KTH ROYAL INSTITUTE
OF TECHNOLOGY

# EP2210 – Performance evaluation of communication networks

Viktoria Fodor
Laboratory for Network and Systems Engineering
School of Electrical Engineering and Computer Science, KTH

Course homepage
https://www.kth.se/social/course/EP2210/

Fall 2018 (HT 2018)

# EP2210 – Performance evaluation of communication networks

Course objectives:

- Advanced networking course
- Discuss mathematical modeling in some main areas of networking
  - Learn techniques to address performance related questions
  - Discuss some of the significant results – and read the original papers
  - Improve our “paper reading” (and writing) skills

# Topics

1. Traffic modeling

2. Multiple access protocols

3. Congestion control

*Can I use simple "random" packet arrival to evaluate my protocol?*

*The random access control I have implemented has zero throughput… what is going on?*

*What limits the throughput of my TCP session?*

# Topics

3. Scheduling

4. Fairness

5. Multimedia communication

*Which packet should be transmitted first, to satisfy the QoS of the applications?*

*Do the users receive a fair service? What is fairness, by the way? Equality?*

*Should I add redundancy, or should I retransmit? Or maybe I should not even try…*

# Course setup

- Scheduled activities:
  - 12 lectures of 2 hours
  - project presentations
- 2 lectures per subject
  - first lecture – introduction and simple models
  - second lecture – advanced models,discussion of papers, phd student presentations
- Continuous examination (5 tests altogether, lectures 3, 5, 7, 9, 12)
- Home assignments (3 home assignments altogether, submitted at lectures 6, 9, 12)
- Project

# Requirements

- Read all the papers
  - covering the lecture and for home reading
- Home assignments
  - questions to answer
  - numerical examples (e.g., matlab)
  - independent solutions, submit on paper copy or send in via mail
  - tell me in advance if you can not submit on time (minus points)
- Tests
  - ca. 20 minutes
  - questions on the lecture material and about the papers (open book/computer)
  - make-up test after the course (missed or weak results)

# Requirements

- Project
  - literature study on mathematical modeling
  - comparative review of 3-5 papers in the area
  - subject selected from subject list or on your own (discuss with the instructor)
  - in groups of ca. 2 students
  - written report of 4-5 pages
  - presentation of the project

# Grading

- Tests: 50%
- Home assignments: 30%
- Project 20% (same for all project members)
  - detailed on the web-page under Projects

- Grading guidelines (approx):
  - 90%:-A, 80%-B, 70%-C, 60%-D, 50%-E, 45%-Fx

# Requirements – graduate students

- Paper presentation (for 9ECTS)
  - select a lecture topic as soon as possible
  - ca. 20 minutes presentation on one of the lectures (second lecture of a topic)
  - short meeting right after todays lecture abut the details

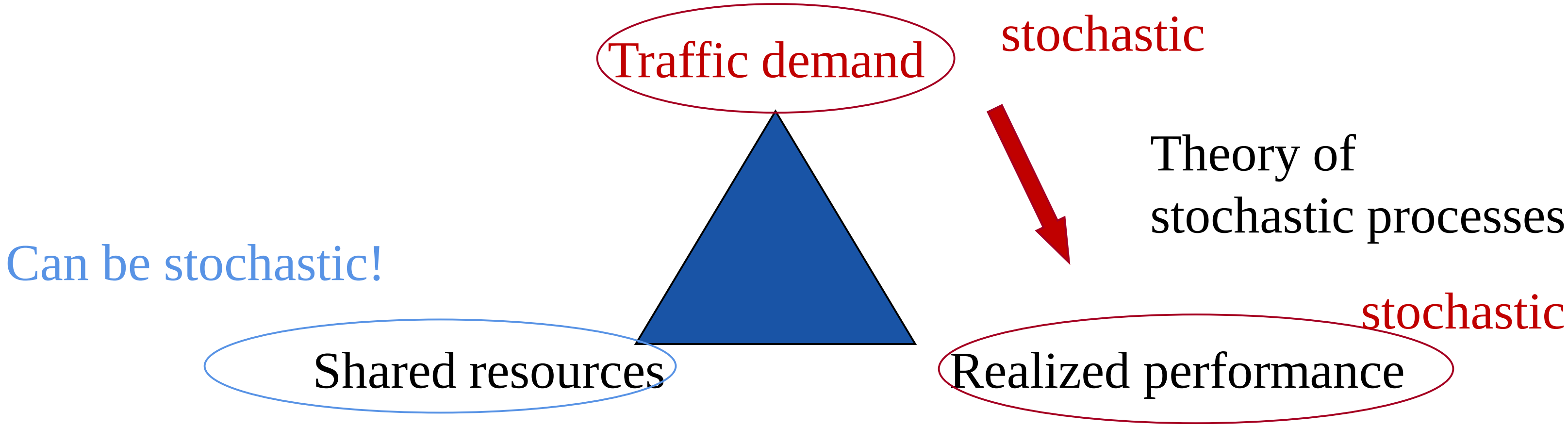
- Small project – during or after the course (for +3ECTS)
  - select a lecture topic
  - prepare a small simulator to support a mathematical model  or problem definition, the simulator could be used for demonstration

# Traffic theory - Traffic models

- Topics:
  - Traffic modeling – traffic objects
  - Markov processes recall
  - Traffic models: markovian and non-markovian models
- Lecture material:
  - A. Adas, “Traffic models in broadband networks,” IEEE Communications Magazine, July 1997.
  - J. Roberts, “Traffic theory and the Internet,” IEEE Communications Magazine, January 2001.
  - V. Frost, B. Melamed, “Traffic modeling for telecommunications networks”, IEEE Communications Magazine, March 1994.
  - I. Kaj, „Stochastic modeling”, 5.2.2-5.3.1.

# Teletraffic theory

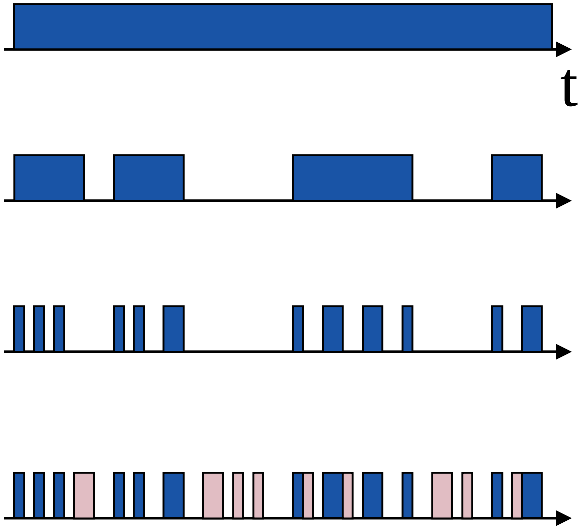
- Teletraffic theory:
  - to model dynamic resource sharing systems
  - to explain the traffic-performance relation



- Traffic: arrival intensity, holding time, packet length (distribution or moments)
- Resources: link bandwidth, router buffer, server capacity
- Performance: utilization, loss, delay, delay variation, perceptual quality

# Traffic modeling

- To describe the network traffic demand
- Statistical characterization
- Traffic objects



- Flow (one instance of communication, TCP or UDP session) — Skype call
- Burst (Active/passive periods) — Talk/listen
- Sequence of packets — IP packets
- Multiplexed packets — IP packets at a router

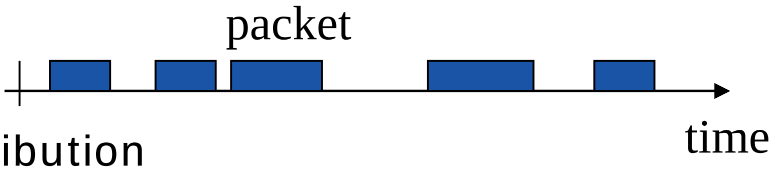
# Traffic modeling

- Packet level – characteristics of the sequence of packets
  - packet arrival process
    - according to some stochastic/deterministic arrival process (e.g. Poisson arrival at a router…)
    - saturated source model: there is always packet to send at the source
  - packet size distribution
- Flow level (burst level is similar too, but rarely used):
  - flow arrival process
    - e.g., flows from all the laptops in a WLAN are generated according to a Poisson process
  - flow duration distribution
  - flow characteristics – how traffic is generated within a flow

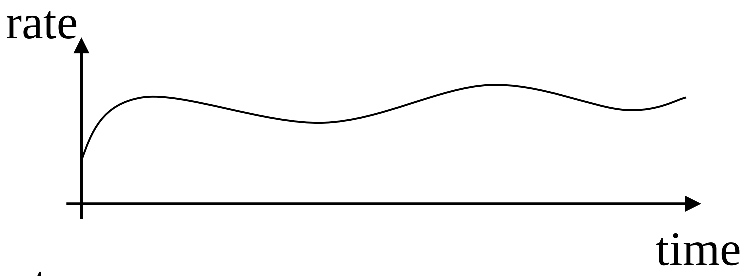
# Flow characteristics

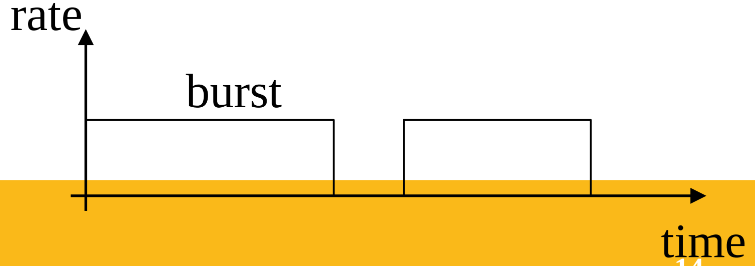
Models that describe the distribution of the sequence of packets for a flow level model

- packet scale model
  - arrival process and packet size distribution
  - queuing theory
  - used typically in this course
  - may lead to very complex models on flow level



- fluid models
  - transmission as a continuous stream
  - parameter: flow rate r(t)
  - system of differential equations
  - often more tractable on the flow level





# Flow types - Terminology

- Flow - one instance of an application
  - Reasonable to classify according to application types
- Elastic flow
  - The application requires the transmission of a given amount of information, some delay is acceptable – that is, transmission is elastic in time
  - E.g., file transfer over TCP
  - Flow characteristics is determined by the transport protocol (e.g., TCP) and the background traffic
- Streaming flow
  - The application has strict delay limitations, late information is dropped
  - E.g., VoIP over UDP
  - Flow characteristics is determined by source characteristics (e.g., coding)

# Traffic modeling

- Should we use packet or flow level models in the following problems?
  - buffer dimensioning – sequence of packets
  - error control – loss of individual packets
    - PACKET LEVEL MODELS
  - video rate control
  - routing
    - FLOW LEVEL MODELS

# Group work

Should we use packet or flow level models in the following problems? In the case of flow level models, what kind of flow characterization is necessary?

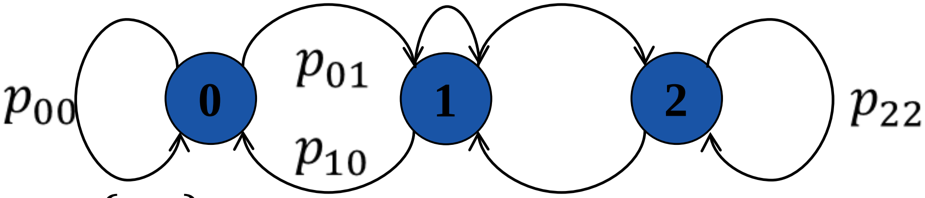
1. What is the probability that a packet collides and therefore needs to be retransmitted when using CSMA/CA protocol?
2. Several Skype calls are using the same communication link. What is the utilization of the link *(utilization={average rate of traffic} / {link transmission rate})*
3. Several flows are multiplexed at a router with limited buffer. What is the probability that consecutive packets of a flow are dropped due to buffer overflow?

# Mathematical modeling

- Recall: Markov chains
- Markovian traffic models
- Home reading: non Markovian models

# Recall – Markov chains

- Basic tools of queuing theory
- Stochastic process
  - Discrete state space
  - Discrete or continuous time (change of state)
  - Markovian property: the future of the process does not depend on the past, only on the present

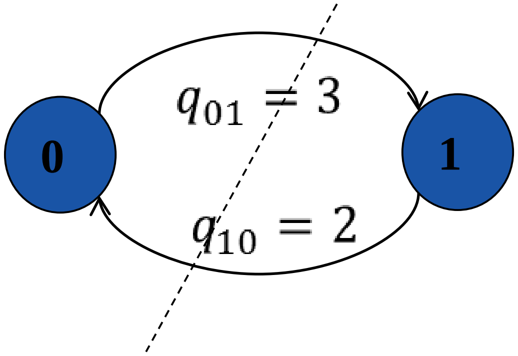


- Discrete time Markov chains
  - State transition probability matrix $\boldsymbol{P} = \{p_{ij}\}$
  - $\underline{p}_{i+1} = \underline{p}_i \boldsymbol{P}$
  - If steady state exists, the stationary state probability is given by $\underline{p} = \underline{p}\boldsymbol{P}$
  - Holding time of a state is geometric with parameter $1 - p_{ii}$ (memoryless)
  - E.g., to model the packet loss process on a link

# Recall – Continuous time Markov chains

- Continuous time Markov chains
  - State transition is possible at any time
  - State transition intensity matrix $\mathbf{Q} = \{q_{ij}\}$, $q_{ii} = -\sum q_{ij}$
  - $\underline{p'(t)} = \underline{p(t)}\mathbf{Q}$
  - If steady state exists, the stationary state probability is given by $\underline{0} = \underline{p}\mathbf{Q}$
  - Holding time of a state is Exponential with parameter $-q_{ii}$, with mean $1/(-q_{ii})$
  - The exponential distribution is memoryless

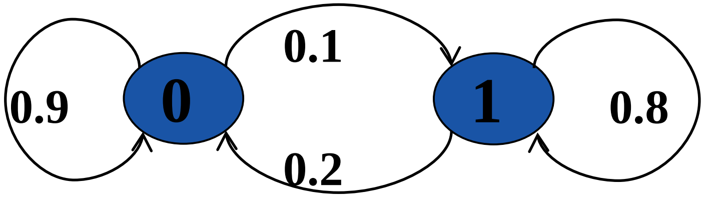
- E.g., good (0) or bad (1) state of a wireless channel



# Recall – Discrete time Markov chains

- E.g., to model the packet loss process at a receiver
  - States: packet received or lost (0,1)
  - Captures the burstiness of the loss process (see Gilbert model later in the course)
    - If a packet is lost (state 1), the next one is lost with probability $p_{11}$
    - If a packet is received (state 0), the next one is received with probability $p_{00}$

→ Packets lost in a raw $\sim Geo(1-p_{11})$, in average 1/ (1-$p_{11}$)



# Markovian traffic modeling

- Traditional telephone networks (from Erlang)
  - Poisson call arrival
  - exponential call duration
  - constant rate

  } ⇒ nice Markovian models (M/M/*/*)

- Similar models are possible for data networks
  - Poisson flow/packet arrival process
  - Exponential flow size (e.g., file length), packet size

# Markovian traffic models

- Poisson process: $P\{N(t)=n\}=e^{-\lambda t}(\lambda t)^n/n!$
- Exponential distribution: $P(X\leq t)=1-e^{-\lambda t}$, $f(t)=\lambda e^{-\lambda t}$

- Recall – some basic results

- Exponentially distributed interarrival and service times
- Possion arrival: exponential interarrival time
- Exponential distribution is memoryless – simple modeling
- Tail function $P(t>T)=e^{-\lambda T}$ – exponential decay in t
  - e.g., the probability that a packet size is larger than *T* decreases exponentially in *T*.
- Consecutive values (interarrival time, service time) are independent, therefore auto-covariance is zero
- $Cov(k)=E[(X_i-E[X])(X_{i+k}-E[X])]=E[X_iX_{i+k}]-E[X]^2=0$

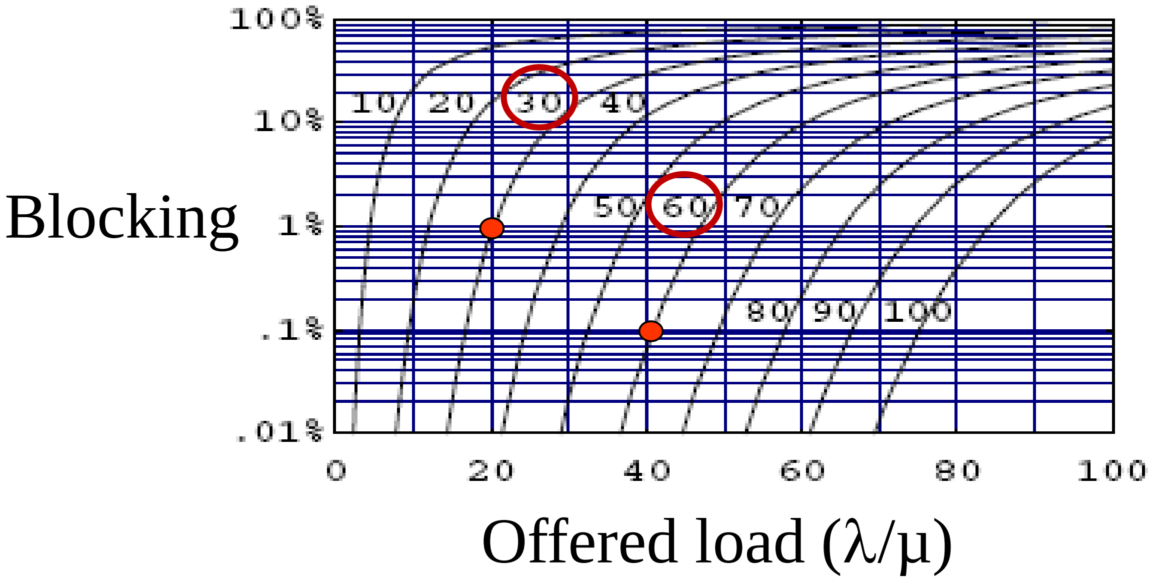
# Markovian traffic models

- Exponential interarrival and service times in queues (M/M/*/*)
- Buffering is efficient, does not cause large delays

- E.g, distribution of the number of users in an M/M/1 queue: $p(n)=(1-\rho)\rho^n$, $\rho=\lambda x$
- $P(n \geq N)=\rho^N$ – the probability that the queue length is at least *N* decays exponentially fast (exponential decay)

# Markovian traffic models

- Multiplexing is efficient, decreases the blocking probability
- E.g, M/M/m/m
    - Multiplexing: higher aggregate arrival intensity → higher offered load
- Blocking given by the Erlang-B curves



B(load,servers)

B(20,30)≈1%

B(40,60)≈0.1%

# Markov modulated models

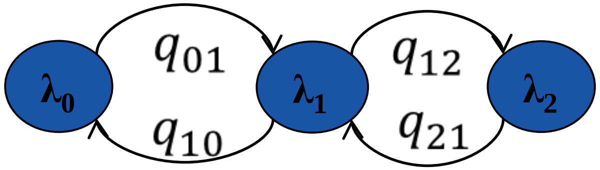
- However, we know that packet arrivals are not Poissonian
  - the arrival rate changes with time (traffic control, coding)
  - immediate result, auto-covariance should not be zero:
    $Cov(k)=E[(X_i-E[X])(X_{i+k}-E[X])]=E[X_iX_{i+k}]- E[X]^2 \neq 0$

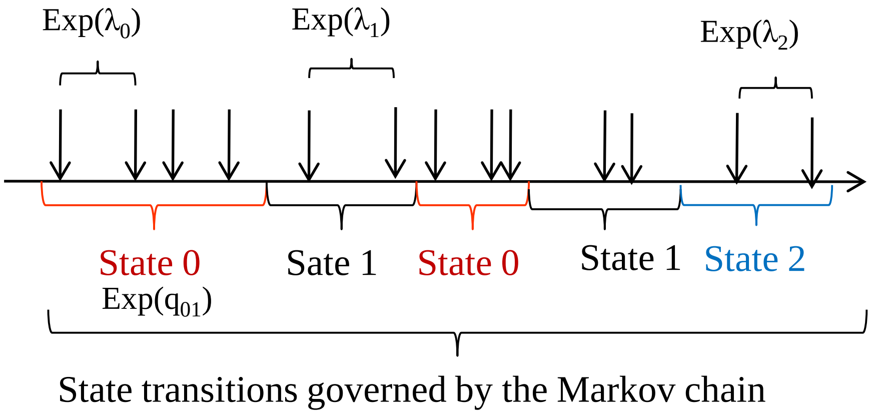
- First step towards modeling traffic sources:
- Markov-modulated traffic models
  - to capture “burstiness” (changing arrival rate)
  - while keeping the simplicity of modeling

# Markov modulated models

## Packet scale models

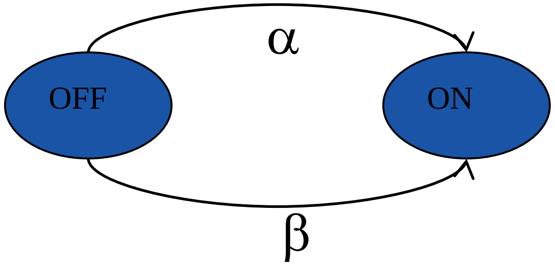
- Markov-modulated Poisson Process (MMPP)
  - A Markov chain is given that describes the state of the source
  - The packet generation process is Poisson in each state, but with different intensity (state i → $\lambda_i$)
  - Burstiness is captured by the state transitions in the Markov chain



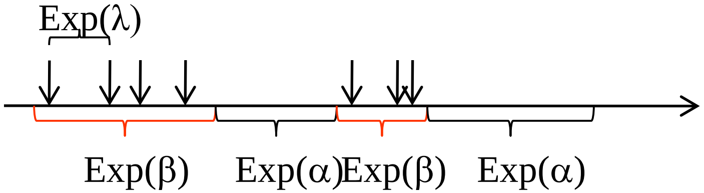


# Markov modulated models

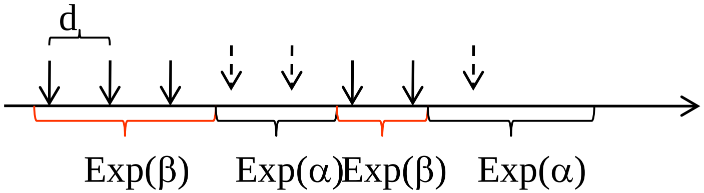
Packet scale models with two states



- Interrupted Poisson Process (IPP)
  - Most simple MMPP
  - two states $\lambda_0=0$, $\lambda_1=\lambda$

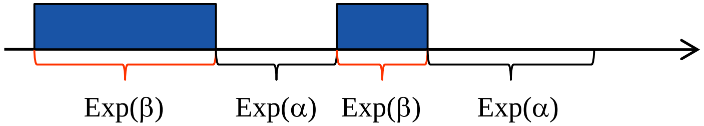


- ON-OFF model
  - two states, no arrivals in state 0 and fixed (d) packet interarrival times in state 1 (deterministic arrival process)

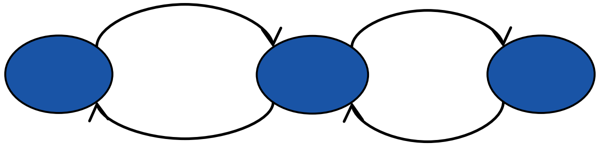


# Markov modulated models

- Fluid models
  - When individual units (e.g., packets) have little impact
- Markov modulated fluid model
  - Traffic as a continuous stream with a parameterized flow rate (state i → $r_i$)
  - Flow rate changes described by a Markov-chain



- Semi Markov models and embedded Markov chains
  - If the state holding times are not Exponential
  - The sequence of states visited can be described with a discrete time Markov chain -> embedded Markov chain



# Markovian traffic models
# Modeling voice traffic

- Compare the average delay at a multiplexer, if
  - Real voice source packets are multiplexed in a simulator
  - Poisson arrival is assumed with the same average rate
  - 2 state MMPP model is used
  - Some advanced technique is used

- Results:
  - Poisson arrival approximation underestimates delays (queue lengths)
  - MMPP seems to fit well at high load regime as well

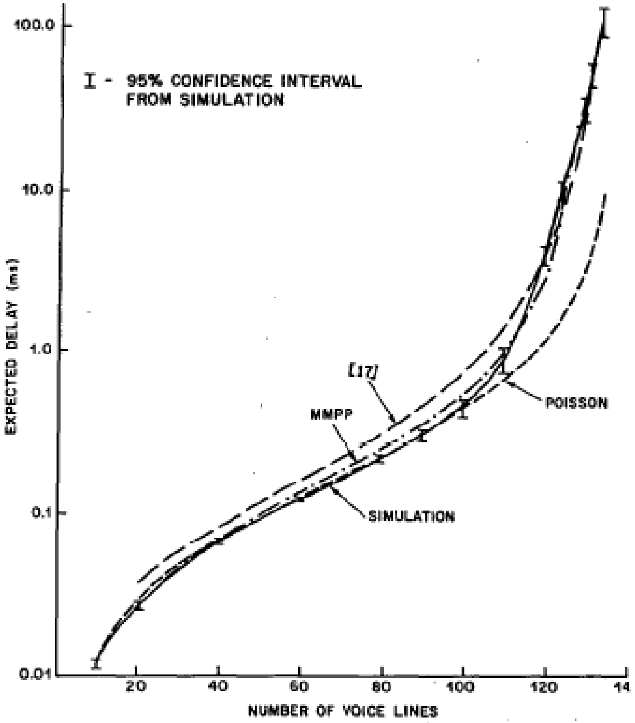


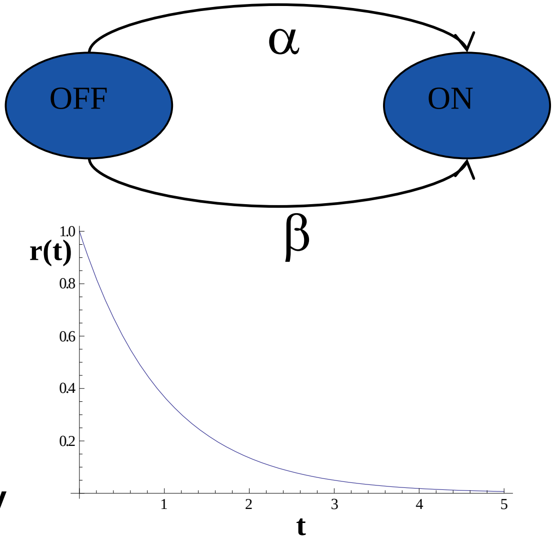
Fig. 2. Expected delay for a packetized voice multiplexer.

# Markovian traffic models

- Auto-covariance and auto correlation function decays exponentially
  - Auto-covariance: $Cov(t)=E[(X_i-E[X])(X_{i+t}-E[X])]$
  - Auto-correlation: $r(t)=Cov(t)/V[X]$

- Simplest example: on-off fluid model
  - The auto-correlation of the state of the system (on or off)
  - $r(t)=e^{-(\alpha+\beta)t}$

- What does it mean: the system has some memory about the past, but only for a short time – (we introduce the concept of short range dependence later)



# Traffic modeling

- Are Markovian traffic models enough to model network traffic sources?
- Or do we need other models?

# Modeling Internet traffic

Read: J. Roberts, “Traffic theory and the Internet,”

“As a first approximation, it is not unreasonable to assume that individual flows also occur as a Poisson process. To ignore the correlation between flow arrivals within the same session is not necessarily significant when the number of sessions is large. It is also true that results derived under the simple Poisson assumption are also often true under more general assumptions.

The size of elastic flows (i.e., the size of the documents transferred) is extremely variable and has a so-called heavy-tailed distribution: most documents are small (a few kilobytes) but the number which are very long tend to contribute the majority of traffic. The precise nature of the size distribution is important in certain circumstances, such as describing the resulting self-similar packet arrival process, and can have a significant impact on performance in some multiplexing schemes.

The duration of streaming flows also generally has a heavy-tailed distribution. Furthermore, the packet arrival process within a variable rate streaming flow is often self-similar.”

# Modeling Internet traffic

- Elastic flows - controlled by congestion control
  - e.g., file transfer
  - arrival of flows: independent activity of a large number of users → *Poisson*
  - size: *heavy tail*
  - traffic characteristics: extreme variability introduced by TCP and heavy tailed flows
  - *self-similar* packet arrival process
- Streaming flows - determined by the source coding
  - arrival of flows: *Poisson*
  - duration: extreme variability, *heavy tail*
  - traffic characteristics (rate): often *self-similar* due to coding
- Conclusion:
  - Simple Markovian or Markov Modulated source models may not work

# Home reading

Home reading for Wednesday next week: A. Adas, “Traffic Models in Broadband Networks”, IEEE Communications Magazine, July 1997

- Markov and Embedded Markov models in detail
  - including the MMPP example for video coding
- Regression models *are not part* of the course material, but are interesting reading
- Long-range dependent traffic models, *not including* fractional ARIMA and fractional Brownian Motion
- See “Reading Assignment” on the course web

# Discrete time Markov chains

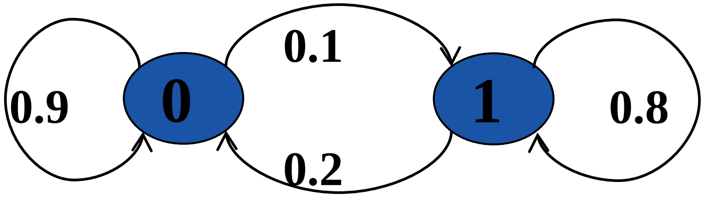
- E.g., to model the packet loss process at a receiver
  - States: packet received or lost (0,1)
  - Captures the burstiness of the loss process (see Gilbert model later in the course)
    - If a packet is lost (state 1), the next one is lost with probability $p_{11}$
    - If a packet is received (state 0), the next one is received with probability $p_{00}$

→ Packets lost in a raw $\sim Geo(1 - p_{11})$, in average 1/ (1-$p_{11}$)

→ Steady state probability of receiving or loosing a packet:

$$\{p_0, p_1\} = \{p_0, p_1\} \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

$$\{p_0, p_1\} = \{p_0, p_1\} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$



- What is the probability that a packet gets lost  $(p_1)$?
- What is the average number of packets lost in a row?

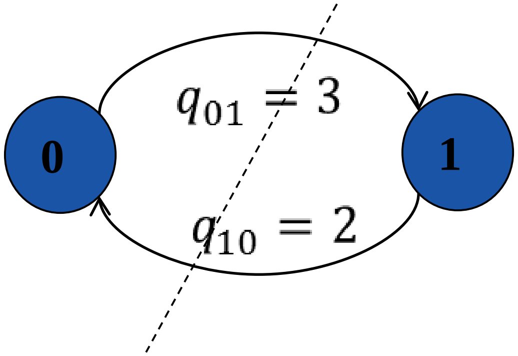
# Continuous time Markov chains

- Continuous time Markov chains
  - State transition is possible at any time
  - State transition intensity matrix $\mathbf{Q} = \{q_{ij}\}$, $q_{ii} = -\sum q_{ij}$
  - $\underline{p}'(t) = \underline{p(t)}\mathbf{Q}$
  - If steady state exists, the stationary state probability is given by $\underline{0} = \underline{p}\mathbf{Q}$
  - Holding time of a state is Exponential with parameter $-q_{ii}$, with mean $1/(-q_{ii})$
  - The exponential distribution is memoryless

- E.g., good (0) or bad (1) state of a wireless channel
- Steady state probabilities:

$$\{0,0\} = \{p_0, p_1\} \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$$

- What is the probability that the system is in state 1?
- What is the average holding time of bad state?



# MMPP traffic models Example

A traffic source is modeled as follows:

- 2 state MMPP
- Transition intensity $q_{12}$=0.5, $q_{21}$=0.5 (transitions per sec)
- Transmission rates: $\lambda_1$=100 packets/s and $\lambda_2$=400 packets/s
- Packet size: 500Bytes

1. Draw the Markov-chain, give all the parameters, give the Q matrix

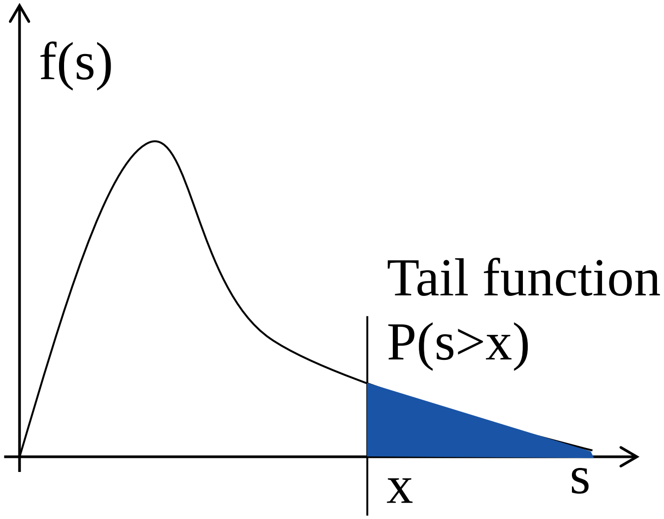
2. What is the mean time in states 1 and 2 respectively?
3. What is the probability that the source is in state 1 (state 2) at an arbitrary point of time?

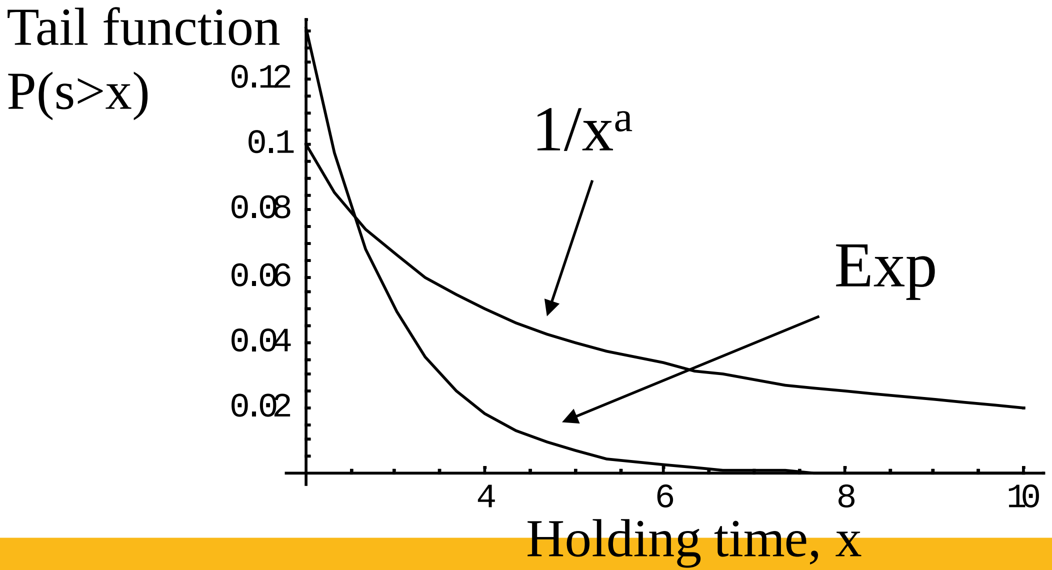
4. What is the average packet interarrival time in state 1?
5. What are the transmission rates in the two states in bit per sec?
6. What is the average transmission rate?

7. If 5 such sources are multiplexed, what is the probability that the instantaneous rate is 8Mbps or larger?

# Heavy-tail distributions, self-similarity, and long-range dependence

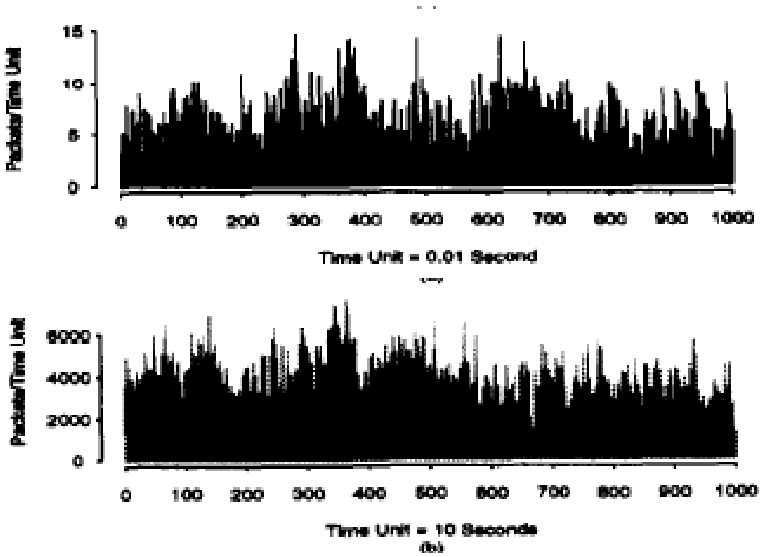
- What are the limitations of Markovian models?
- Example 1
  - Telephone call holding time measurements (holding time, s)
    - Exponential assumption: $P(s>x)=e^{-\mu x}$
    - Statistics (for large s): $P(s>x) \sim x^{-\alpha}$, $\alpha>0$
  - Decay is slower than exponential: heavy-tail distribution



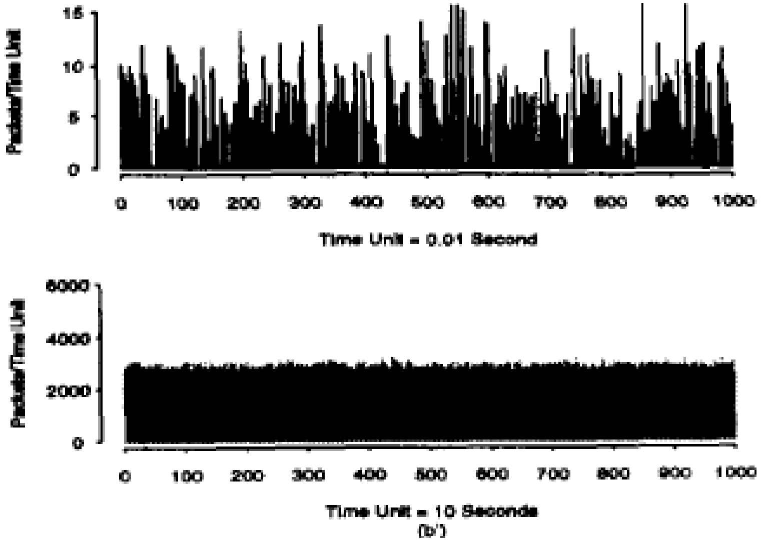


# Heavy-tail distributions, self-similarity, and long-range dependence

- Example 2
  - Packet arrivals in 40 hours Ethernet traffic (Bellcore ’89)
  - Number of packet arrivals in increasing time intervals



Ethernet measurement

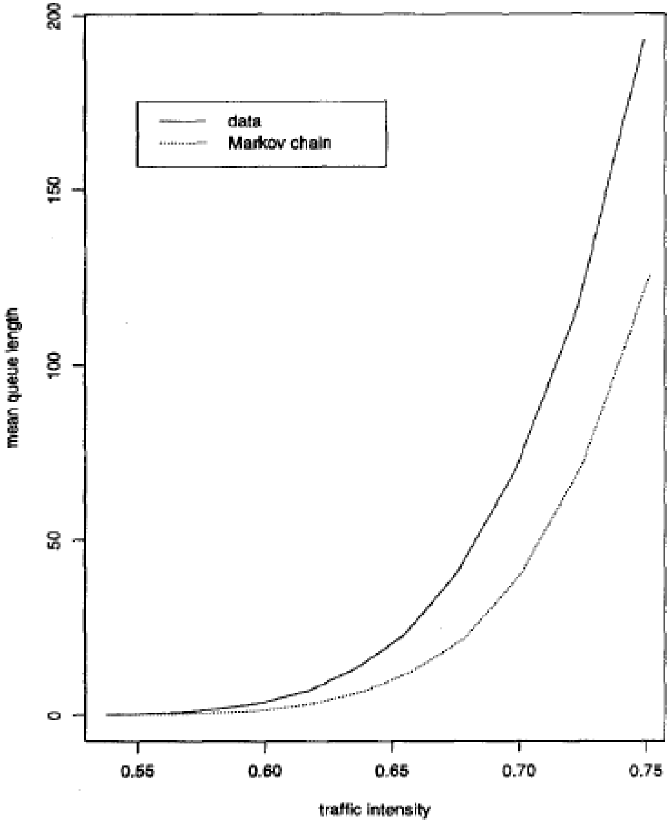


Poisson

- Self-similar nature of packet arrival process

# Heavy-tail distributions, self-similarity, and long-range dependence

- Example 3 – the effect of (long range dependent) self similarity
- LRD-SS source characteristics changes the network performance significantly.
  - E.g., mean queue-length at routers/multiplexers
  - Blocking and loss probabilities
- Therefore
  - we have to take it into account at the performance evaluation
  - have to understand how it emerges and whether it is possible to avoid it



End of first lecture.

# Traffic modeling - recall

- Teletraffic theory – performance triangle
- Traffic modeling
  - Traffic objects: flow (elastic, streaming), burst, packet
  - Modeling levels:
    - Packet level: arrival process, packet length distribution
    - Flow level: arrival process, flow length distribution, flow characteristics (packet scale or fluid)
- Classical traffic models
  - Poisson arrival, exponential service time (packet, flow length) – M/M/*/*
    - Simple model
    - Exponential decay (interarrival time, service time, queue length), no time correlation of input parameters
    - Efficient multiplexing, efficient buffering
- Markov modulated traffic models
  - Poisson arrival, exponential service time (packet, flow length) – M/M/*/*
    - Still tractable model
    - Captures burstiness, auto-covariance is not zero, but decays Exponentially
    - Seem to model well the effect of burstiness on buffering

# Warm up

Model a traffic source with a two state MMPP. The average packet generation rate is 100 packets per second in high intensity periods and 10 packets per second in low intensity periods. The average time in high respectively low intensity periods is 10-10 seconds.

- Give the distribution of the holding times in the two states.
- Give the Markov chain governing the source, with all the parameters.
- Calculate the average transmission rate in terms of packets per second.

# Modeling Internet traffic

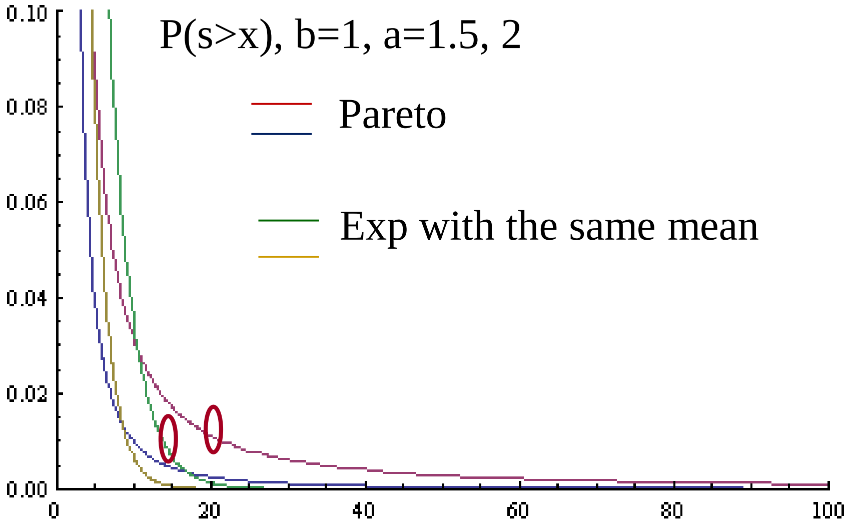
- Elastic flows - controlled by congestion control
  - e.g., file transfer
  - arrival of flows: independent activity of a large number of users → *Poisson*
  - size: *heavy tail*
  - traffic characteristics: extreme variability introduced by TCP and heavy tailed flows
  - *self-similar* packet arrival process
- Streaming flows - determined by the source coding
  - arrival of flows: *Poisson*
  - duration: extreme variability, *heavy tail*
  - traffic characteristics (rate): often *self-similar* due to coding
- Conclusion:
  - Simple Markovian or Markov Modulated source models may not work

# Heavy-tail distributions, self-similarity, and long-range dependence

- We have to address the followings
  - what is heavy-tail distribution
  - what is self-similarity (and related: what is long range dependence)
  - how are these related to each other
  - when is it possible to apply Markovian models

# Heavy-tail distributions

- Exponential distribution: $P(X>x)=e^{-\mu x}$
- Heavy-tail distribution:
  - $P(X>x) \sim x^{-a}$, $x \to \infty$, $a>0$
  - the asymptotic shape is hyperbolic
- Pareto distribution: often used heavy tail distribution (e.g., for file size length):
  - $f(x)= ab^a/(x^{a+1})$,
    - $a>0$ (shape),
    - b is the minimum possible value (base)
  - $P(X>x)=1-F(x)=(b/x)^a$
  - $E[X]=ab/(a-1)$ for $a>1$ otherwise the mean is not finite



# Heavy-tail distributions – Waiting for the bus revisited

- Distribution of remaining service time (remaining time to wait for the bus…)

$$R_t(x) = P(X > x + t | x > t) = \frac{\bar{F}(x+t)}{\bar{F}(t)}$$

- Exponential distribution: $P(X > x) = e^{-\mu x}$ , $R_t(x) = e^{-\mu x}$ (the memoryless property)

- Pareto distribution:

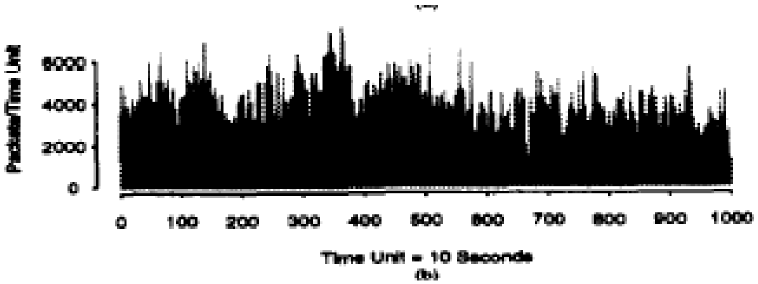
$$f(x) = \frac{ab^a}{x^{a+1}}, \mathrm{P(X > x)} = \left(\frac{b}{x}\right)^a$$

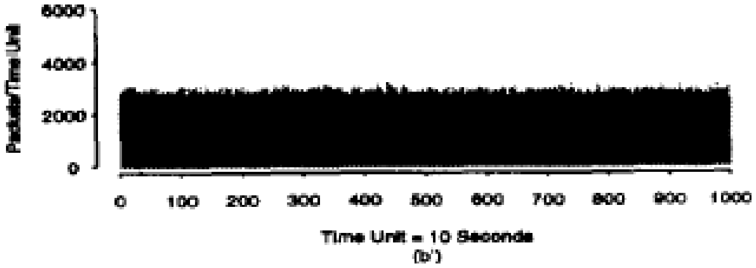
$$R_t(x) = \frac{\left(\frac{b}{x+t}\right)^a}{\left(\frac{b}{t}\right)^a} = \left(1 + \frac{x}{t}\right)^{-a}$$

- That is, the remaining service time increases with t, the time already spent on service!

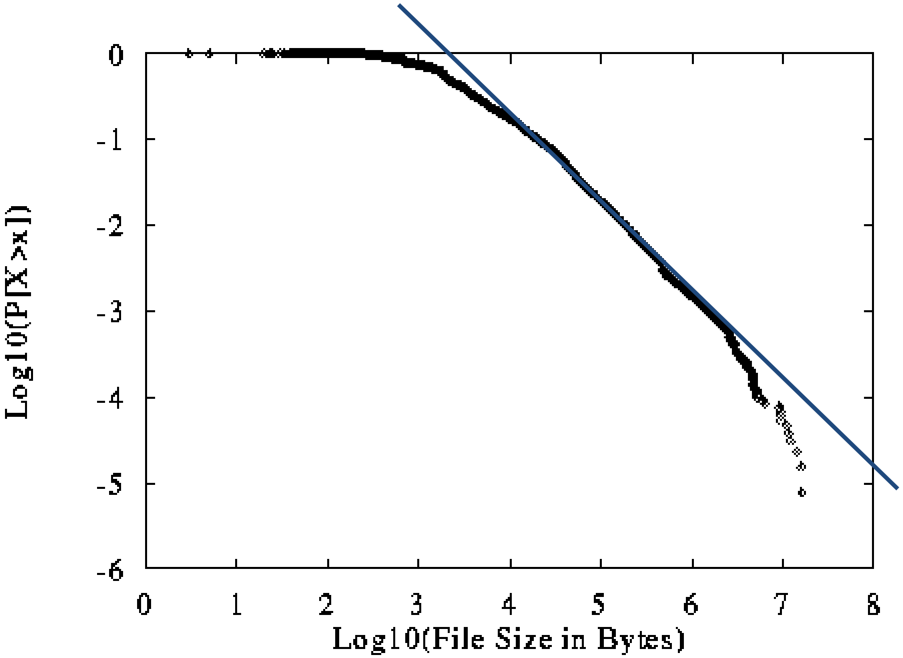
# Heavy-tail distributions

- Why is heavy-tail a problem?
- Can be proved:
  - Superposition of ON-OFF processes where the distribution of the ON periods is heavy tailed (e.g., Pareto) gives long-range dependent self-similar process

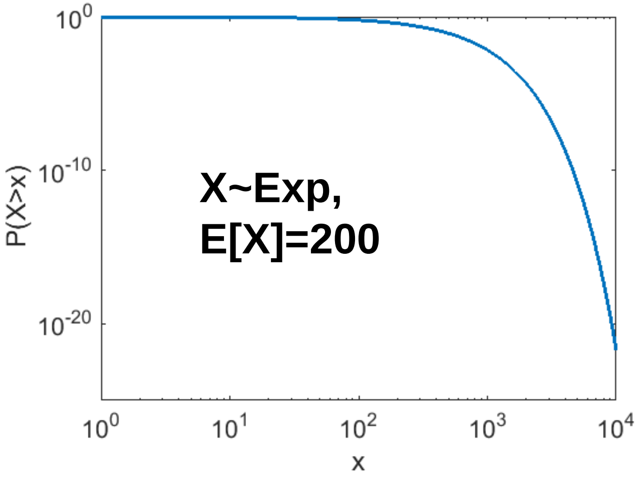


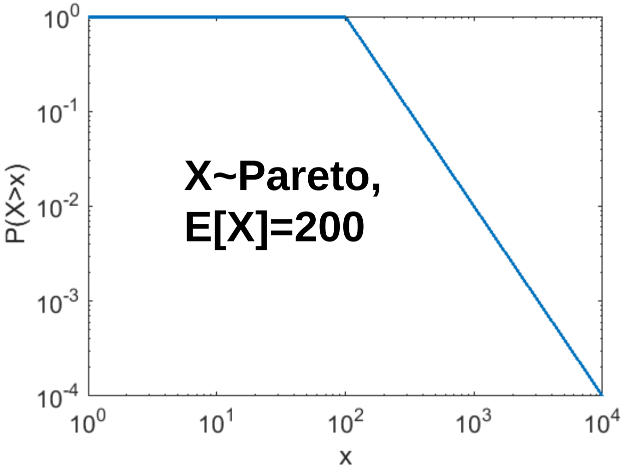


# Heavy tail plots

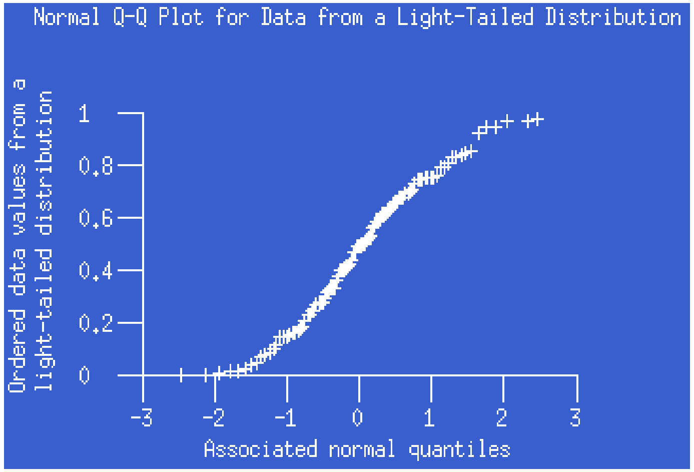


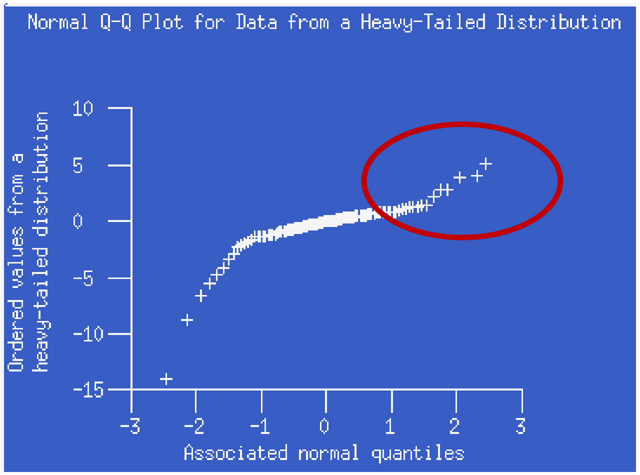
The figure shows a nearly linear plot on the log-log scale, which reflects a decay ~ $x^{-a}$, that is, heavy tail.

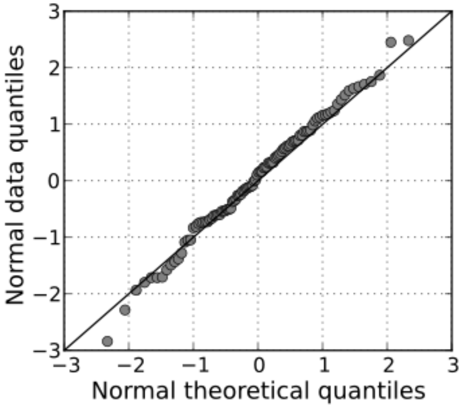




# Heavy tail plots







"Normal normal qq" by Skbkekas - Own work. Licensed under CC BY-SA 3.0 via Commons -

Q-Q plot: compares ordered sequence of samples from two distributions.

The up-going curve on the Q-Q plot reflects that the tail is heavier than the one of the normal distribution.

# Heavy-tail distributions, self-similarity, and long-range dependence

- Group A:
  - Define long range dependence
  - Give example of processes that are short and that are long range dependent. (It is enough the characterize the process with the auto-correlation function.)

- Group B:
  - Define self-similarity
  - Give the auto-correlation function of self-similar processes
  - Explain when is a self-similar process also long-range dependent.

Give a short summary on the white board.

# Long-range dependence

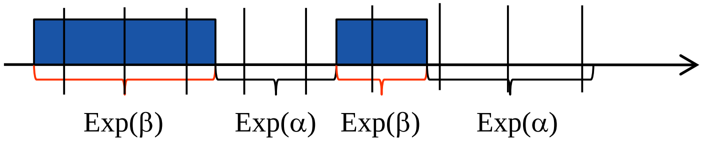
- Consider $X_i$ stochastic process, i=1,2,3 (discrete time)
  - Discrete process, samples from a continuous time process or integral over the interval
  - E[X], V[X] finite
  - Auto-covariance: Cov(k)=E[($X_i$-E[X])($X_{i+k}$-E[X])]
  - Auto-correlation: r(k)=Cov(k)/V[X]
- Short-range dependent:
  - $\sum_{k=1}^{\infty} r(k) < \infty$: the consecutive samples are correlated, but the correlation decreases fast with k
- Long-range dependent:
  - $\sum_{k=1}^{\infty} r(k) = \infty$: the consecutive samples are correlated, the correlation is preserved for long period.
- Note: long-range dependence is an asymptotic definition for large lags (k).

# Long-range dependence

- Short-range dependent:
  - $\sum_{k=1}^{\infty} r(k) < \infty$: the consecutive samples are correlated, but the correlation disappears fast
- Long-range dependent:
  - $\sum_{k=1}^{\infty} r(k) = \infty$: the consecutive samples are correlated, the correlation can be preserved for long period.

- MMPP is short range dependent. E.g., on-off fluid:

$$r(k) = e^{-(\alpha+\beta)k} \quad \text{(geometric serie)}$$



$$\sum_{k=1}^{\infty} r(k) = \sum_{k=1}^{\infty} e^{-(\alpha+\beta)k} < \infty \quad \text{since } e^{-(\alpha+\beta)} < 1$$

- So, what is the relationship between long range dependence and self-similarity?

# Self-similarity

- Consider $X_i$ stochastic process, i=1,2,3 (discrete time)
- E[X], V[X] finite

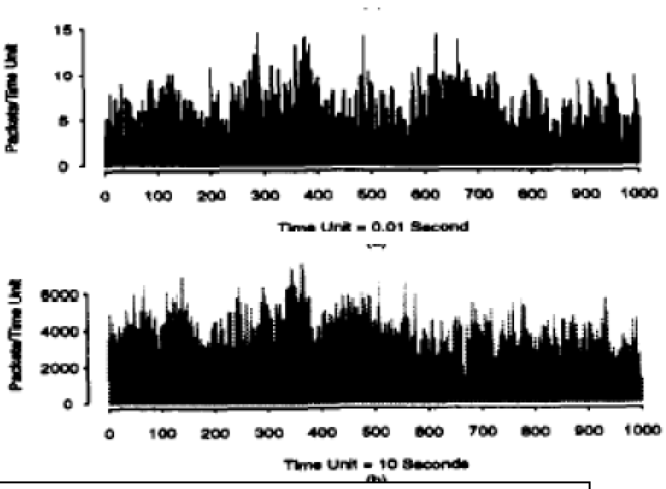
**time unit: 0.01s → $X_1$,$X_2$…**

$$X_1, X_2, \ldots$$

$$X_1^{(2)} = \frac{X_1 + X_2}{2}, X_2^{(2)} = \frac{X_3 + X_4}{2}$$

$$X_1^{(m)} = \frac{X_1 + \cdots + X_m}{m}, X_2^{(m)} = \frac{X_{m+1} + \cdots + X_{2m}}{m}$$

$$r^{(m)}(k) = \frac{Cov^m(k)}{V\left[X^m\right]}$$



**time unit: 10s → $X_1^{(1000)}$, $X_2^{(1000)}$…**

- (Second order) Self-similar: auto-correlation $r^m(k)=r(k)$, for all m and k
- Asymptotically self-similar: if above true for large m and k

# Self-similarity and long-range dependence (LRD)

- Second-order self-similar: $r^m(k)=r(k)$, for all m and k
- r(k) has specific form (can be proved):

$$r(k) \sim H(2H-1)\frac{1}{k^{2(1-H)}}, \qquad 0<H<1,\ H\neq 0.5$$

- H: Hurst parameter, the parameter of a self-similar process
- Self-similarity and LRD

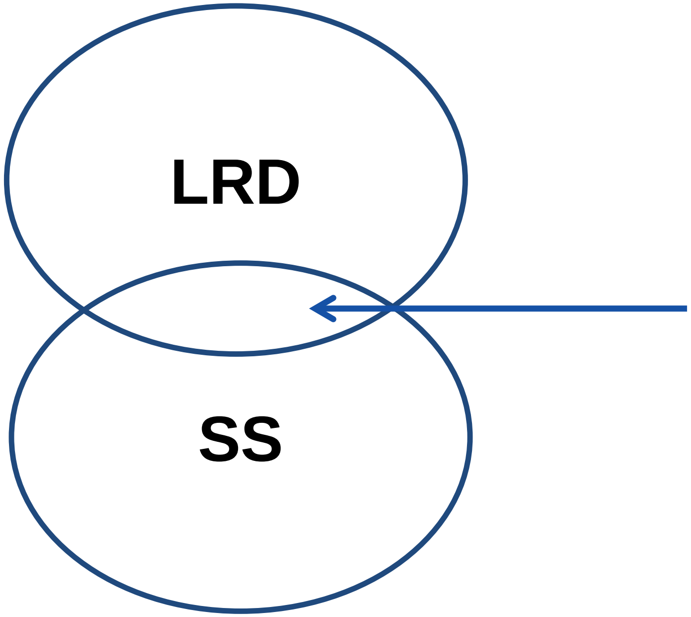
$$\sum_{k=1}^{\infty} r(k) = H(2H-1)\sum_{k=1}^{\infty}\frac{1}{k^{2(1-H)}} \quad \text{is a hyper-harmonic serie}$$

$$\sum_{k=1}^{\infty} r(k) = \infty \ \text{ if } 2(1\text{-H}) < 1 \rightarrow \text{H} > 0.5$$

→ A self-similar process is LRD if 0.5<H<1. This is the interval when SS makes trouble.

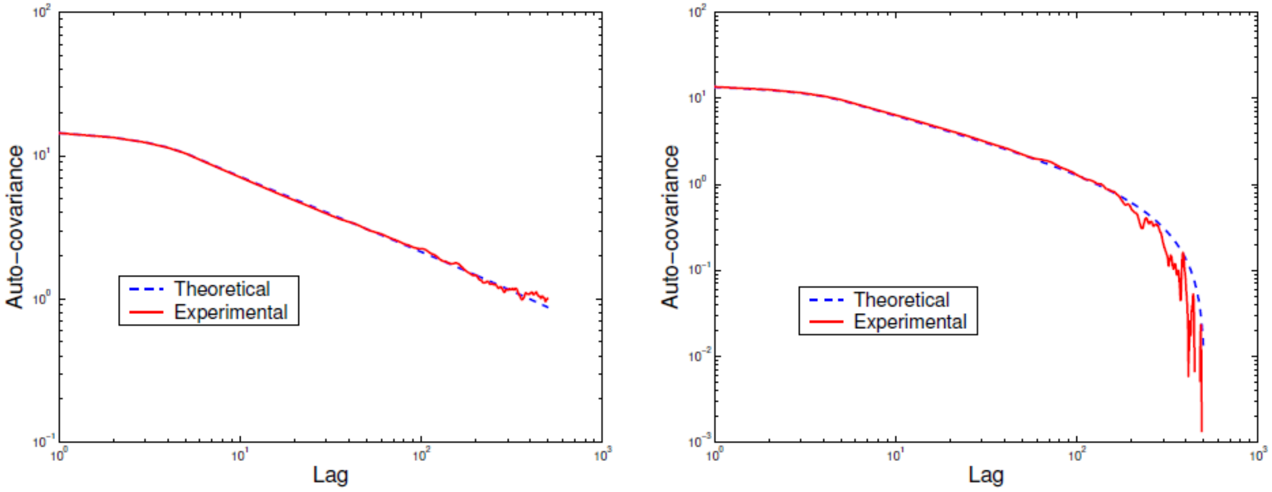
- Often the terms self-similarity and long-range dependence are used for the same thing.

# Heavy tail, self-similarity (SS) and long-range dependence (LRD)



- LRD-SS is the “problematic area
- Multiplexed heavy tail on-off sources give LRD-SS process

# Long-range dependence



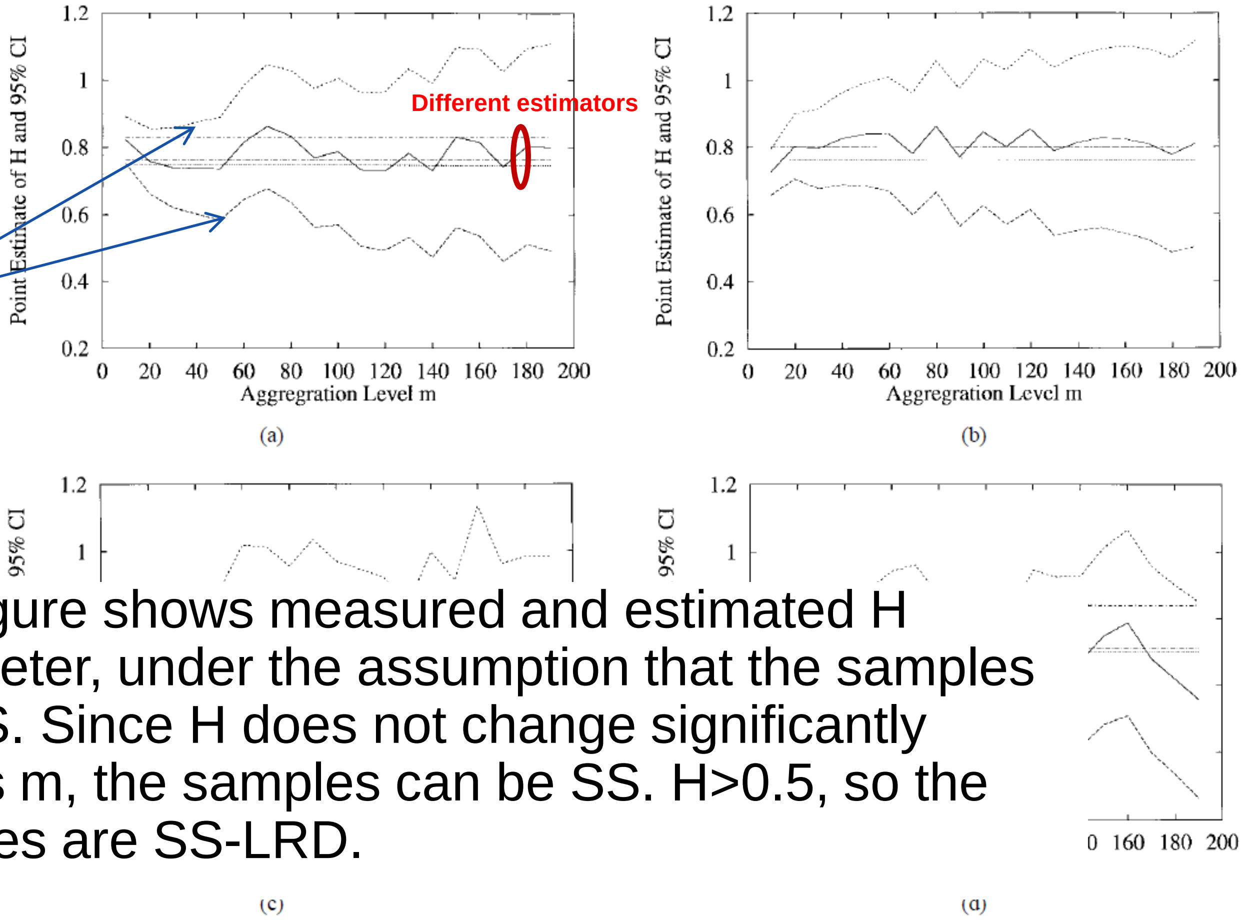
(a) Pareto Service Time

(b) Pareto+Exponential Service Time

The figure shows the auto-covariance (r(k)) as a function of the lag (k). Linear line in the log-log scale shows slow decay in r(k), which may result in long range dependence. In contrast, r(k) diminishes fast in the second figure, showing short range dependence.

# Self similarity

High load



Different estimators

CI: confidence interval

The figure shows measured and estimated H parameter, under the assumption that the samples are SS. Since H does not change significantly across m, the samples can be SS. H>0.5, so the samples are SS-LRD.

Low load

# Markovian vs. SS/LRD models

How should we choose traffic model for performance evaluation?

# Examples

W. E. Leland, M. S. Taqqu, W. Willinger and D. V. Wilson, "On the self-similar nature of Ethernet traffic (extended version)," in *IEEE/ACM Transactions on Networking*, vol. 2, no. 1, pp. 1-15, Feb. 1994.

**Abstract:** We demonstrate that Ethernet LAN traffic is statistically self-similar, that none of the commonly used traffic models is able to capture this fractal-like behavior, that such behavior has serious implications for the design, control, and analysis of high-speed, cell-based networks, and that aggregating streams of such traffic typically intensifies the self-similarity ("burstiness") instead of smoothing it. These conclusions are supported by a rigorous statistical analysis of hundreds of millions of high quality Ethernet traffic measurements collected between 1989 and 1992, coupled with a discussion of the underlying mathematical and statistical properties of self-similarity and their relationship with actual network behavior. The authors also present traffic models based on self-similar stochastic processes that provide simple, accurate, and realistic descriptions of traffic scenarios expected during B-ISDN deployment.

# Examples

M. E. Crovella and A. Bestavros, "Self-similarity in World Wide Web traffic: evidence and possible causes," in *IEEE/ACM Transactions on Networking*, vol. 5, no. 6, pp. 835-846, Dec. 1997.

**Abstract:**The notion of self-similarity has been shown to apply to wide-area and local-area network traffic. We show evidence that the subset of network traffic that is due to World Wide Web (WWW) transfers can show characteristics that are consistent with self-similarity, and we present a hypothesized explanation for that self-similarity. Using a set of traces of actual user executions of NCSA Mosaic, we examine the dependence structure of WWW traffic. First, we show evidence that WWW traffic exhibits behavior that is consistent with self-similar traffic models. Then we show that the self-similarity in such traffic can be explained based on the underlying distributions of WWW document sizes, the effects of caching and user preference in file transfer, the effect of user "think time", and the superimposition of many such transfers in a local-area network.

# Examples

Qingyun Liu, Xiaohan Zhao, Walter Willinger, Xiao Wang, Ben Y. Zhao, and Haitao Zheng, ”Self-Similarity in Social Network Dynamics,” ACM Trans. Model. Perform. Eval. Comput. Syst. vol. 2, no. 1, October 2016

Analyzing and modeling social network dynamics are key to accurately predicting resource needs and system behavior in online social networks. The presence of statistical scaling properties, that is, self-similarity, is critical for determining how to model network dynamics. In this work, we study the role that self-similarity scaling plays in a social network edge creation (that is, links created between users) process, through analysis of two detailed, time-stamped traces, a 199 million edge trace over 2 years in the Renren social network, and 876K interactions in a 4-year trace of Facebook. Using wavelet-based analysis, we find that the edge creation process in both networks is consistent with self-similarity scaling, once we account for periodic user activity that makes edge creation process non-stationary. producing desired properties in both temporal patterns and graph structural features.

# Examples

Tsybakov, B. , Georganas, N.D., “Overflow and losses in a network queue with a self-similar input,” *Queueing Systems, Theory and Applications,* 2000.

**Abstract:** …considers a discrete time queuing system that models a communication network multiplexer which is fed by a self-similar packet traffic. The model has … an input traffic which is an aggregation of independent source-active periods having Pareto-distributed lengths and arriving as Poisson batches. The new asymptotic upper and lower bounds to the buffer-overflow and packet-loss probabilities  are obtained.

# Examples

J. Liebeherr, A. Burchard and F. Ciucu, "Delay Bounds in Communication Networks With Heavy-Tailed and Self-Similar Traffic," in *IEEE Transactions on Information Theory*, vol. 58, no. 2, pp. 1010-1024, Feb. 2012.

Abstract:Traffic with self-similar and heavy-tailed characteristics has been widely reported in communication networks, yet, the state-of-the-art of analytically predicting the delay performance of such networks is lacking. This work addresses heavy-tailed traffic that has a finite first moment, but no second moment, and presents end-to-end delay bounds for such traffic. The derived performance bounds are non-asymptotic in that they do not assume a steady state, large buffer, or many sources regime. The analysis follows a network calculus approach where traffic is characterized by envelope functions and service is described by service curves. The system model is a multi-hop path of fixed-capacity links with heavy-tailed self-similar cross traffic at each node. A key contribution of the paper is a probabilistic sample-path bound for heavy-tailed arrival and service processes, which is based on a scale-free sampling method.

# Examples

St Robert, J-Y Le Boudec, On a Markov modulated chain exhibiting self-similarities over finite timescale, *Performance Evaluation,*
vol 27–28, pp 159-173,1996.

Abstract: Recent papers have pointed out that data traffic exhibits self-similarity, but self-similarity is observed only on a finite timescale. In order to account for that, we introduce the concept of pseudo long-range dependencies. In this paper, we describe a Modulated Markov process producing self-similarity on a finite timescale; the process is quite easy to manipulate and depends only on three parameters (two real numbers and one integer). An advantage of using it is that it is possible to re-use the well-known analytical queuing theory techniques developed in the past in order to evaluate network performance.

# Examples

A. Thummler, P. Buchholz and M. Telek, "A Novel Approach for Phase-Type Fitting with the EM Algorithm," in *IEEE Transactions on Dependable and Secure Computing*, vol. 3, no. 3, pp. 245-258, July-Sept. 2006.

Abstract: The representation of general distributions or measured data by phase-type distributions is an important and nontrivial task in analytical modeling. Although a large number of different methods for fitting parameters of phase-type distributions to data traces exist, many approaches lack efficiency and numerical stability. In this paper, a novel approach is presented that fits a restricted class of phase-type distributions, namely, mixtures of Erlang distributions, to trace data.

# Markovian vs. SS/LRD models

- How should we choose traffic model for performance evaluation?
  - SS/LRD
    - complex models, possible to use for simulation but mathematical models are not that tractable
    - LRD captures asymptotic behavior but not short time characteristics
  - Markovian models
    - can capture correlations on arbitrary – finite – time scale
    - easier to use in mathematical models
- We have to choose models according to the dominant time scale we consider.

# Summary (1/2)

- Network traffic modeling
  - Flows, bursts and packets
  - Elastic and streaming flows
  - Packet scale and fluid models for flow characterization
- Markovian traffic models
  - Markov modulated traffic models
  - The rate is modulated by a Markov chain to capture burstiness
  - Can describe short term correlation

# Summary (2/2)

- Long-range dependence, self-similarity and heavy-tail
  - Asymptotic characteristics
  - Heavy-tail: the tail function of the distribution has only hyperbolic decrease: $P(s>x) \sim x^{-a}$, $x \to \infty$,
    multiplexing heavy-tail flows leads to self-similarity
  - Long-range dependence: correlation is preserved over long timescales: $\sum_{k=1}^{\infty} r(k) = \infty$
  - Self-similarity: the correlation is preserved irrespective of time aggregation: $r^m(k) = r(k)$
  - Self similarity is characterized by H, the Hurst parameter, and the SS process is LRD if $0.5<H<1$
  - LRD-SS flows lead to inefficient multiplexing and long queues