the K-map in your answer sheets.

18 Design of Combinational Circuit

1. Truth Table

Q3	Q2	Q1	Q0	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

$Q_3Q_2 =$ / $Q_1Q_0 =$	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	X	X	X	X
10	0	0	X	X

2. K-Map and 3. Boolean expressions

Format: Sum of products

Q1, Q0

Q3, Q2	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	x	x	x	x
10	0	0	x	x

$\overline{Q2}\,Q1 + Q2\,Q0$

Format: Product of sums

Q1, Q0

| Q3, Q2	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	x	x	x	x
10	0	0	x	x
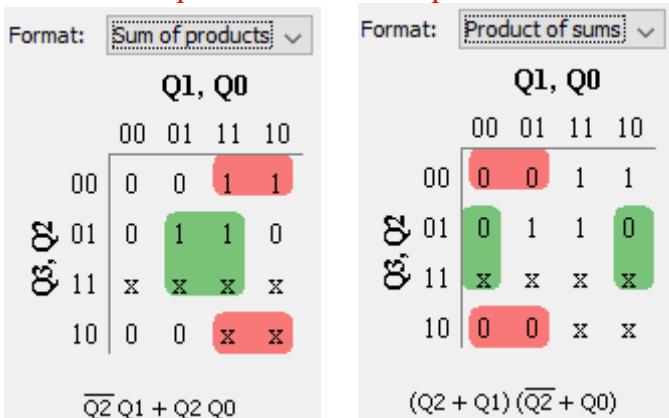
$(Q2 + Q1)\,(\overline{Q2} + Q0)$

4. Use SOP for NAND only (inverters are ok if you note that they can be made with a NAND) No deductions if not simplest possible.

For SOP draw $Y = \overline{\overline{\overline{Q2} \cdot Q1 + Q2 \cdot Q0}} = \overline{\overline{Q2} \cdot Q1 \cdot \overline{Q2 \cdot Q0}}$
(left figure)

For POS draw $Y = \overline{\overline{\overline{(Q2 + Q1) \cdot (\overline{Q2} + Q0)}}} = \overline{\overline{\overline{Q2} \cdot \overline{Q1}} \cdot \overline{Q2 \cdot \overline{Q0}}}$

(right figure, note the double inversion bars, extra inverters/NANDs are needed)
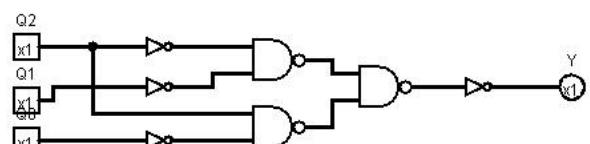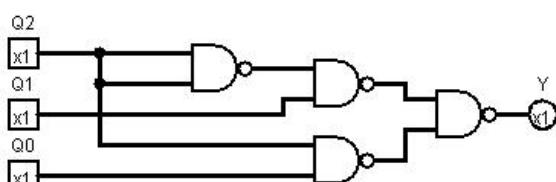
19 Analysis of FSM

**Swedish:** Analysera vad nedanstående tillståndsmaskin (FSM) utför.
1. Ta fram Boolska uttryck för nästa tillstånd.
2. Rita K-Maps för q1+ och q0+.
3. Rita tillståndstabell.
4. Rita tillståndsdiagram.

Använd ordningen q1 q0 b a (det finns inga utsignaler förutom tillståndsvariablerna)

English: Analyze the state machine (FSM) below.
1. Derive Boolean expressions for next state.
2. Draw K-Maps for q1+ and q0+.
3. Draw a state table.
4. Draw a state diagram.

Use the order q1 q0 b a (there are no outputs except for the state variables)
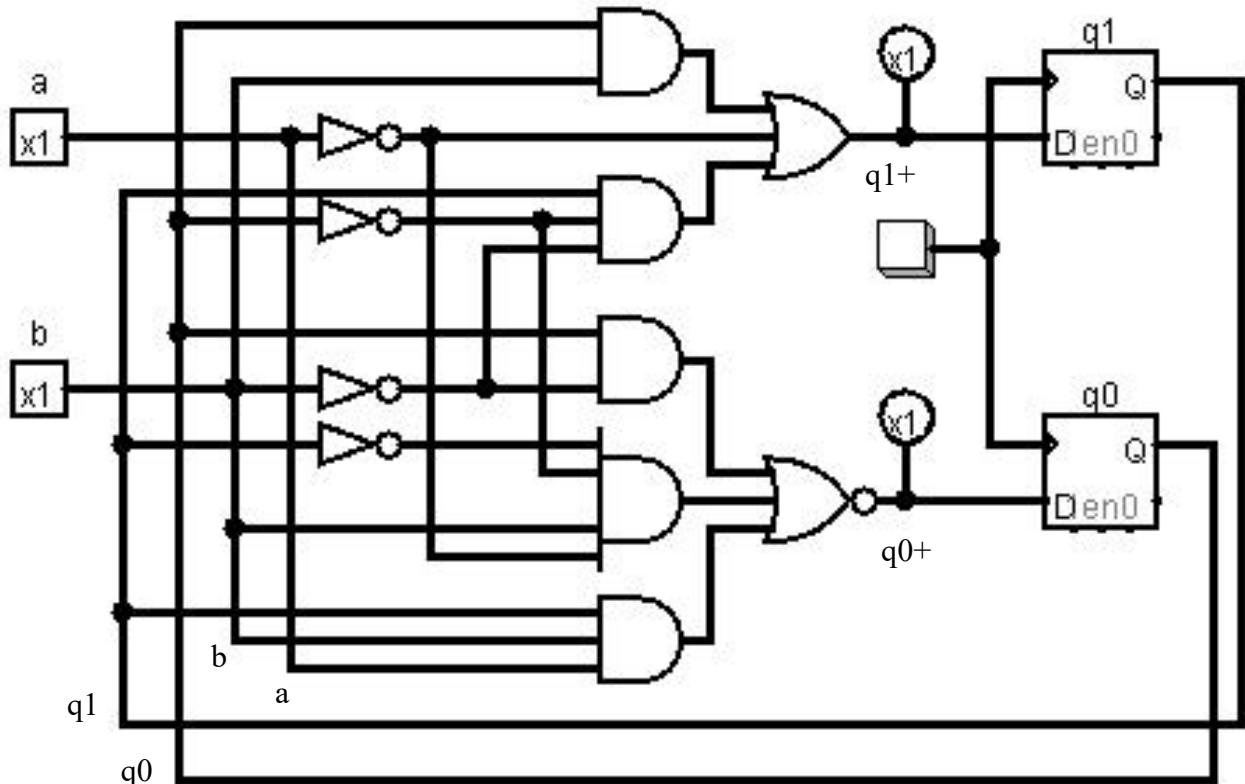
1. Boolean expressions for next state q1+ and q0+, Note that q0+ has a NOR gate.

$$q_1^+ = q_0 \cdot b + \overline{a} + q_1 \cdot \overline{q_0} \cdot \overline{b}$$

$$q_0^+ = \overline{q_0 \cdot \overline{b} + \overline{q_1} \cdot \overline{q_0} \cdot b \cdot \overline{a} + q_1 \cdot b \cdot a}$$

2. K-Maps for q1+ and q0+

$$q_1^+ = f(q_1, q_0, b, a) \qquad q_0^+ = f(q_1, q_0, b, a)$$

$q_1 q_0$ \ ba	00	01	11	10
00	1	0	0	1
01	1	0	1	1
11	1	0	1	1
10	1	1	0	1

$q_1 q_0$ \ ba	00	01	11	10
00	1	1	1	0
01	0	0	1	1
11	0	0	0	1
10	1	1	0	1

3. State table

$$q_1^+ q_0^+ = f(q_1, q_0, b, a)$$

$q_1 q_0$ \ ba	00	01	11	10
00	1|1	0|1	0|1	1|0
01	1|0	0|0	1|1	1|1
11	1|0	0|0	1|0	1|1
10	1|1	1|1	0|0	1|1

4. State diagram
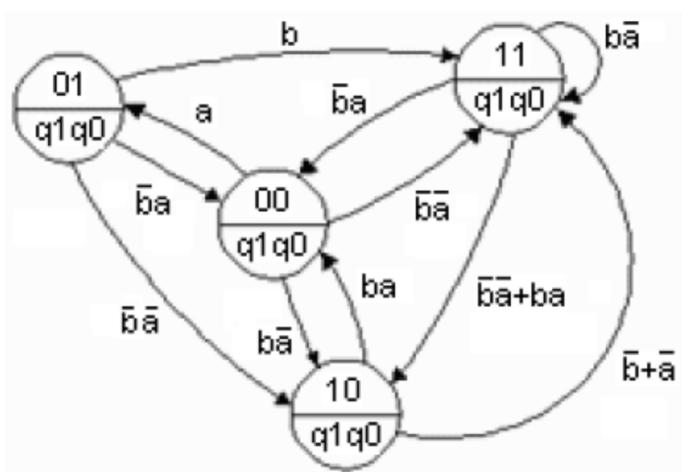
Other notations on the arrows are also possible
If you find this state diagram familiar, you probably had FSM 21 in Lab 3

20 Design of FSM

Swedish: Konstruera en tillståndsmaskin (FSM) enligt tillståndsdiagrammet nedan.
1. Rita tillståndstabell.
2. Ta fram K-map för nästa tillstånd.
3. Ta fram minimerade uttryck för nästa tillstånd. Använd "X" = don't care.
4. Rita kretsschema för en FSM med DFFs och vilka grindar som helst.

English: Design a state machine (FSM) according to the state diagram below.
1. Draw a state table.
2. Derive K-maps for next state.
3. Derive minimized expressions for next state. Use "X" = don't care.
4. Draw the FSM circuit diagram with DFFs and any gates.
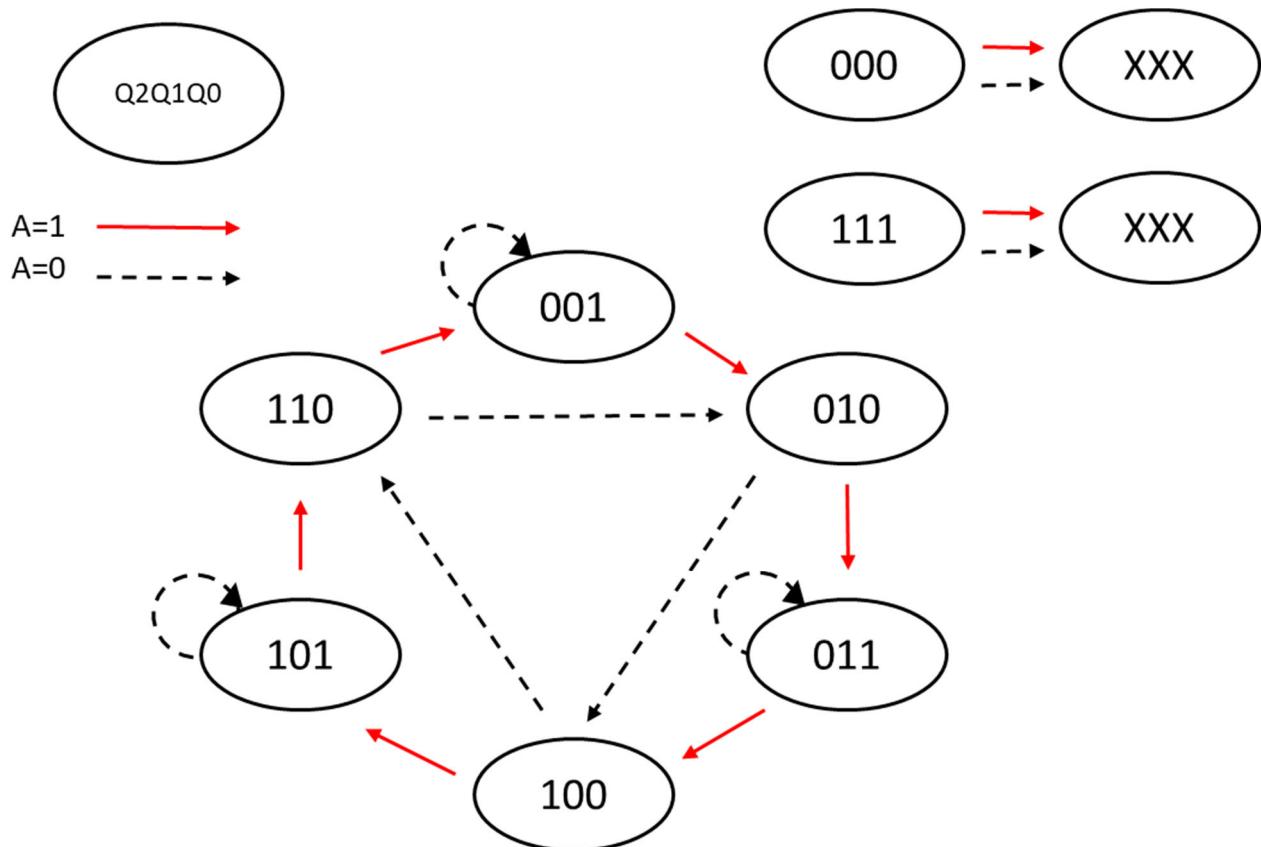
Rita om K-map i dina inlämnade svar.

Redraw the K-map in your answer sheets.

(Answer on next page)

1. State Table

Present state			Next state A = 0			Next state A = 1		
Q2	Q1	Q0	Q2+	Q1+	Q0+	Q2+	Q1+	Q0+
0	0	0	X	X	X	X	X	X
0	0	1	0	0	1	0	1	0
0	1	0	1	0	0	0	1	1
0	1	1	0	1	1	1	0	0
1	0	0	1	1	0	1	0	1
1	0	1	1	0	1	1	1	0
1	1	0	0	1	0	0	0	1
1	1	1	X	X	X	X	X	X

2. K-Maps for next state Q2+, Q1+, and Q0+

3. Minimized Boolean expressions for next state (select SoP or PoS)

Output: Q2p
Format: Sum of products

$\overline{A}\,\overline{Q2}\,\overline{Q0} + Q2\,\overline{Q1} + A\,Q1\,Q0$

Output: Q1p
Format: Sum of products

$\overline{A}\,Q1\,Q0 + \overline{A}\,Q2\,\overline{Q0} + A\,\overline{Q2}\,\overline{Q0} + A\,\overline{Q1}\,Q0$

Output: Q0p
Format: Sum of products

$\overline{A}\,Q0 + A\,\overline{Q0}$

Output: Q2p
Format: Product of sums

$(Q2 + Q1)(A + Q2 + \overline{Q0})(\overline{Q2} + \overline{Q1})(\overline{A} + Q2 + Q0)$

Output: Q1p
Format: Product of sums

$(A + Q2 + Q0)(A + Q1 + \overline{Q0})(\overline{A} + \overline{Q1} + \overline{Q0})(\overline{A} + \overline{Q2} + Q0)$

Output: Q0p
Format: Product of sums

$(A + Q0)(\overline{A} + \overline{Q0})$
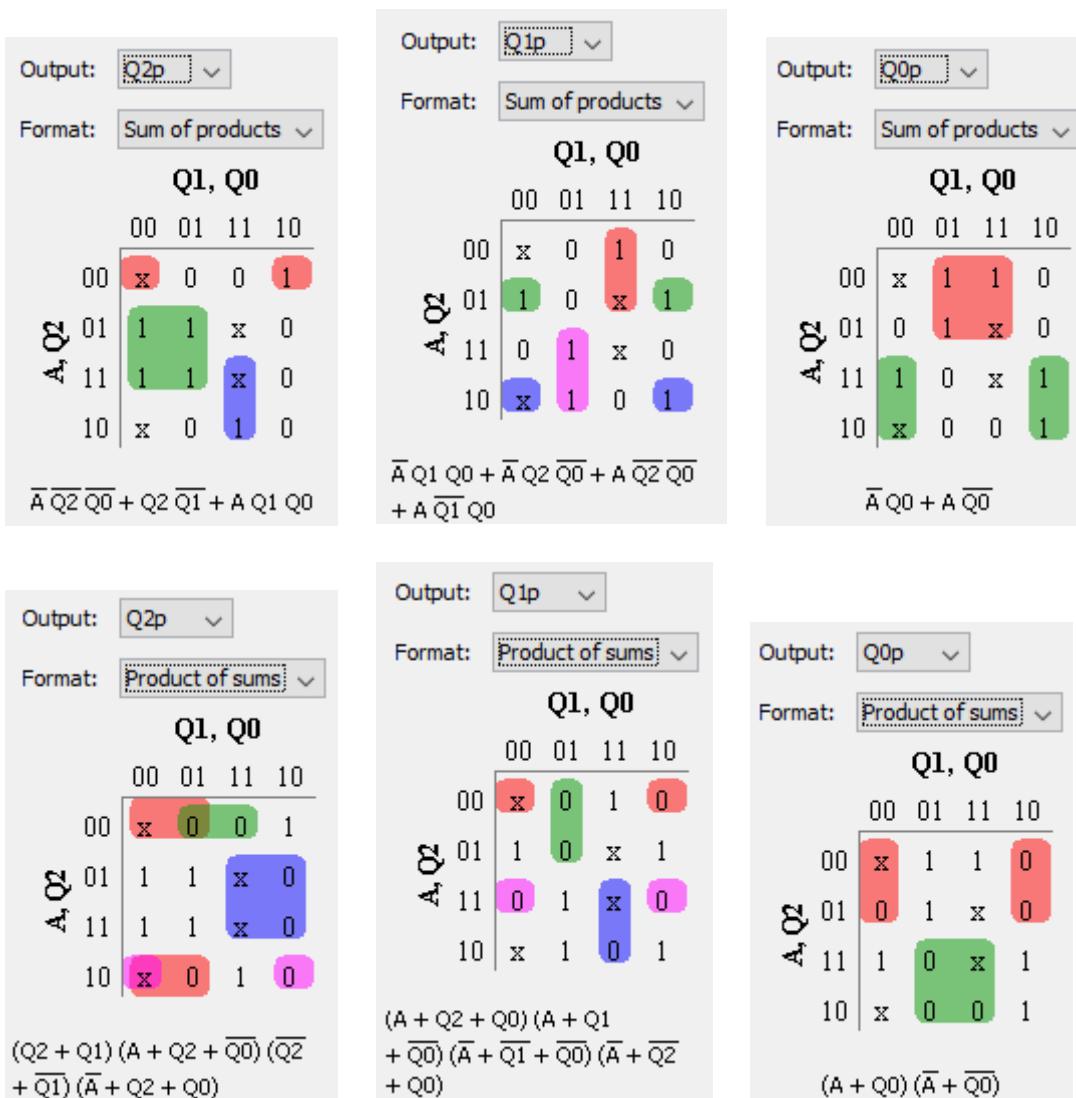
4. Schematic for FSM with DFFs (and SoP)
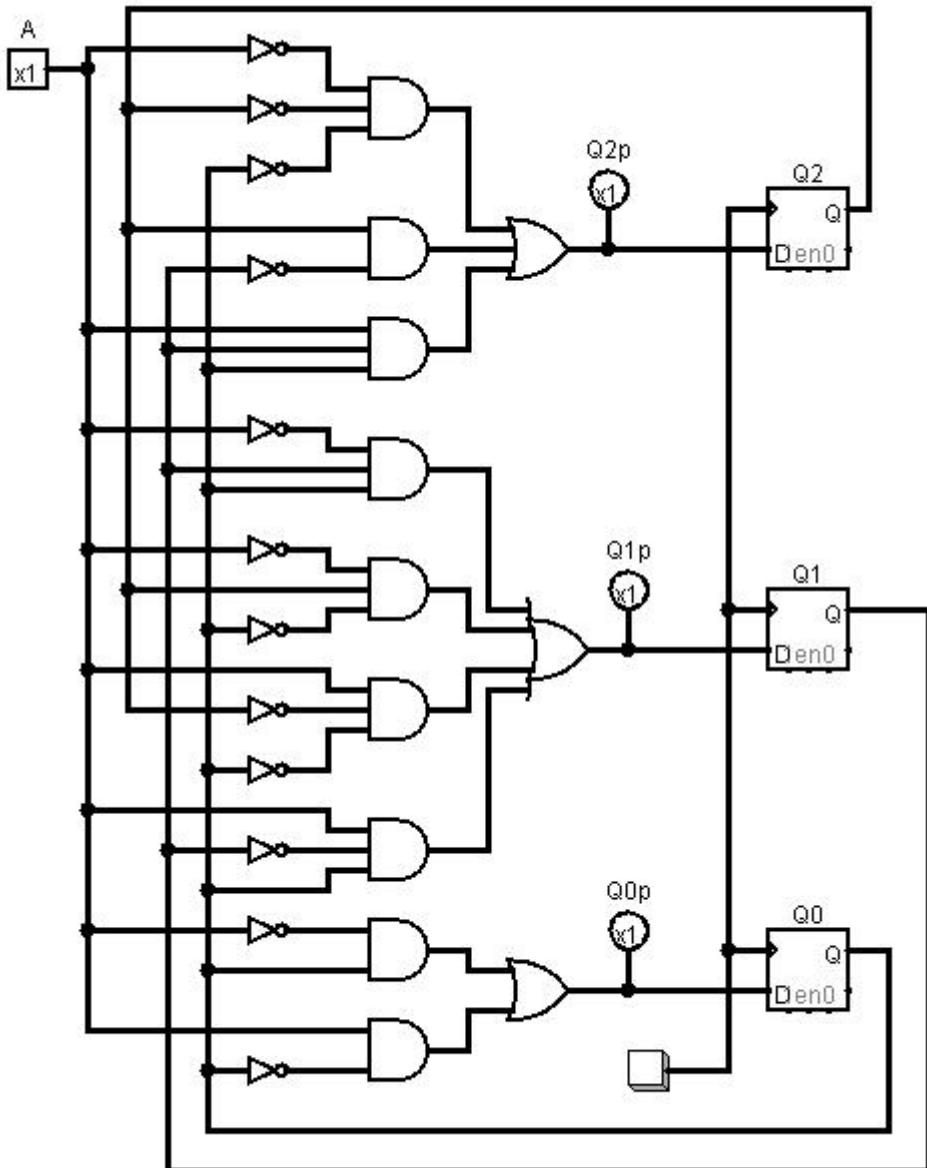
5. Final state table and state diagram for states 000 and 111 (not required)

Present state			Next state A = 0			Next state A = 1		
Q2	Q1	Q0	Q2+	Q1+	Q0+	Q2+	Q1+	Q0+
0	0	0	1	0	0	0	1	1
0	0	1	0	0	1	0	1	0
0	1	0	1	0	0	0	1	1
0	1	1	0	1	1	1	0	0
1	0	0	1	1	0	1	0	1
1	0	1	1	0	1	1	1	0
1	1	0	0	1	0	0	0	1
1	1	1	0	1	1	1	0	0