KTH Teknikvetenskap

# Suggested solutions for sample exam 1

*Version Preparatory course in mathematics*

**SF0003 Introductory Course in Mathematics**
**August 2017**

1. Write $\frac{1}{6} - \frac{3}{10} + \frac{13}{15}$ over a common denominator. The answer should be reduced as far as possible.

   *Suggested solution:* The smallest common denominator is $2 \cdot 3 \cdot 5 = 30$, so we have

   $$\frac{1}{6} - \frac{3}{10} + \frac{13}{15} = \frac{5}{30} - \frac{9}{30} + \frac{26}{30} = \frac{22}{30} = \frac{11}{15}.$$

2. Determine the coefficients in front of $x$ and $x^2$ when the expression $(x+3)(x^2+2x-1)(19x^3 - x^2 + 1)$ is completely expanded.

   *Suggested solution:* Instead of expanding the whole expression and identifying the coefficients of $x$ and $x^2$, respectively, it is enough to find the combinations of one term from the first, second and third parenthesis that gives an $x$ and $x^2$ term, respectively, when multiplied together.

   To obtain an $x$ term there are two combinations,

   $$(x+3)(x^2+2x-1)(19x^3-x^2+1) = \cdots + x \cdot (-1) \cdot 1 + \cdots ,$$

   $$(x+3)(x^2+2x-1)(19x^3-x^2+1) = \cdots + 3 \cdot 2x \cdot 1 + \cdots ,$$

   and the $x$ term will therefore be $-x + 6x = 5x$, i.e. the coefficient of $x$ is 5.

   There are three combinations that result in $x^2$ terms,

   $$(x+3)(x^2+2x-1)(19x^3-x^2+1) = \cdots + x \cdot 2x \cdot 1 + \cdots ,$$

   $$(x+3)(x^2+2x-1)(19x^3-x^2+1) = \cdots + 3 \cdot x^2 \cdot 1 + \cdots ,$$

   $$(x+3)(x^2+2x-1)(19x^3-x^2+1) = \cdots + 3 \cdot (-1) \cdot (-x^2) + \cdots ,$$

   and in total the $x^2$ term is $2x^2 + 3x^2 + 3x^2 = 8x^2$, i.e. the coefficient of $x^2$ is 8.

3. Determine the intersection point between the line $x + 2y - 4 = 0$ and the line $x = 10$.

   *Suggested solution:* Since the intersection point is on the line $x = 10$ it has the $x$-coordinate 10. The $y$-coordinate of the intersection point satisfies $x + 2y - 4 = 10 + 2y - 4 = 0$, or $6 + 2y = 0$, which gives $y = -3$. The intersection point is thus $(10, -3)$.

4. Solve the equation $3\sqrt{3 - x} = 5 - x$.

   *Suggested solution:* If $3\sqrt{3 - x} = 5 - x$ it follows that

$$(3\sqrt{3 - x})^2 = 9(3 - x) = (5 - x)^2 = x^2 - 10x + 25$$

   or

$$0 = x^2 - 10x + 25 + 9x - 27 = x^2 - x - 2$$

   which is satisfied for $x = -1$ and $x = 2$. Since we have taken the square of the equation we cannot be sure that these are solutions also to the original equation.

   For $x = -1$ we have

$$3\sqrt{3 - (-1)} = 3\sqrt{4} = 6 = 5 - (-1),$$

   so this is a solution.

   For $x = 2$ we have

$$3\sqrt{3 - 2} = 3\sqrt{1} = 3 = 5 - 3,$$

   so this is also a solution.

   The solutions to the equations are thus $x = -1$ and $x = 2$.

5. Determine the centre and the radius of the circle given by the equation $x^2 - 2x + y^2 + 2y = 1$.

   *Suggested solution:* We complete the square and find

$$\begin{aligned} 1 &= x^2 - 2x + y^2 + 2y \\ &= x^2 - 2x + 1 - 1 + y^2 + 2y + 1 - 1 \\ &= (x - 1)^2 - 1 + (y + 1)^2 - 1, \end{aligned}$$

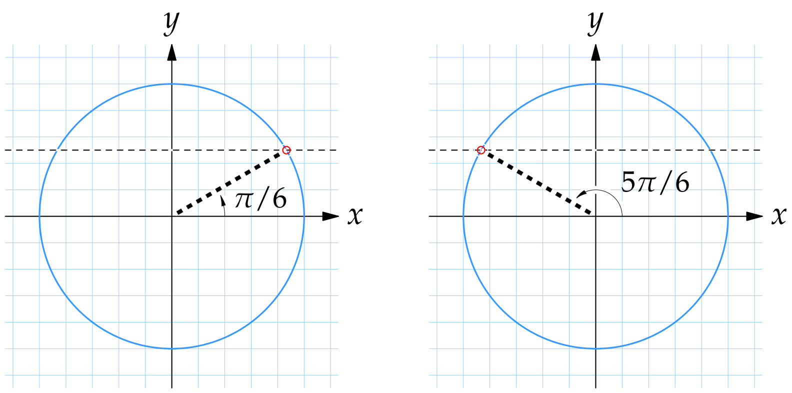
   or

$$3 = (x - 1)^2 + (y + 1)^2.$$

   This is the equation for a circle with centre $(x, y) = (1, -1)$ and radius $r = \sqrt{3}$.

6. Solve the equation $\sin 5x = \frac{1}{2}$.

   *Suggested solution:* We begin by determining the solutions to the equation when $0 \le 5x \le 2\pi$. According to the unit circle there are two such solutions: $5x = \pi/6$ and $5x = \pi - \pi/6 = 5\pi/6$.



We obtain the remaining solutions by adding integer multiples of $2\pi$ to the two solutions above,

$$5x = \frac{\pi}{6} + 2n\pi \quad \text{and} \quad 5x = \frac{5\pi}{6} + 2n\pi,$$

where $n$ is an arbitrary integer, or after dividing by 5,

$$x = \frac{\pi}{30} + \frac{2n\pi}{5} \quad \text{and} \quad x = \frac{\pi}{6} + \frac{2n\pi}{5},$$

where $n$ is an arbitrary integer.