

Suggested solutions for sample exam 2

Version Preparatory course in mathematics

SF0003 Introductory Course in Mathematics

August 2016

1. Simplify $\frac{\frac{1}{3} - \frac{2}{7}}{\frac{9}{4} - \frac{1}{3}}$ by writing over a common denominator. The result should be reduced as far as possible.

Suggested solution: We have

$$\begin{aligned} \frac{\frac{1}{3} - \frac{2}{7}}{\frac{9}{4} - \frac{1}{3}} &= \frac{21}{21} \cdot \frac{\frac{1}{3} - \frac{2}{7}}{\frac{9}{4} - \frac{1}{3}} = \frac{7 - 6}{21(\frac{9}{4} - \frac{1}{3})} \\ &= \frac{12}{12} \cdot \frac{1}{21(\frac{9}{4} - \frac{1}{3})} = \frac{12}{21(27 - 4)} = \frac{12}{21 \cdot 23} = \frac{4}{7 \cdot 23}. \end{aligned}$$

2. Simplify $\frac{3}{x} - \frac{7}{x+1} + \frac{4x-1}{x^2+x}$ by writing over a common denominator. The result should be reduced as far as possible.

Suggested solution: We have $x^2 + x = x(x+1)$, so

$$\begin{aligned} \frac{3}{x} - \frac{7}{x+1} + \frac{4x-1}{x^2+x} &= \frac{3(x+1)}{x(x+1)} - \frac{7x}{x(x+1)} + \frac{4x-1}{x(x+1)} \\ &= \frac{3(x+1) - 7x + (4x-1)}{x(x+1)} \\ &= \frac{2}{x(x+1)}. \end{aligned}$$

3. Complete the square to determine the smallest value obtained by the polynomial $x^2 + 3x + 4$.

Suggested solution: We complete the square in the polynomial and get

$$\begin{aligned} x^2 + 3x + 4 &= x^2 + 2 \cdot \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 4 \\ &= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 4 \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{7}{4}. \end{aligned}$$

Since the first term here is always greater than or equal to zero we find that the smallest value is obtained when it is equal to zero (and $x = -3/2$). The smallest value obtained by this polynomial is thus $7/4$.

4. Simplify $\ln 81 - \ln 9 - \ln 3$.

Suggested solution: We have

$$\ln 81 - \ln 9 - \ln 3 = \ln \frac{81}{9 \cdot 3} = \ln 3.$$

5. Determine the equation for the circle which has centre $(-1, 2)$ and contains the point $(2, 6)$.

Suggested solution: The radius r of the circle is equal to the distance from its centre to given point on the circle. We thus have

$$r^2 = (2 - (-1))^2 + (6 - 2)^2 = 3^2 + 4^2 = 25,$$

or $r = 5$. The equation of the circle is

$$(x - (-1))^2 + (y - 2)^2 = 5^2,$$

or

$$(x + 1)^2 + (y - 2)^2 = 25.$$

6. Suppose that $-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$ and that $\sin v = a$. Express $\sin\left(\frac{\pi}{2} - v\right)$ in terms of a .

Suggested solution: The addition formula for sin tells us that

$$\sin\left(\frac{\pi}{2} - v\right) = \sin\left(\frac{\pi}{2}\right) \cos(-v) + \cos\left(\frac{\pi}{2}\right) \sin(-v) = \sin\left(\frac{\pi}{2}\right) \cos(v) - \cos\left(\frac{\pi}{2}\right) \sin(v).$$

Since $\cos\left(\frac{\pi}{2}\right) = 0$ and $\sin\left(\frac{\pi}{2}\right) = 1$ this gives us

$$\sin\left(\frac{\pi}{2} - v\right) = \cos(v).$$

The pythagorean identity then gives

$$\cos^2(v) = 1 - \sin^2(v) = 1 - a^2,$$

and

$$\cos(v) = \pm \sqrt{1 - a^2}.$$

Since $-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$ we have $\cos(v) \geq 0$, and we conclude that

$$\sin\left(\frac{\pi}{2} - v\right) = \cos(v) = \sqrt{1 - a^2}.$$