KTH Teknikvetenskap

Suggested solutions for sample exam 2

Version Preparatory course in mathematics

SF0003 Introductory Course in Mathematics
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1. Simplify $\dfrac{\frac{1}{3}-\frac{2}{7}}{\frac{9}{4}-\frac{1}{3}}$ by writing over a common denominator. The result should be reduced as far as possible.

   Suggested solution: We have

$$\begin{aligned}\frac{1/3-2/7}{9/4-1/3} &= \frac{21}{21}\cdot\frac{1/3-2/7}{9/4-1/3} = \frac{7-6}{21(9/4-1/3)}\\ &= \frac{12}{12}\cdot\frac{1}{21(9/4-1/3)} = \frac{12}{21(27-4)} = \frac{12}{21\cdot 23} = \frac{4}{7\cdot 23}.\end{aligned}$$

2. Simplify $\dfrac{3}{x}-\dfrac{7}{x+1}+\dfrac{4x-1}{x^2+x}$ by writing over a common denominator. The result should be reduced as far as possible.

   Suggested solution: We have $x^2+x = x(x+1)$, so

$$\begin{aligned}\frac{3}{x}-\frac{7}{x+1}+\frac{4x-1}{x^2+x} &= \frac{3(x+1)}{x(x+1)}-\frac{7x}{x(x+1)}+\frac{4x-1}{x(x+1)}\\ &= \frac{3(x+1)-7x+(4x-1)}{x(x+1)}\\ &= \frac{2}{x(x+1)}.\end{aligned}$$

3. Complete the square to determine the smallest value obtained by the polynomial x^2+3x+4.

   Suggested solution: We complete the square in the polynomial and get

$$\begin{aligned}x^2+3x+4 &= x^2+2\cdot\frac{3}{2}x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+4\\ &= \left(x+\frac{3}{2}\right)^2-\frac{9}{4}+4\\ &= \left(x+\frac{3}{2}\right)^2+\frac{7}{4}.\end{aligned}$$

   Since the first term here is always greater than or equal to zero we find that the smallest value is obtained when it is equal to zero (and $x=-3/2$). The smallest value obtained by this polynomial is thus $7/4$.

4. Simplify $\ln 81 - \ln 9 - \ln 3$.

   Suggested solution: We have

$$\ln 81 - \ln 9 - \ln 3 = \ln \frac{81}{9 \cdot 3} = \ln 3.$$

5. Determine the equation for the circle which has centre $(-1,2)$ and contains the point $(2,6)$.

   Suggested solution: The radius r of the circle is equal to the distance from its centre to given point on the circle. We thus have

$$r^2 = (2-(-1))^2 + (6-2)^2 = 3^2 + 4^2 = 25,$$

   or $r = 5$. The equation of the circle is

$$(x-(-1))^2 + (y-2)^2 = 5^2,$$

   or

$$(x+1)^2 + (y-2)^2 = 25.$$

6. Suppose that $-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$ and that $\sin v = a$. Express $\sin\left(\frac{\pi}{2} - v\right)$ in terms of a.

   Suggested solution: The addition formula for sin tells us that

$$\sin(\frac{\pi}{2} - v) = \sin(\frac{\pi}{2})\cos(-v) + \cos(\frac{\pi}{2})\sin(-v) = \sin(\frac{\pi}{2})\cos(v) - \cos(\frac{\pi}{2})\sin(v).$$

   Since $\cos(\frac{\pi}{2}) = 0$ and $\sin(\frac{\pi}{2}) = 1$ this gives us

$$\sin(\frac{\pi}{2} - v) = \cos(v).$$

   The pythagorean identity then gives

$$\cos^2(v) = 1 - \sin^2(v) = 1 - a^2,$$

   and

$$\cos(v) = \pm\sqrt{1-a^2}.$$

   Since $-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$ we have $\cos(v) \geq 0$, and we conclude that

$$\sin(\frac{\pi}{2} - v) = \cos(v) = \sqrt{1-a^2}.$$