The Fifth International Conference on Proton-emitting Nuclei (PROCON2015)
IMP Lanzhou China, 6th to 10th July, 2015

Alpha decay as a probe of the structure of neutron-deficient nuclei around *Z=82*

and a short tour to proton emitters

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Collaborators:

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A.N. Andreyev (York), M. Huyse, P. Van Duppen (KU Leuven, Belgium)
DS Delion (Bucharest)

Outline

- **The Geiger-Nuttall law and experimental status of alpha decay studies: The Gamow interpretation**
- **Microscopic two-step description of alpha decay;**
  Clustering (formation) + Penetration
- **Smooth behavior of alpha formation amplitudes and their Abrupt changes;**
- **Relation to the pairing collectivity;**
- **Influence of the Z=82 shell closure**
- **Superallowed decays and neutron-proton correlation; proton decay**
- **Summary**

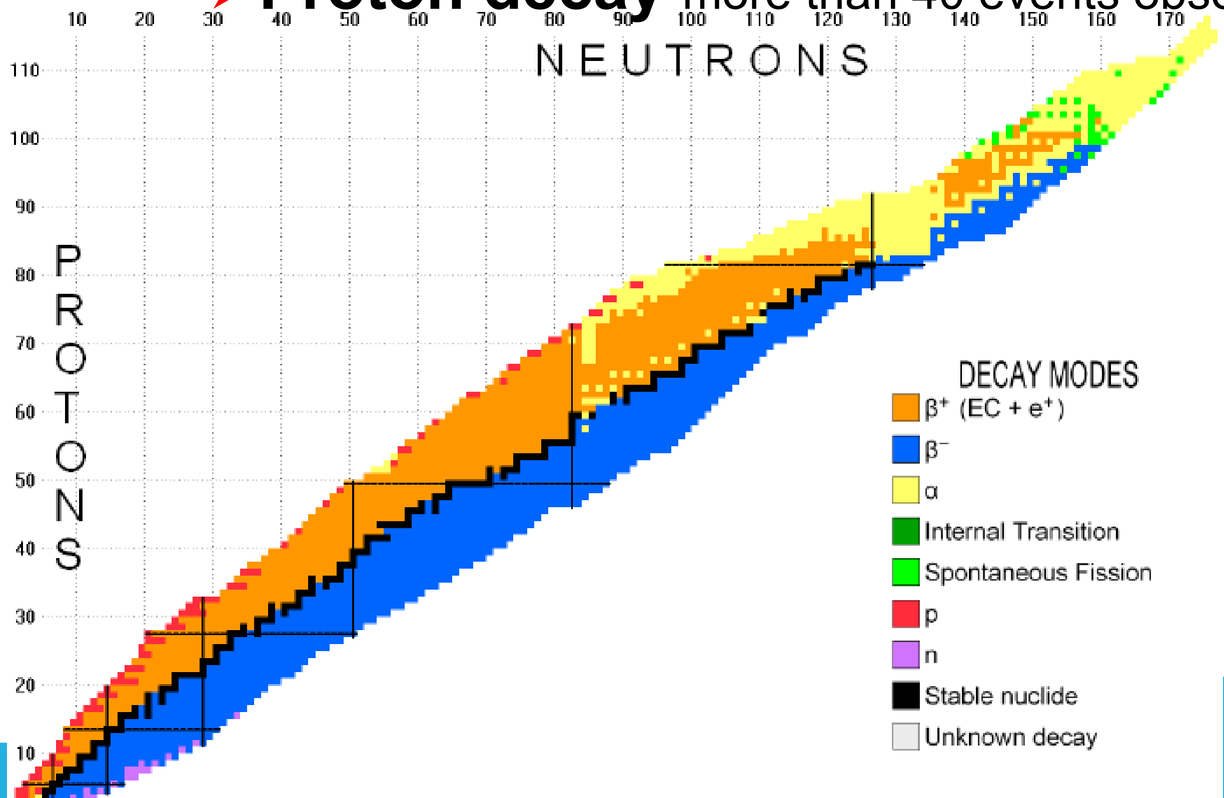
Experimental Progress

- **α radioactivity** ~ 400 events observed in A > 150 nuclei;
  - **α decay of N≈Z nuclei (2000s-)**
  - **one of the oldest subjects and long history of success**

- **Heavier cluster decays** 11 events observed in trans-lead nuclei

^{223}Ra(^{14}C), Rose and Jones, Nature 307, 245 (1984);

- **Proton decay** more than 40 events observed in the rare-earth region.



G. Audi et al. / Nuclear Physics A 729 (2003) 3-

- **New decay modes**
  - ✧ **Di-proton decay**
  - ✧ **Neutron decay**
  - ✧ **^{12}C cluster decay**

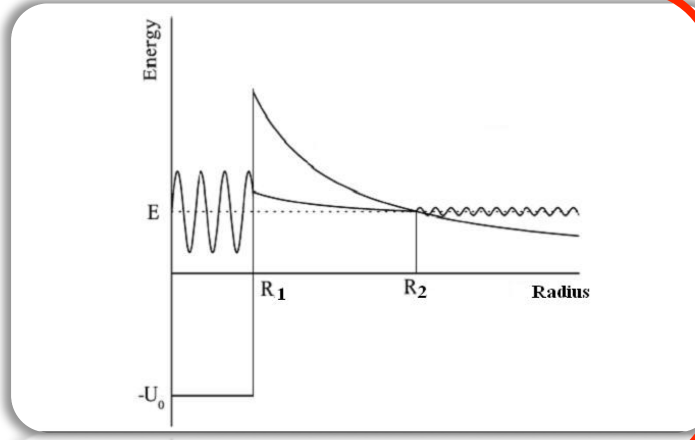
Theoretical understanding

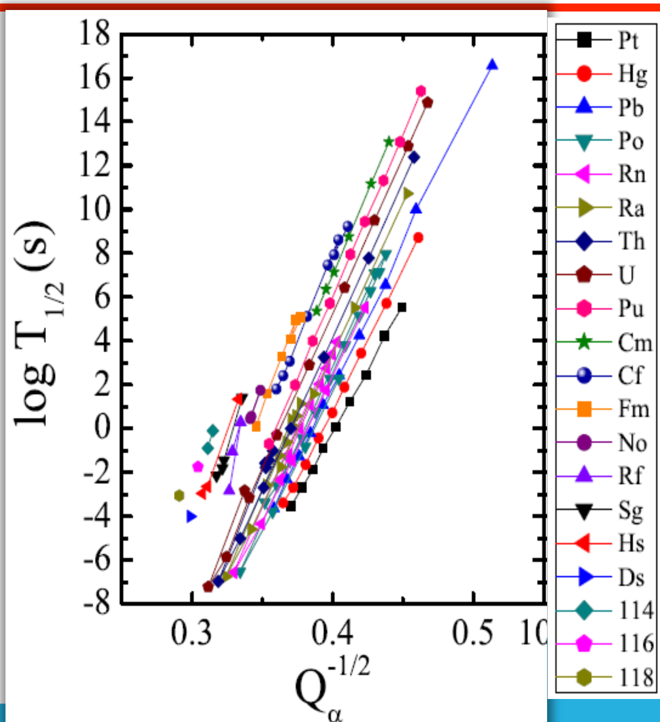
➢Quantum tunneling interpretation

G. Gamow, Z. Phys. 51, 204 (1928).

R. W. Gurney and E. U. Condon, Nature 122, 439 (1928).

$$P = \exp\left\{-2\int_{R_1}^{R_2}\sqrt{\frac{2\mu}{\hbar^2}|V_C(r)-Q_\alpha|\mathrm{d}r}\right\},$$





The Geiger-Nuttall law of alpha decays

$$\log_{10} T_{1/2} = AQ_\alpha^{-1/2} + B$$

H. Geiger and J. M. Nuttall, Philos. Mag. 22, 613 (1911).

Theoretical understanding

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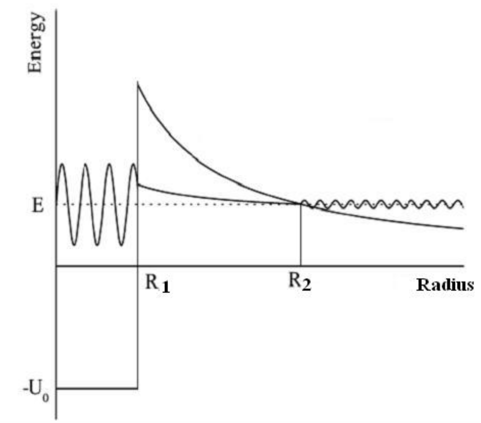
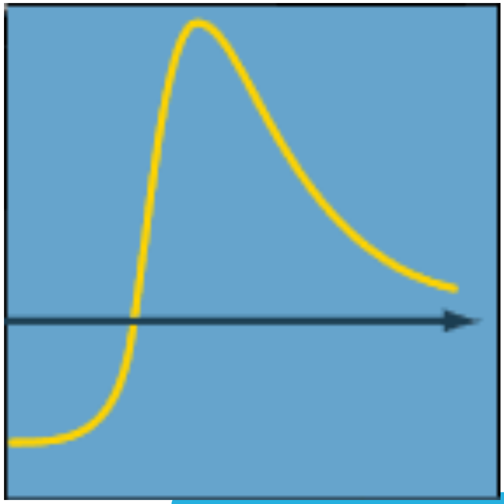


"Standard" picture of the alpha decay process



$$\lambda = \ln 2/T = \nu S P_s$$

➢**The alpha particle is not a basic constitute of the atomic nucleus**

❖**Alpha particle model of atomic nucleus before the neutron was discovered**

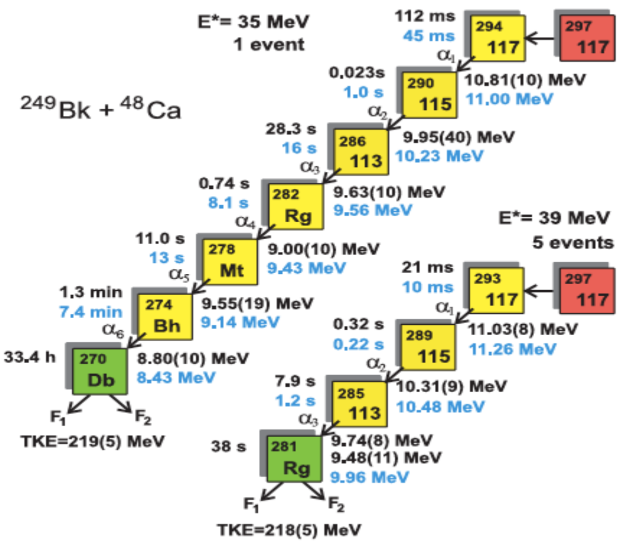
❖**Light nuclei may exhibit profound alpha clustering structure**

➢**The Gamow theory does not carry nuclear structure information**

➢**Spectroscopic factor is not an observable**

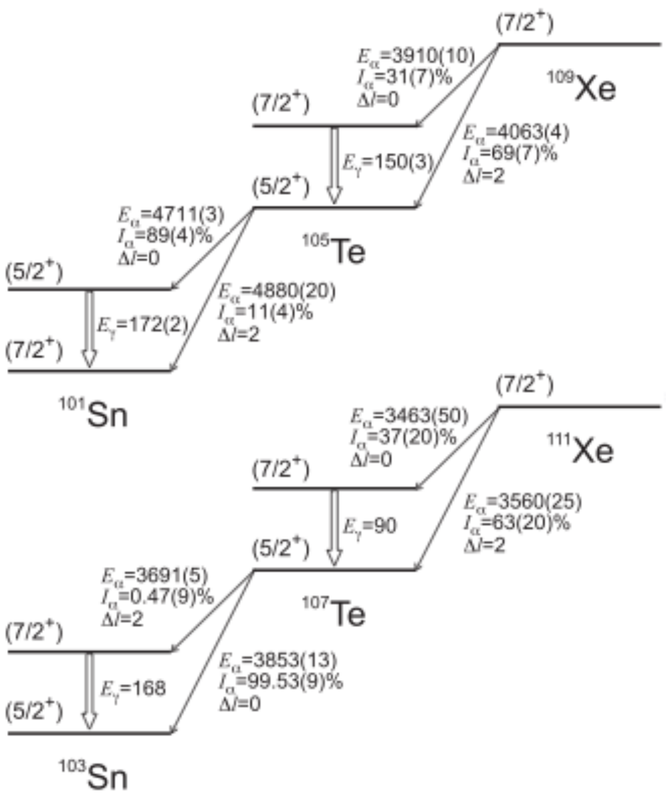
Alpha decay as an experimental tool

SHE probe

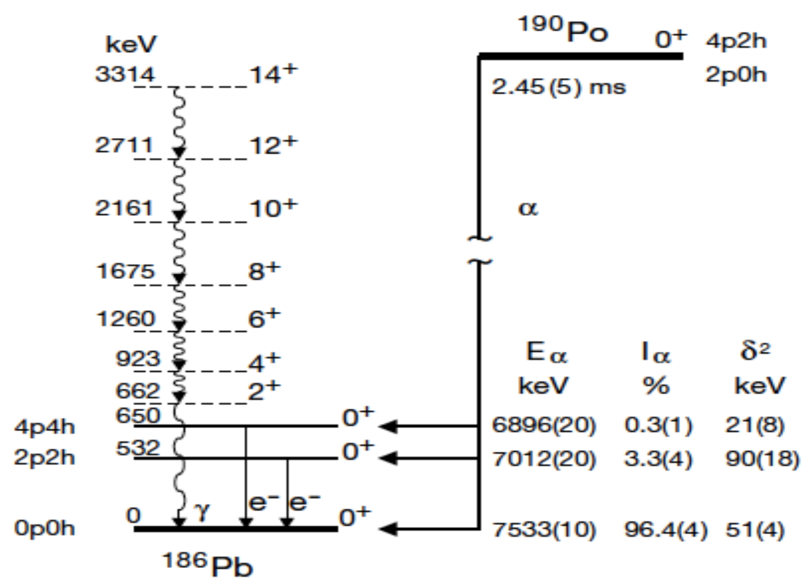


Yu. Ts. Oganessian et al., PhysRevLett.104.142502 (2010)

Decays of N~Z nuclei



Fine structure



S.N.Liddick et al., Phys.Rev.Lett. 97, 082501 (2006)
D.Seweryniak et al., Phys.Rev. C 73, 061301 (2006)
I. Darby et al., Phys. Rev. Lett. 105, 162502 (2010).

A.N. Andreyev et al., Nature 405 430 (2000)

Microscopic description of alpha decay

R.G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_c} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi,\rho)}{RF_c(R)} \right|^2 ,$$

ν is the outgoing velocity of the emitted particle

$F_c(R)$ is the formation amplitude

H_l^+ is the Coulomb-Hankel function The penetrability is proportional to $|H_l^+(\chi,\rho)|^{-2}$.

R is the distance between the center of mass of the cluster and daughter nucleus which divides the decay process into an internal region and complementary external region.

The Coulomb function is 'well' understood from the Gamow theory.

$$H_0^+(\chi,\rho) \approx (\cot\beta)^{1/2} \exp[\chi(\beta - \sin\beta\cos\beta)] ,$$

$\chi = 2Z_cZ_de^2/\hbar v$ $\qquad$ $\rho = \mu v R/\hbar$ $\qquad$ $\cos^2\beta = \frac{\rho}{\chi} = \frac{Q_cR}{e^2Z_cZ_d}.$

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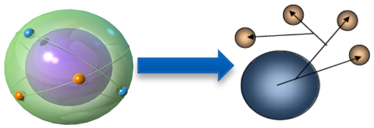
R is the distance between the center of mass of the cluster and daughter nucleus which divides the decay process into an internal region and complementary external region.

F(R) describes the formation amplitude of the alpha particle inside the nucleus

Yes, it depend on the radius.

$$m \rightarrow d + \alpha$$



$$\mathcal{F}_l(R) = \int d\mathbf{R} d\xi_d d\xi_\alpha [\Psi(\xi_d)\phi(\xi_\alpha)Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_\alpha, \mathbf{R}),$$

Shell Model

H.J. Mang, PR 119,1069 (1960); I. Tonozuka, A. Arima, NPA 323, 45 (1979).

BCS approach

HJ Mang and JO Rasmussen, Mat. Fys. Medd. Dan. Vid. Selsk. (1962)

DS Delion, A. Insolia and RJ Liotta, PRC46, 884(1992).

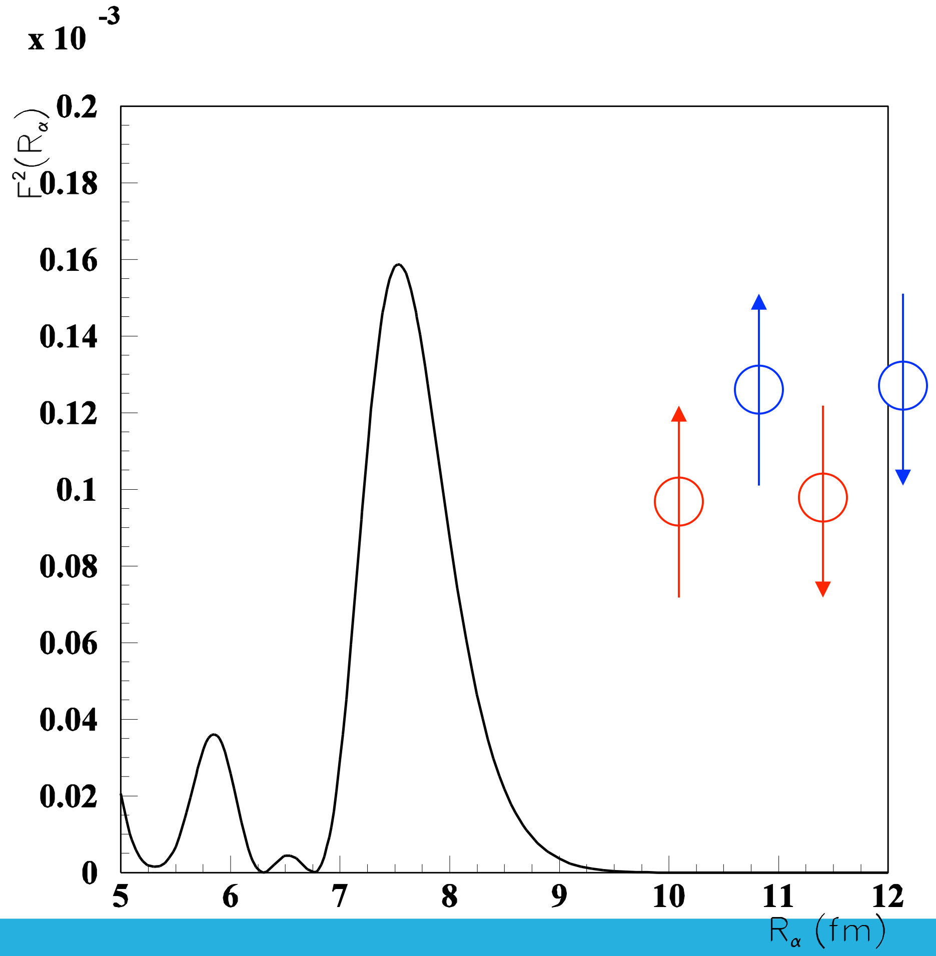
Decay width is the flux of outgoing particles

$$\Gamma = \hbar v \left| \frac{\Psi_{\text{int}}(R)}{\Psi_{ext}(R)} \right|^2 = \hbar v |A|^2$$

The width does not depend on the matching radius *R* because both functions satisfy the same Schrödinger equation

$$\Psi_{ext}(R) = \frac{H_l^{(+)}(kR)}{R} \xrightarrow[R\to\infty]{} \frac{e^{i(kR+\sigma_l)}}{R}$$

Significant alpha clustering at the nuclear surface



Formation amplitude:

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_c} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi, \rho)}{RF_c(R)} \right|^2,$$

- Can be extracted from experimental data in a 'model-independent' way;
- Can be calculated microscopically;
- F(R) does not depend on the Q value

PHYSICAL REVIEW VOLUME 113, NUMBER 6 MARCH 15, 1959

Alpha-Decay Barrier Penetrabilities with an Exponential Nuclear Potential: Even-Even Nuclei*

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Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California
(Received October 27, 1958)

$$\lambda = \delta^2 P / h,$$

$$-\int_{R_i}^{R_0} \frac{(2M)^{\frac{1}{2}}}{\hbar} \left[V(r) + \frac{2Ze^2}{r} + \frac{\hbar^2}{2mr^2} l(l+1) - E \right]^{\frac{1}{2}} dr,$$

$$V(r) = -1100 \exp\left\{ -\left[\frac{r - 1.17A^{\frac{1}{3}}}{0.574} \right] \right\} \text{ Mev},$$

Lambda is the decay constrant

Alpha formation probability from experiments

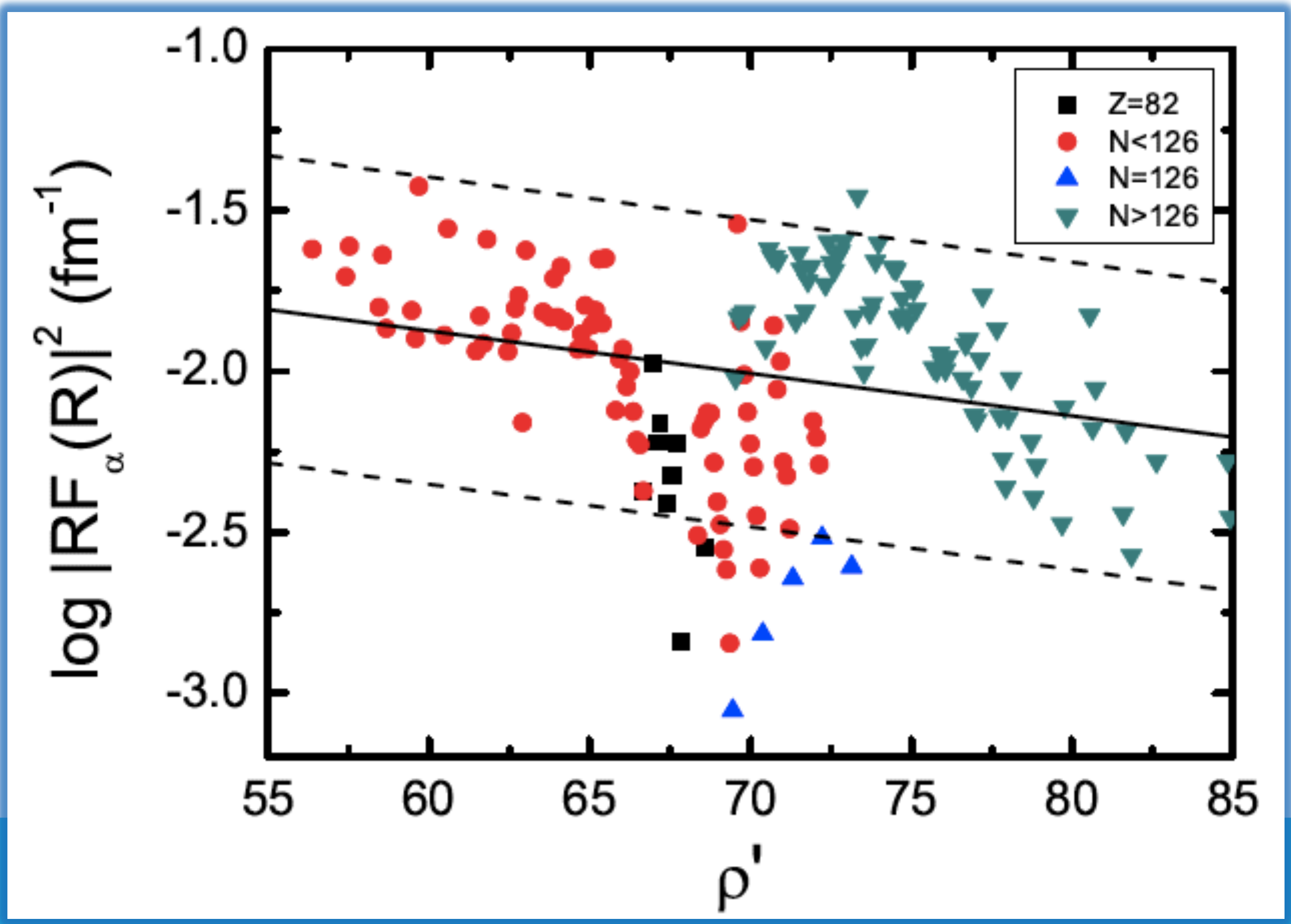
$$\log|RF(R)|^{-2} = \log T_{1/2}^{\mathrm{Expt.}} - \log\left[\frac{\ln 2}{\nu}|H_0^+(\chi,\rho)|^2\right],$$

R should be large enough that the nuclear interaction is negligible, i.e., at the nuclear surface.

R=1.2($A_d^{1/3}+A_c^{1/3}$)

$$\chi' = \frac{\hbar}{e^2\sqrt{2M}}\chi = Z_c Z_d\sqrt{\frac{A}{Q_c}},$$

$$\rho' = \frac{\hbar}{\sqrt{2MR_0e^2}}(\rho\chi)^{1/2} = \sqrt{AZ_cZ_d(A_d^{1/3}+A_c^{1/3})},$$



CQ et al, Phys.Rev.C80,044326 (2009); 81,064319 (2010).

Alpha formation probability from experiments

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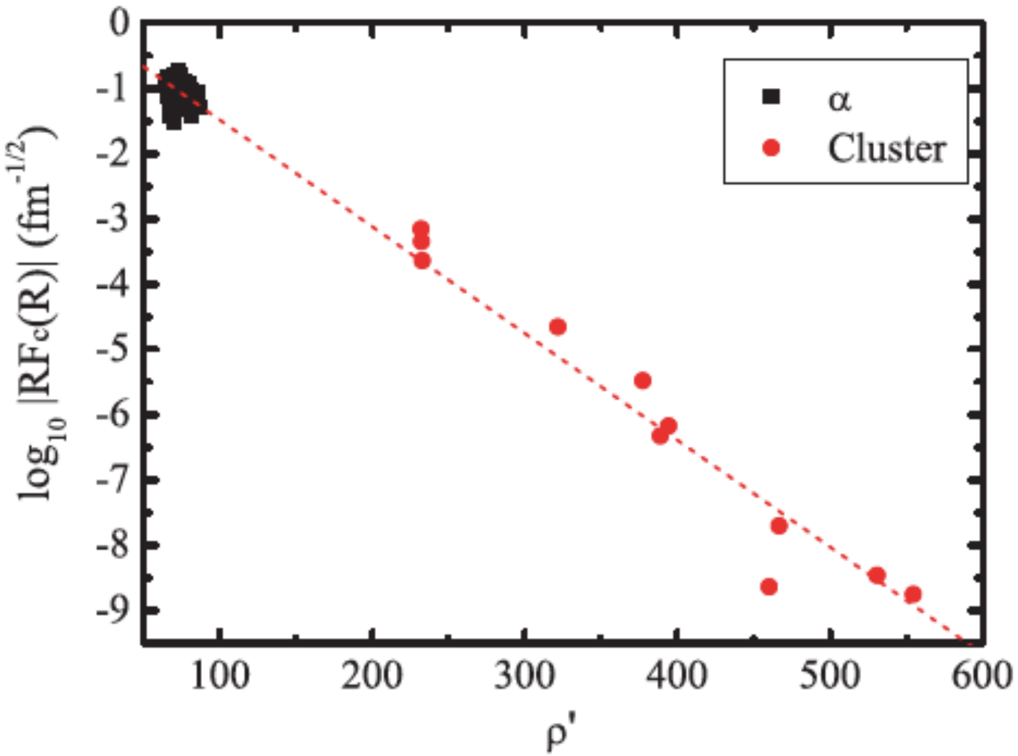
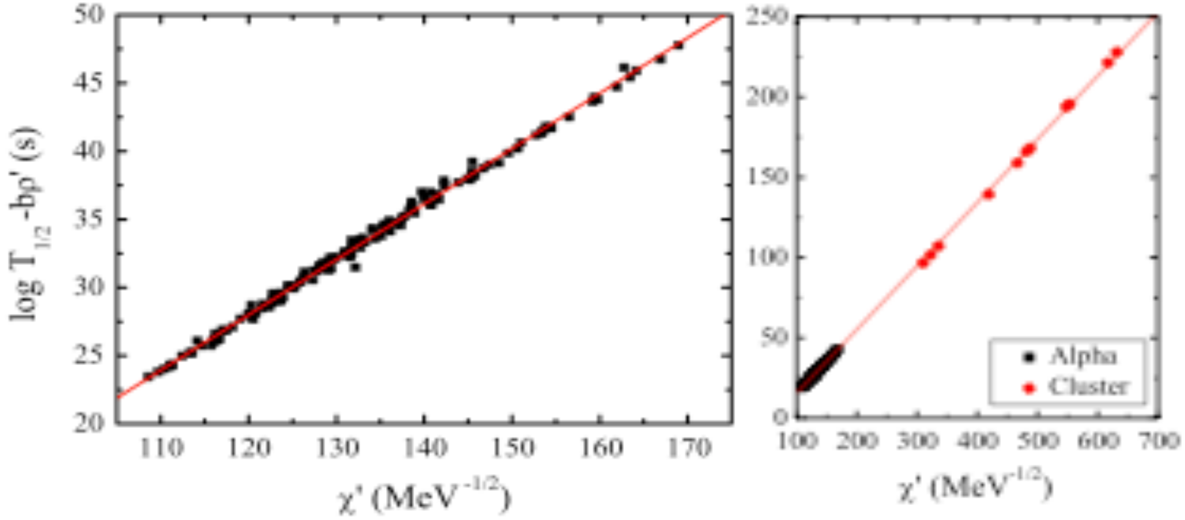
R=1.2($A_d^{1/3}$+$A_c^{1/3}$)



CQ et al, Phys.Rev.C80,044326 (2009); 81,064319 (2010).

Generalization of the Geiger-Nuttall law

Universal decay law of alpha and cluster decays



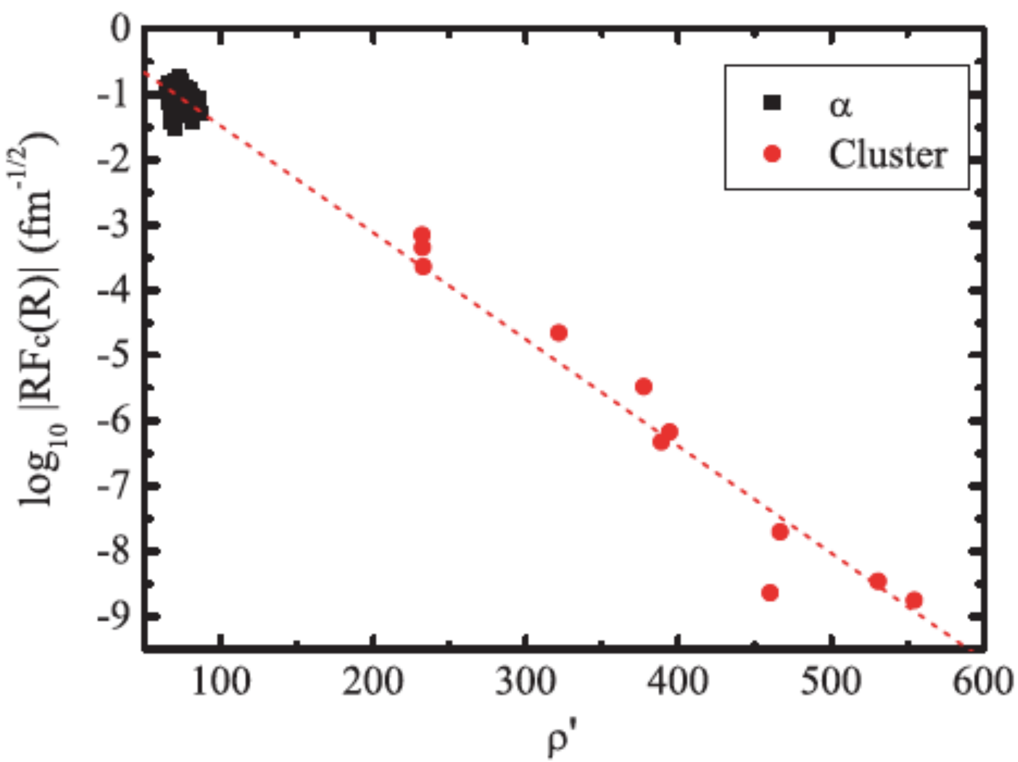
$$\log T_{1/2} = a\chi' + b\rho' + c$$

$$= 2aZ/\sqrt{A_{\alpha d}}Q_{\alpha}^{-1/2} + b\sqrt{2A_{\alpha d}Z(A_d^{1/3} + 4^{1/3})} + c,$$

$$A(Z) = 2aZ/\sqrt{A_{\alpha d}} \text{ and } B(Z) = b\rho' + c.$$

CQ, F.R.Xu,R.J.Liotta,R.Wyss, PRL103, 072501 (2009), PRC80,044326 (2009)

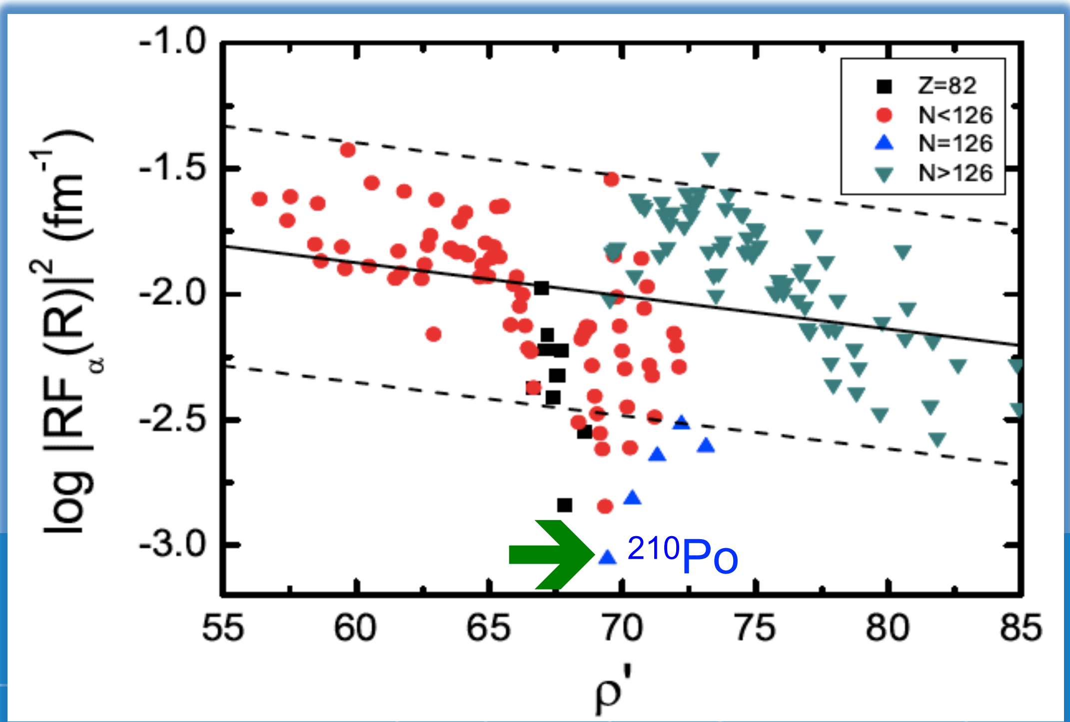
- On the logarithm scale the differences in the formation probabilities are usually small fluctuations along the straight lines predicted by the Geiger-Nuttall law;
- The smooth trend is a consequence of the smooth transition in the nuclear structure that is often found when going from a nucleus to its neighboring nuclei ->BCS.
- Proton and neutron **pairing correlations** crucial in reproducing the alpha clustering in heavy nuclei



➢a division occurs between decays corresponding to *N <126 and N >126;*

➢Sudden change *at N = 126;*

➢The case that shows the most significant hindrance corresponds to the α decay of the nucleus ^{210}Po, one order of magnitude smaller than that of ^{212}Po.

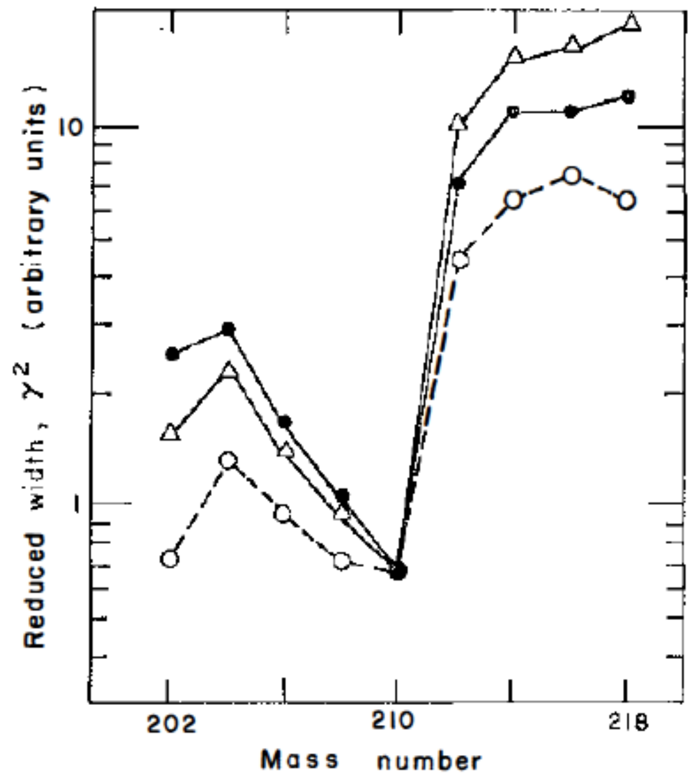


Earlier works

H.J. Mang, Phys. Rev. **119**, 1069 (1960).

neutron wave functions, however, belong to different single-particle states. Since the single-particle states that form the closed shell are more strongly bound than the ones outside the closed shell, it is to be expected that the wave functions corresponding to the latter are larger at the edge of the nucleus. Therefore, the reduced width of Po^{212} should be larger than the reduced width of Po^{210}. That this explanation is correct shows in a calcula-

Reduced widths [similar to F(R)] of Po isotopes as a function of A



- *Calculations underestimated the formation probability by several orders of magnitude due to the limited model space employed.*
- *$S_n(^{206}Po)>S_n(^{208}Po)>S_n(^{210}Po)$.*

Theoretical explanation

^{210}Po vs ^{212}Po (The later is the textbook example of alpha emitter)

$$|^{212}\mathrm{Po}(\alpha_4)\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\alpha_4)|^{210}\mathrm{Pb}(\alpha_2)\otimes^{210}\mathrm{Po}(\beta_2)\rangle$$

If we neglect the proton-neutron interaction

$$|^{212}\mathrm{Po}(\alpha,\mathrm{g.s.})\rangle = |^{210}\mathrm{Pb}(2\nu,\mathrm{g.s.})\otimes|^{210}\mathrm{Po}(2\pi,\mathrm{g.s.})\rangle,$$

$$|^{210}\mathrm{Po}(\alpha,\mathrm{g.s.})\rangle = |^{206}\mathrm{Pb}(2\nu^{-1},\mathrm{g.s.})\otimes|^{210}\mathrm{Po}(2\pi,\mathrm{g.s.})\rangle.$$

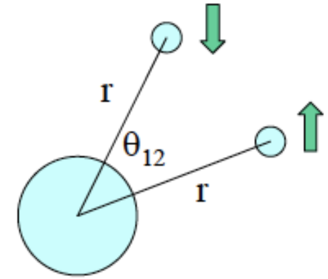
$$\mathscr{F}_\alpha(R;^{212}\mathrm{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\mathbf{r}_1\mathbf{r}_2;^{210}\mathrm{Pb(gs)})\Psi(\mathbf{r}_3\mathbf{r}_4;^{210}\mathrm{Po(gs)}),$$

$$\mathscr{F}_\alpha(R;^{210}\mathrm{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\mathbf{r}_1\mathbf{r}_2;^{206}\mathrm{Pb(gs)})\Psi(\mathbf{r}_3\mathbf{r}_4;^{210}\mathrm{Po(gs)}).$$

Two-body clustering

CQ et al, Phys.Rev.C 81,064319 (2010).

Two-body clustering



Configuration mixing from higher lying orbits is important for clustering at the surface

$$\Psi_2(\mathbf{r_1},\mathbf{r_2}) = (\chi_1\chi_2)_0\Phi_2(r_1,r_2,\theta_{12}) = (\chi_1\chi_2)_0\frac{1}{4\pi}\sum_p\sqrt{\frac{2j_p+1}{2}}X_p\phi_p(r_1)\phi_p(r_2)P_{l_p}(\cos\theta_{12}),$$

r_1=9fm

^{210}Pb

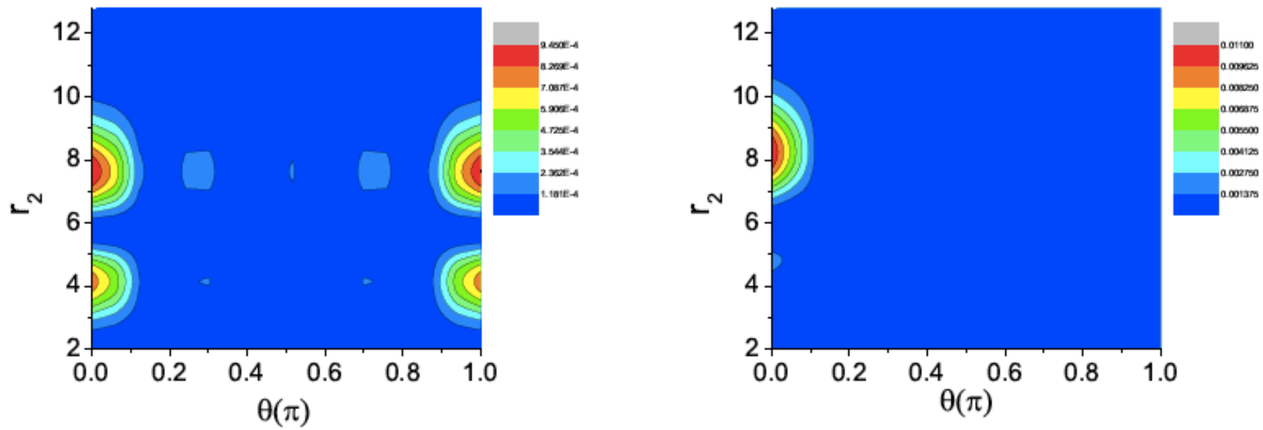
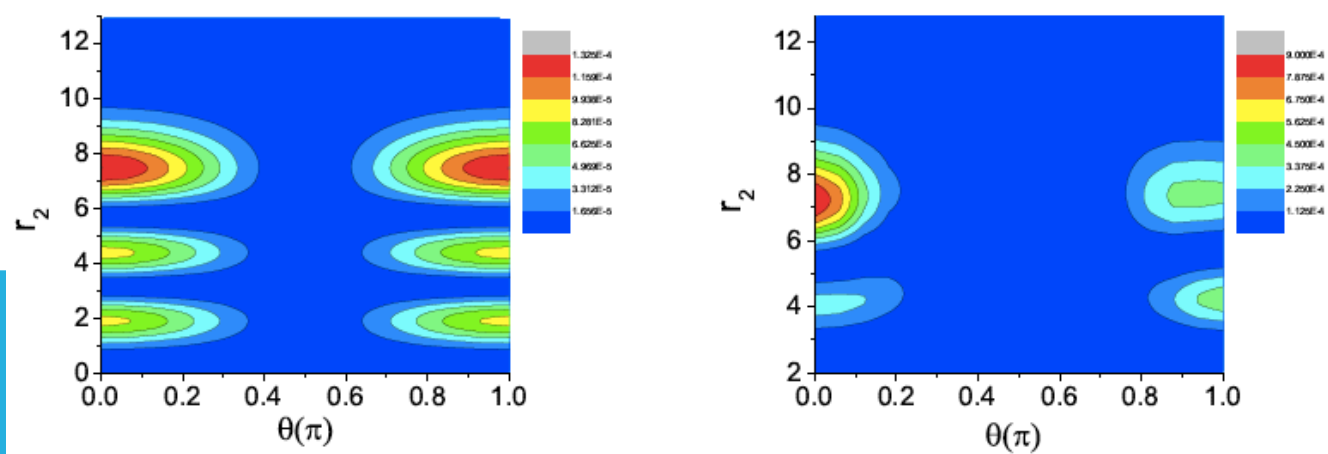


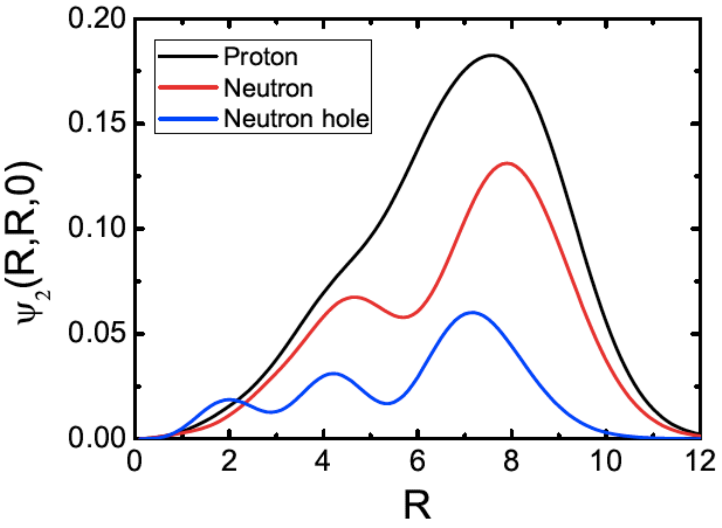
FIG. 10: (color online). The square of the two-neutron wave function $|\Psi_{2\nu}(r_1,r_2,\theta)|^2$ with $r_1 = 9$ fm as a function of r_2 and θ. Left: the leading configuration; Right: 4 major shells

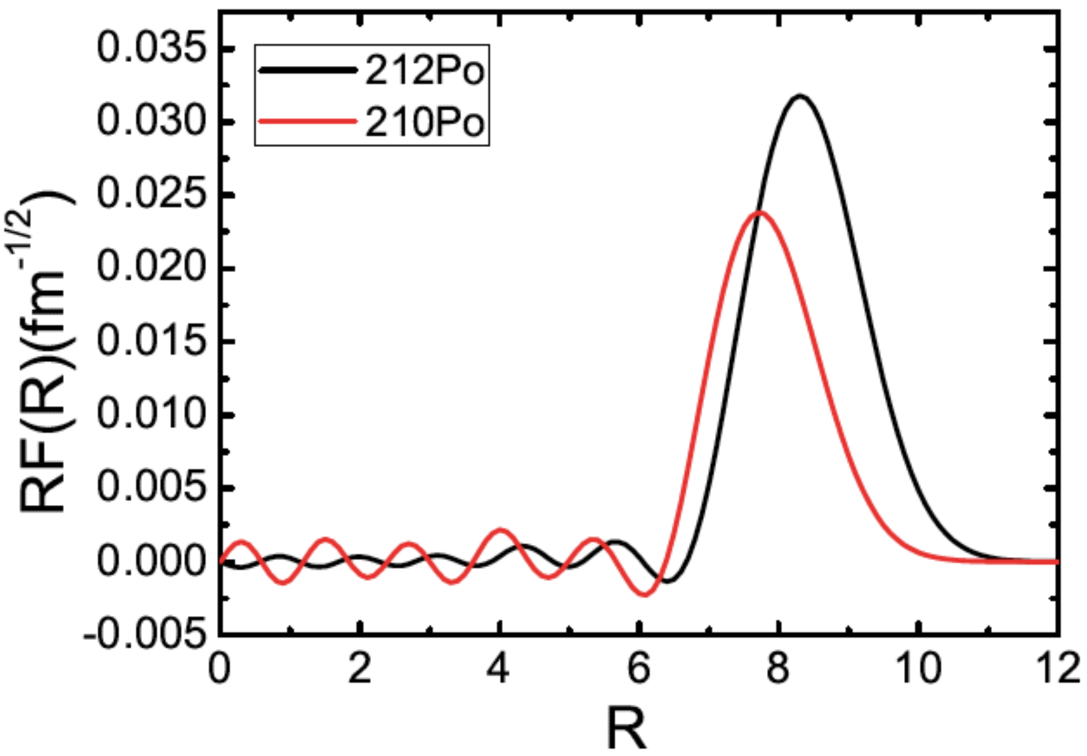
^{206}Pb



Alpha formation amplitude

$$F_\alpha(R) = \sqrt{\frac{1}{4\pi}} \int dr_\alpha \phi_\alpha(r_\alpha) \Psi_{2\pi}(\mathbf{r_1}, \mathbf{r_2}) \Psi_{2\nu}(\mathbf{r_3}, \mathbf{r_4}),$$

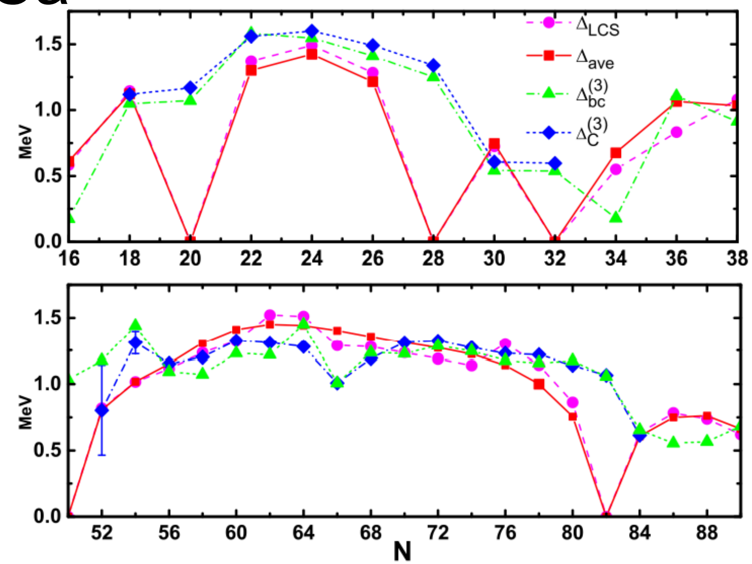




- **Alpha particle is formed on the nuclear surface;**
- **The clustering induced by the pairing mode is inhibited if the configuration space does not allow a proper manifestation of the pairing collectivity.**

Pairing gap

Ca



Sn

Two-body wave function from HFB

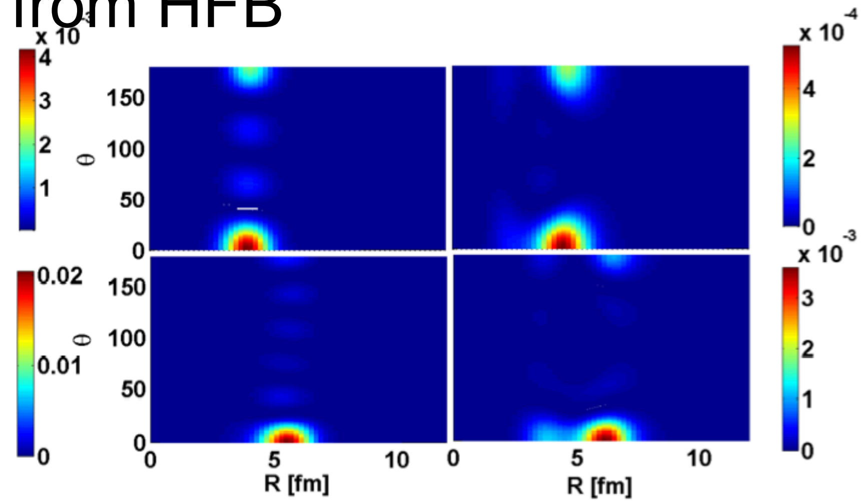
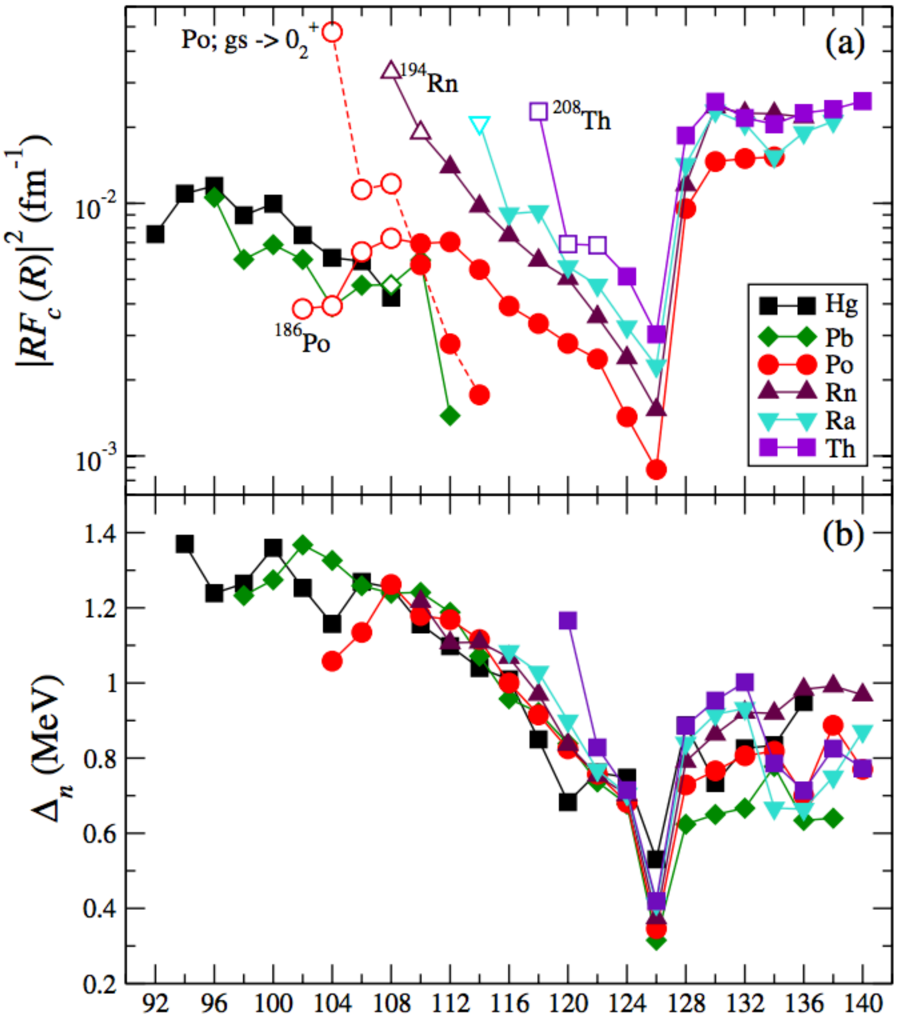


Figure 5: Upper: The two-neutron correlation plots for ^{46}Ca (left) and ^{54}Ca (right). Lower: Same as upper but for ^{128}Sn (left, 4 holes) and ^{136}Sn (right, 4 particles). Notice that the scale is different.

S. Changizi, CQ, Phys. Rev. C 91, 024305 (2015); Nucl. Phys. A 940, 210 (2015)

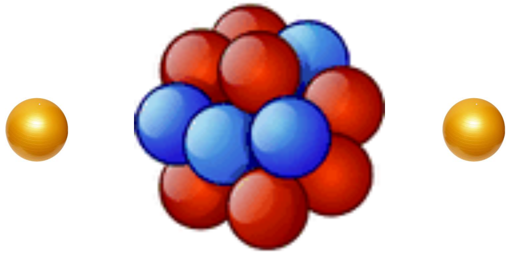
Pairing gap and the alpha formation

Larger pairing energy => Enhanced two-particle clustering at the nuclear surface



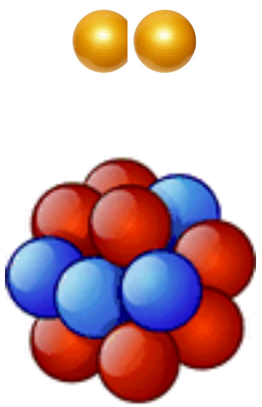
$$\Delta_n(Z,N) = \frac{1}{2}[B(Z,N) + B(Z,N-2) - 2B(Z,N-1)].$$

A.N. Andreyev CQ et al., PhysRevLett.110.242502 (2013).



No Pairing

$$\Delta = G\sum_i u_i v_i,$$



’Strong’ pairing

Exact diagonalization for the pairing Hamiltonian with applications to alpha and di-neutron clustering

- Shell model calculations restricted to the v=0 subspace
- There are as many independent solutions as states in the v=0 space.
- **Valid for any forms of pairing.**

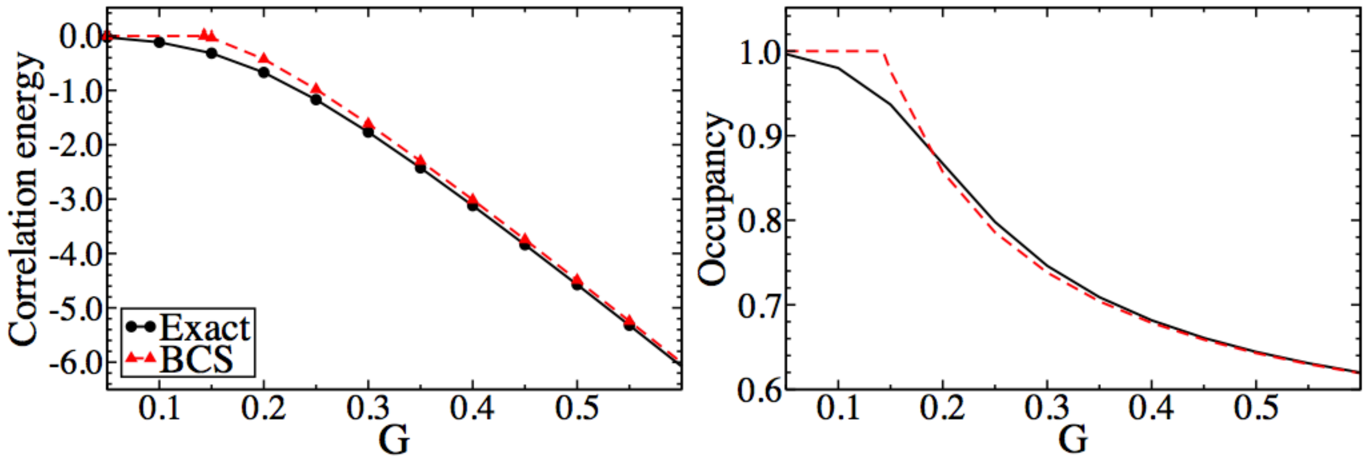
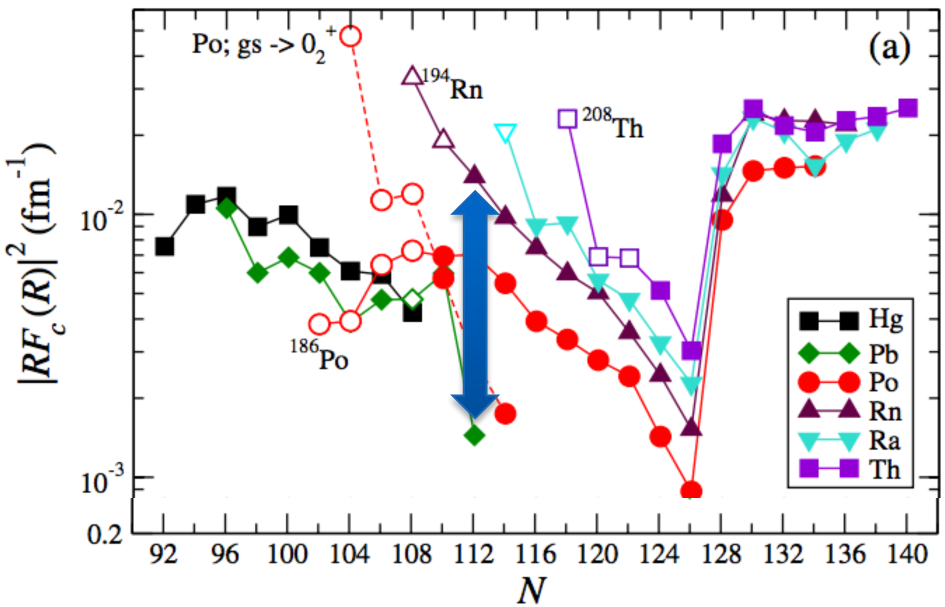
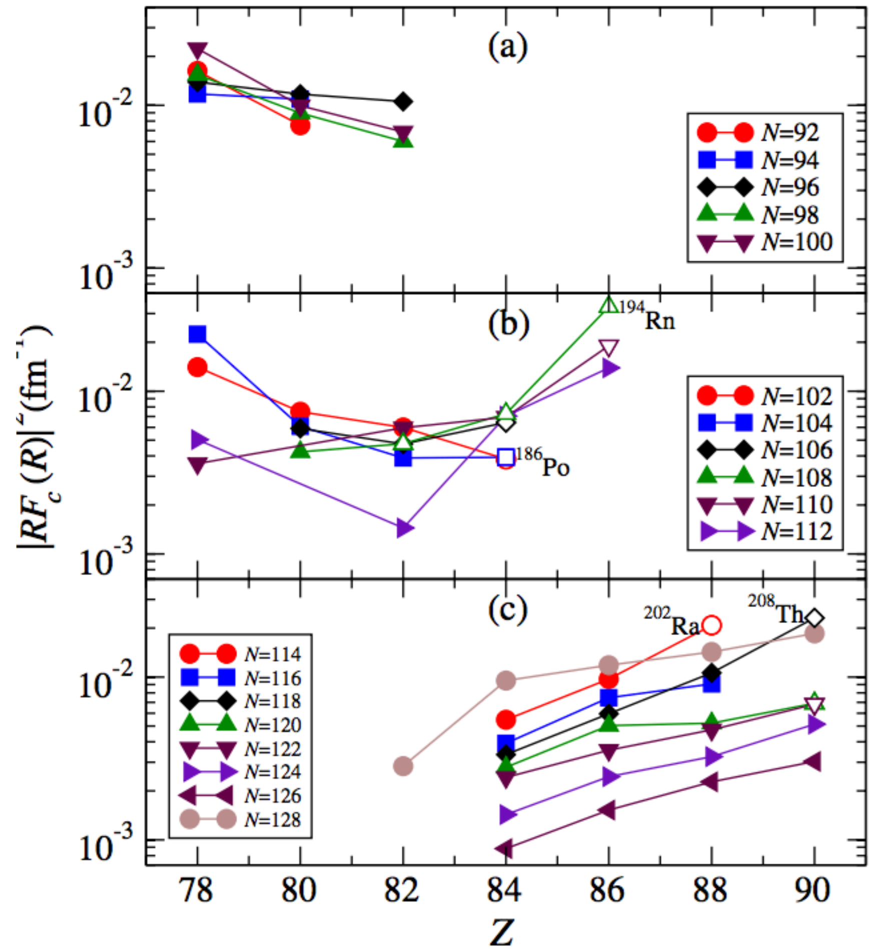


FIG. 1. Left: Comparison between the correlation energies from the BCS model and exact solution for a half-occupied system with $N = 4$ pairs and two $\Omega = 4$ orbitals separated by one unit. Right: The corresponding occupancy of the lower level from the two calculations.

CQ, to be published

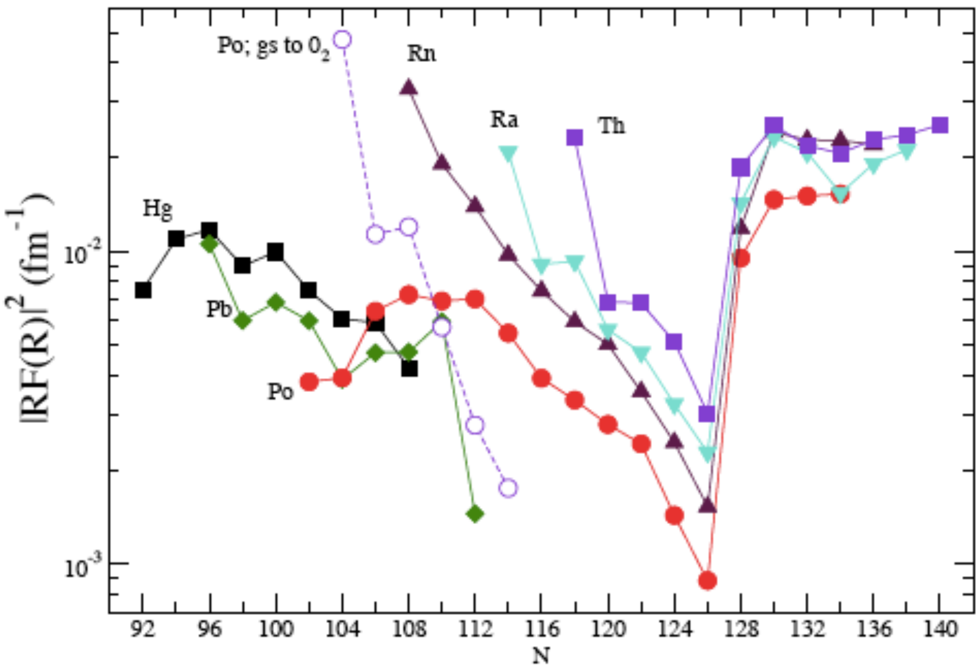
Alpha decay of neutron-deficient nuclei and Z=82 shell closure





A.N. Andreyev CQ et al., PhysRevLett.110.242502 (2013).

Decay to excited states



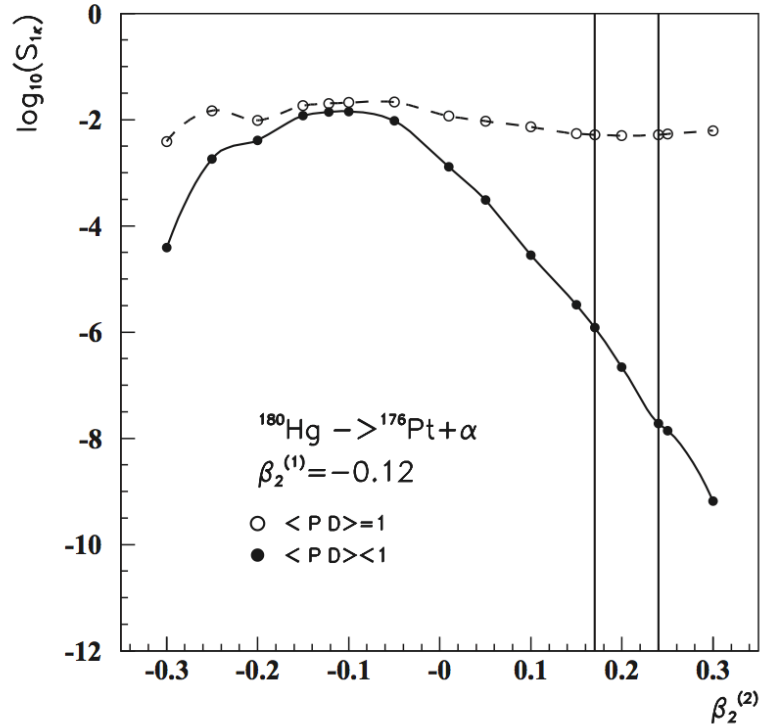


FIG. 2. Spectroscopic factor for the transition ^{180}Hg(gs) → ^{176}Pt(0_2^+) + α between BCS states with $k = 1$ in Eq. (4). The parent (initial) state corresponds to an oblate deformation with $\beta_2^{(1)}B = -0.12$. The daughter (final) state carries a deformation $\beta_2^{(2)}$ as given in the abscissa. The dashed curve was obtained by considering the overlap between the parent and daughter BCS states as unity (i.e., $\langle P|D\rangle = 1$) while for the full curve that overlap is the one provided by our calculation. Vertical lines denote the deformations provided by the PES calculation for the minima corresponding to the 0_1^+ ($k' = 1$) and 0_2^+ ($k' = 2$) states.

DS Delion CQ et al PRC 90, 061303(R) (2014)

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Shape coexistence in atomic nuclei

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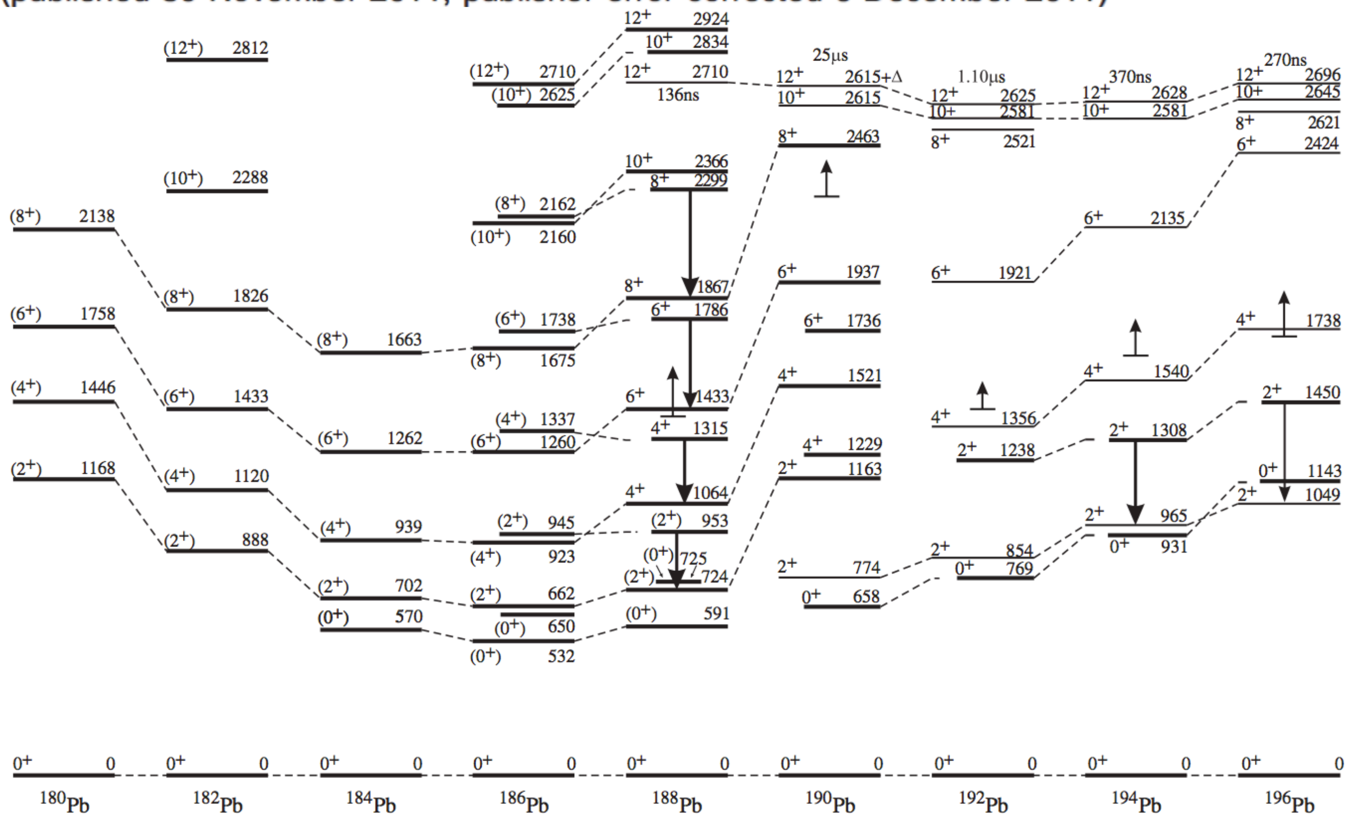
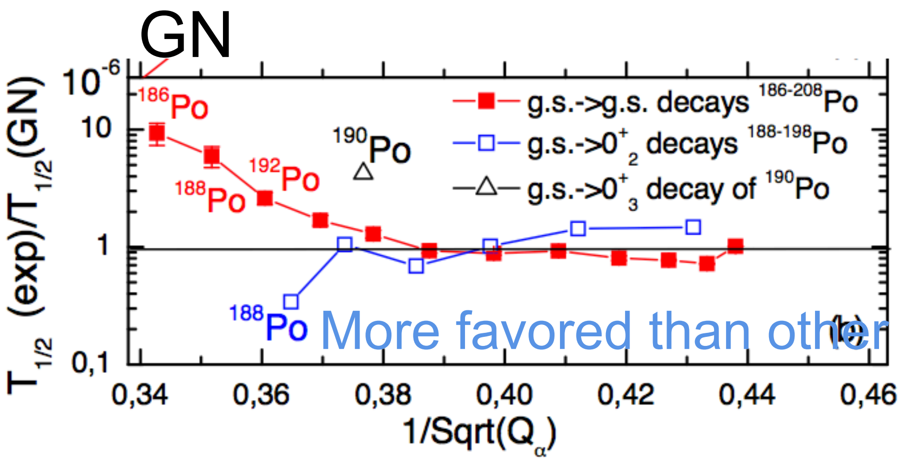
(published 30 November 2011; publisher error corrected 6 December 2011)



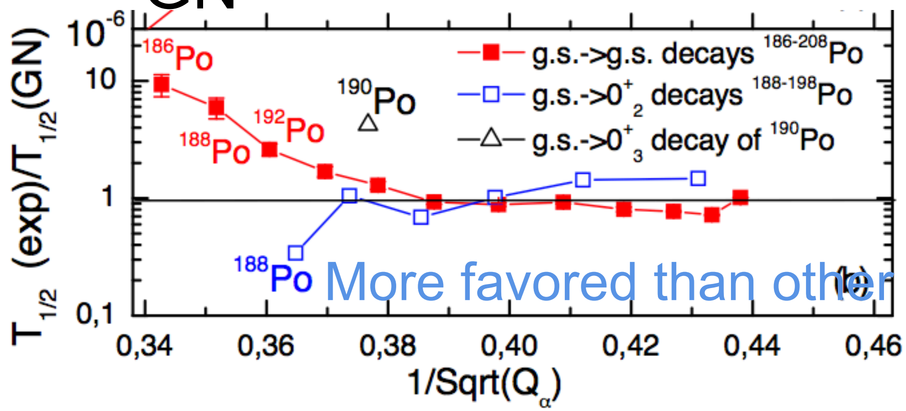
Large deviation from the GN law in neutron-deficient Po isotopes



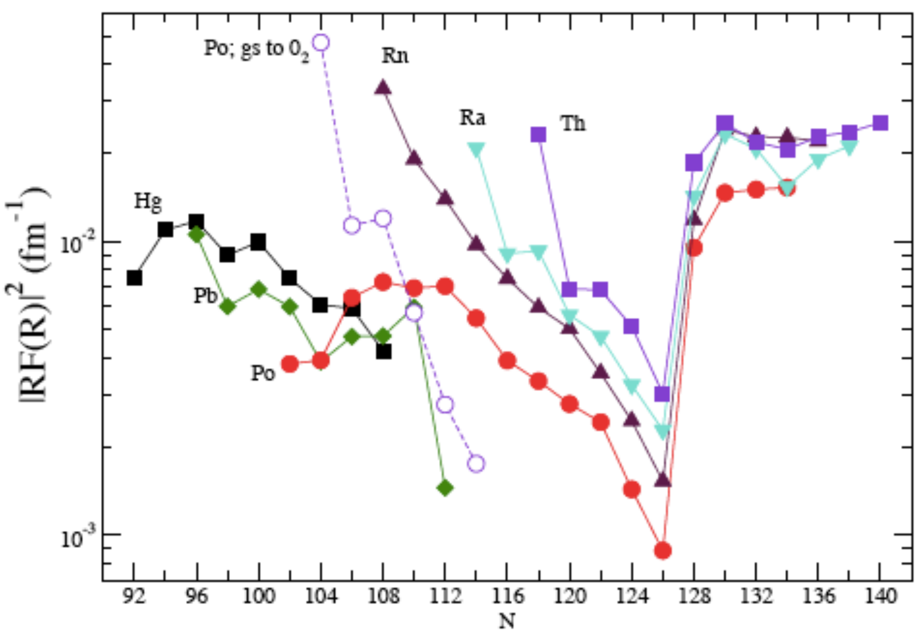
More favored than other Po isotopes?

Large deviation from the GN law in neutron-deficient Po isotopes

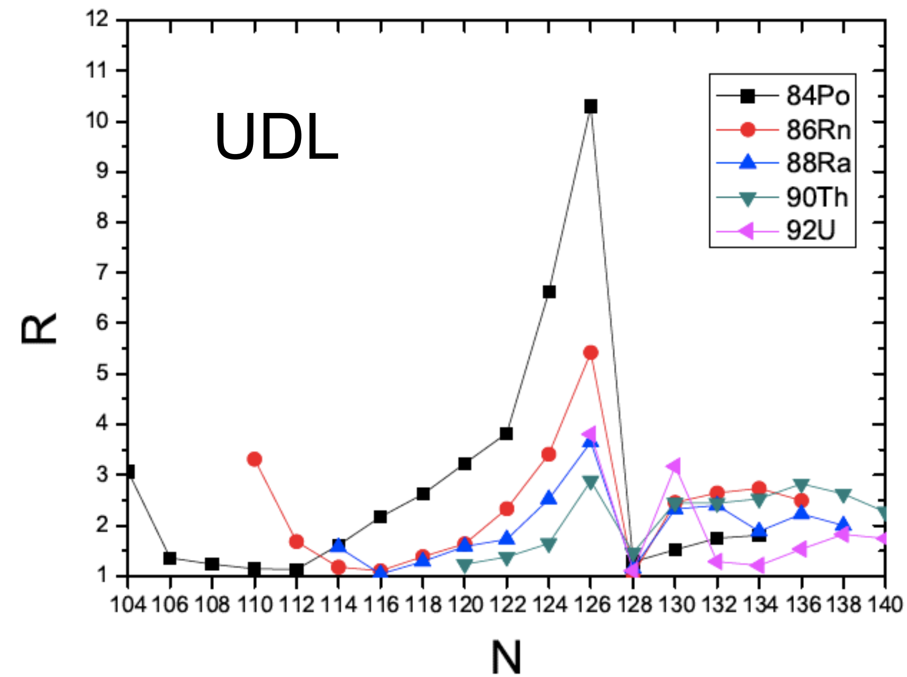
GN



More favored than other Po isotopes?

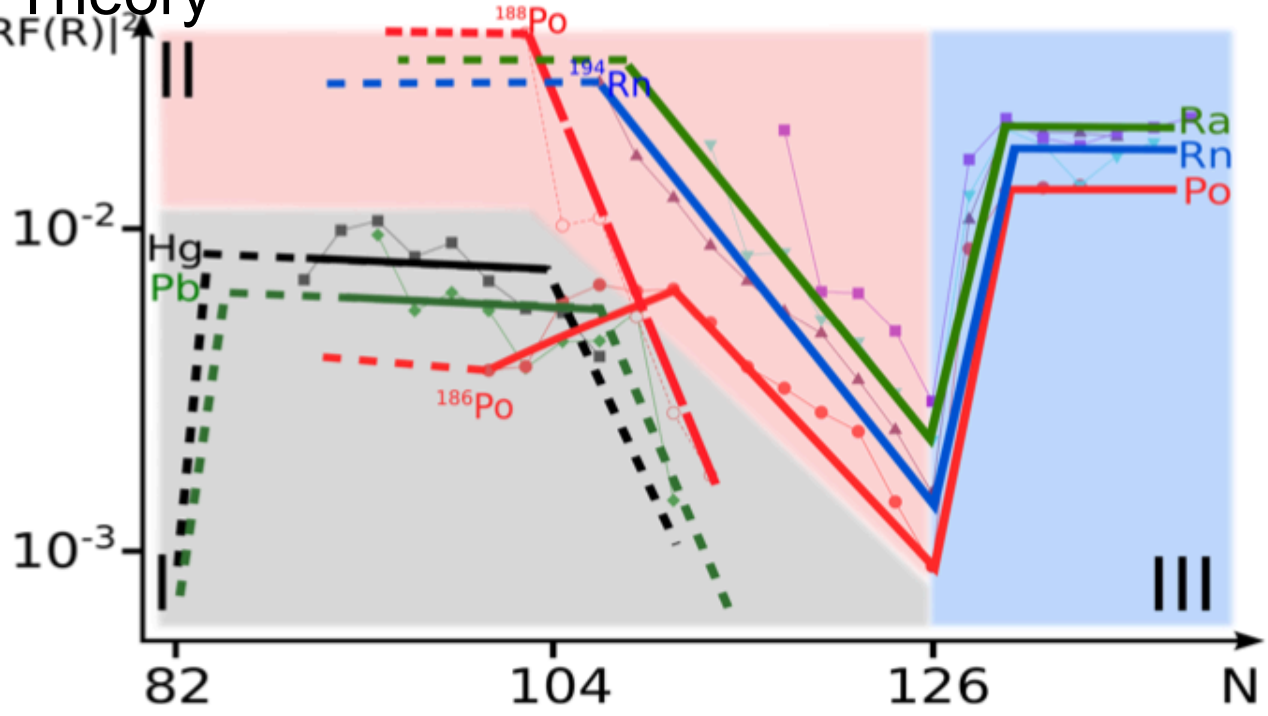


UDL



Where the alpha formation saturate?

Solid: Observed
Dashed: Theory



C. Qi, A.N. Andreyev, M. Huyse, R.J. Liotta, P. Van Duppen, R. Wyss Phys. Lett. B 734, 203 (2014)

‘Superallowed’ alpha decay around N=Z nuclei

PRL 97, 082501 (2006) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2006

Discovery of ^{109}Xe and ^{105}Te: Superallowed α Decay near Doubly Magic ^{100}Sn

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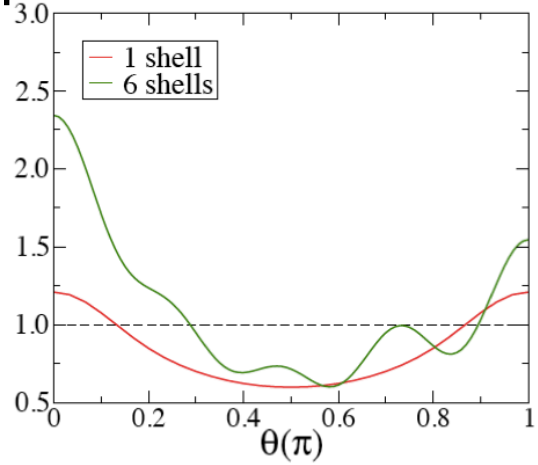
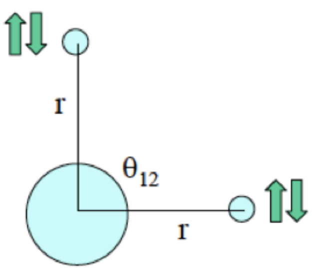
Shell model calculations on the alpha formation amplitude in N=Z nuclei.

The four-body (alpha) wave function can be written as

$$|\gamma_4\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\gamma_4)|\alpha_2 \otimes \beta_2\rangle,$$

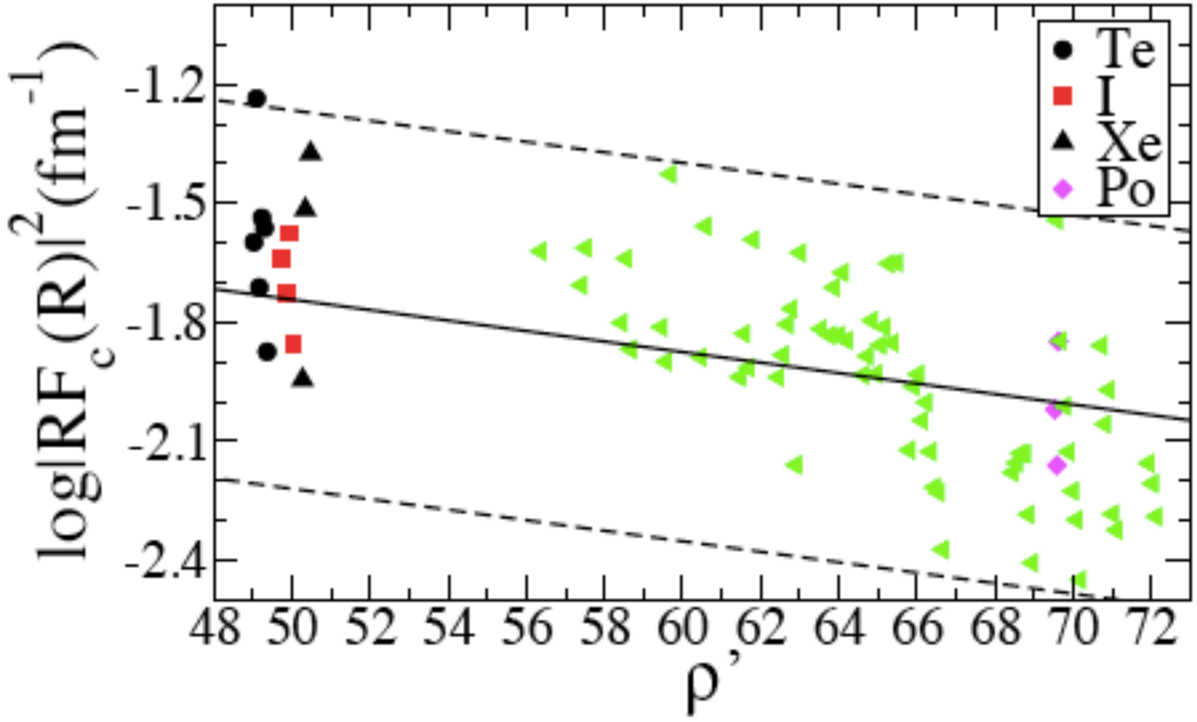
where α_2 and β_2 denote proton and neutron wave functions, respectively.





Relative angular distribution of four-particle wave function with (solid lines) and without (dashed line) neutron-proton interactions.

Alpha formation properties in N~Z nuclei

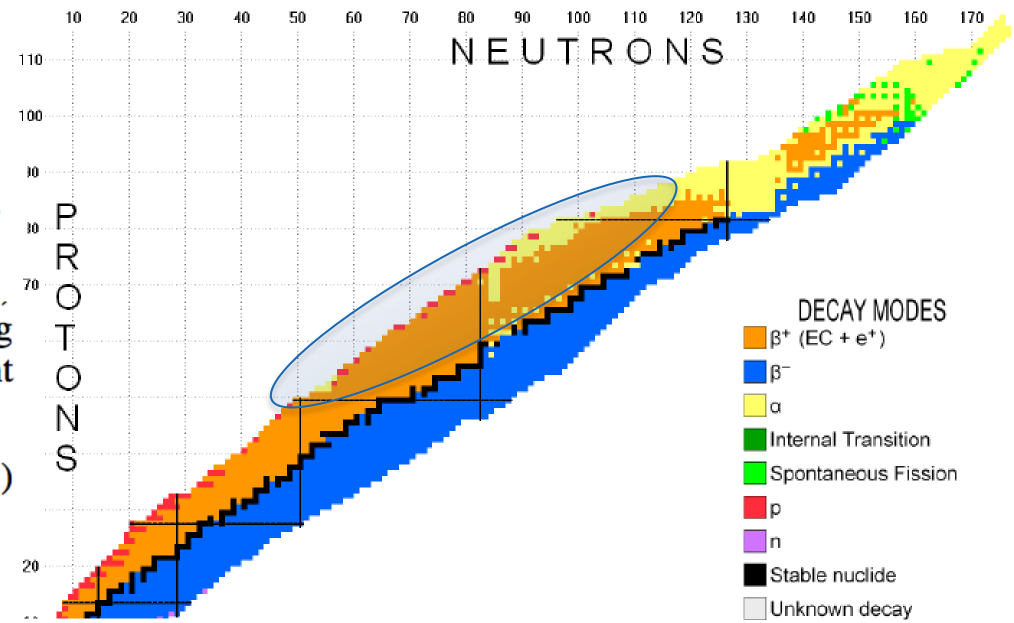


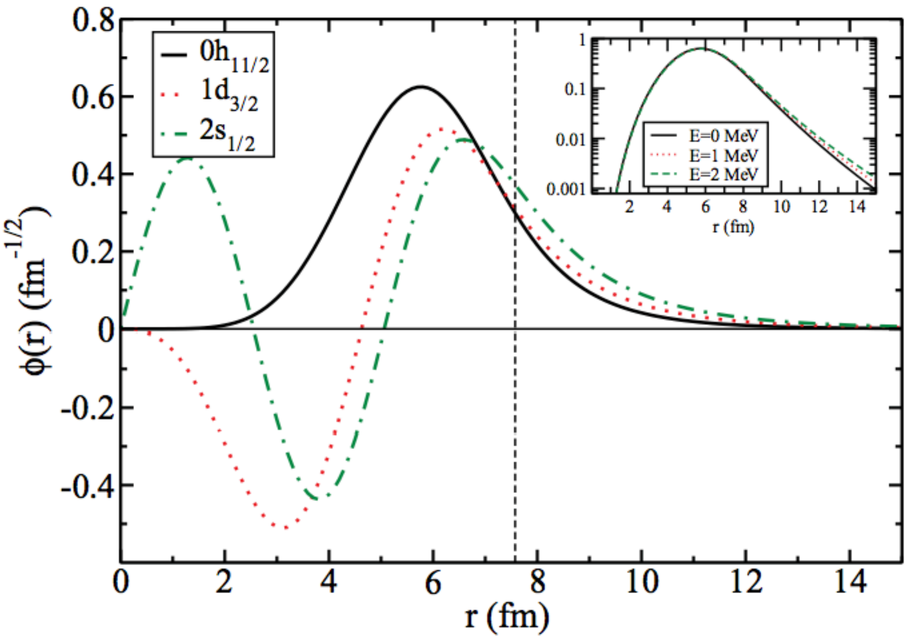
Proton decay

$$\mathcal{F}_l(R) = \int d\mathbf{R}\, d\xi_d [\Psi(\xi_d)\xi_p Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_p, \mathbf{R}),$$

that $\mathcal{F}_l(R)$ would indeed be the wave function of the outgoing particle $\psi_p(R)$ if the mother nucleus would behave at the point R as

$$\Psi_m(\xi_d, \xi_p, \mathbf{R}) = [\Psi(\xi_d)\xi_p \psi_p(R) Y_l(\mathbf{R})]_{J_m M_m}. \qquad (3)$$





C. Qi, D.S. Delion, R.J. Liotta, and R. Wyss, 85, 011303(R) (2012)

Formation vs 'u'

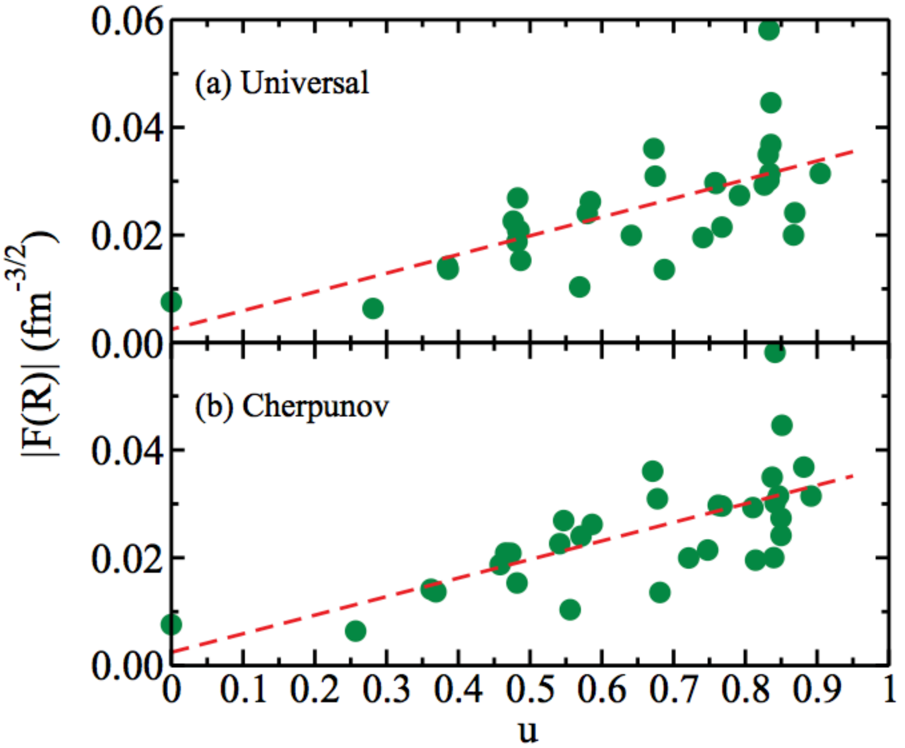


FIG. 4. (Color online) The formation amplitudes $|F_l(R)|$ extracted from experimental data for proton decays of nuclei $N \geqslant 75$ ($Z > 67$) as a function of u calculated from BCS calculations using for the Woods-Saxon mean field the universal parameters [23] (upper) and the Cherpunov parameters [24] (lower).

Summary

- Microscopic studies of the alpha decay;
- An abrupt change in alpha formation amplitudes is noted around the N=126 shell closure;
- Effect of pairing collectivity;
- Influence of the neutron-proton correlation on alpha formation
- ***Alpha decay as a powerful probe for nuclear structure in neutron-deficient nuclei***

Thank you!