Theoretical Nuclear Physics

(SH2011, Second cycle, 6.0cr)

Comments and corrections are welcome!

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The course contains 12 sections

1- 4 Introduction

◇Basic Quantum Mechanics concepts

◇Basic nuclear physics concepts: Pairing, single-particle excitations, square well, Magnetic resonances

5-11 Nuclear shell model

◇Single-particle model and the spin-orbit interaction(5)

◇Nuclear deformation and the Nilsson model, rotation(6)

◇Second quantization and Hartree-Fock (7-8)

◇Two-particle system, LS and jj coupling(9)

◇Isospin and neutron-proton coupling scheme(10)

◇One-nucleon operators, gamma transition (10)

◇beta decay, 14C-dating β decay(11)

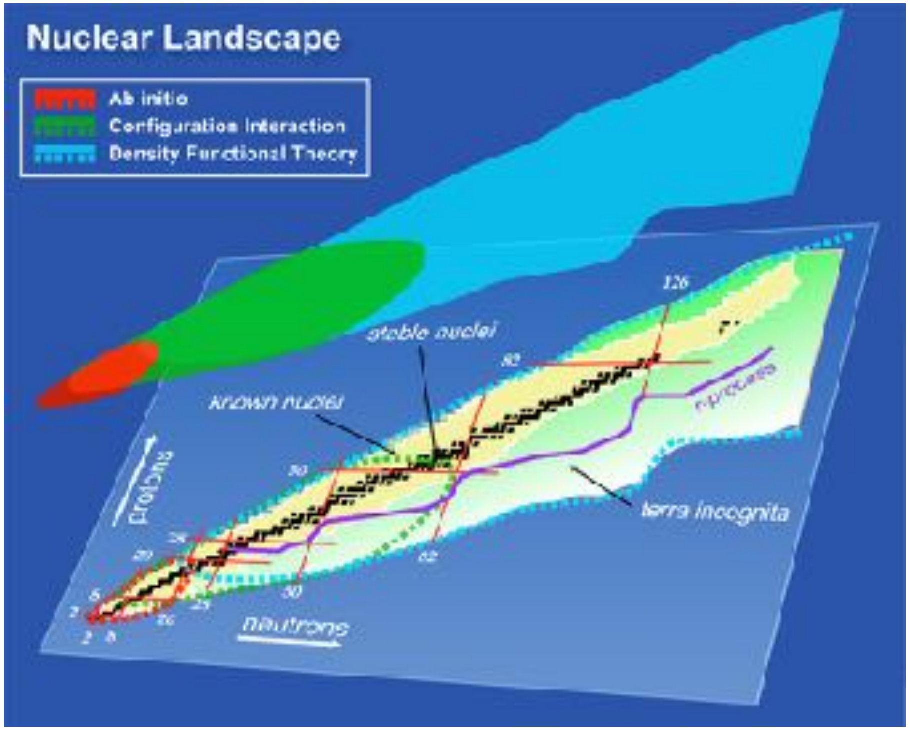
12 Summary and applications (12)

Why Focus on Microscopic Nuclear Theories?

- Much of the progress in the past 50 years has been based on empirical models (most with microscopic degrees of freedom) tuned to experimental data
  - Single-particle model
  - Nuclear coupling schemes (pairing, np coupling)
  - Hartree-Fock, RPA and TDA
- The physics of nuclei impacts the programs because nuclei are the source of the energy and they are important diagnostics
  - Fission
  - Decay
  - Nuclear reactions
  - Astrophysics

Why Focus on Microscopic Nuclear Theories?

- Modern theories
  - No-core shell model and Ab inito theories for light nuclei
  - Large scale shell model for medium nuclei
  - Modern energy density functional approaches (Hartree-Fock) for heavy nuclei



➢NN interaction (tensor, three-body…)
➢Novel coupling schemes
➢Novel decays

What we introduced in this course is the basis of all these practices.

Connections to computational science

1Teraflop=10^{12} flops
1peta=10^{15} flops
1exa=10^{18} flops

Jaguar at ORNL 2.3petaflops

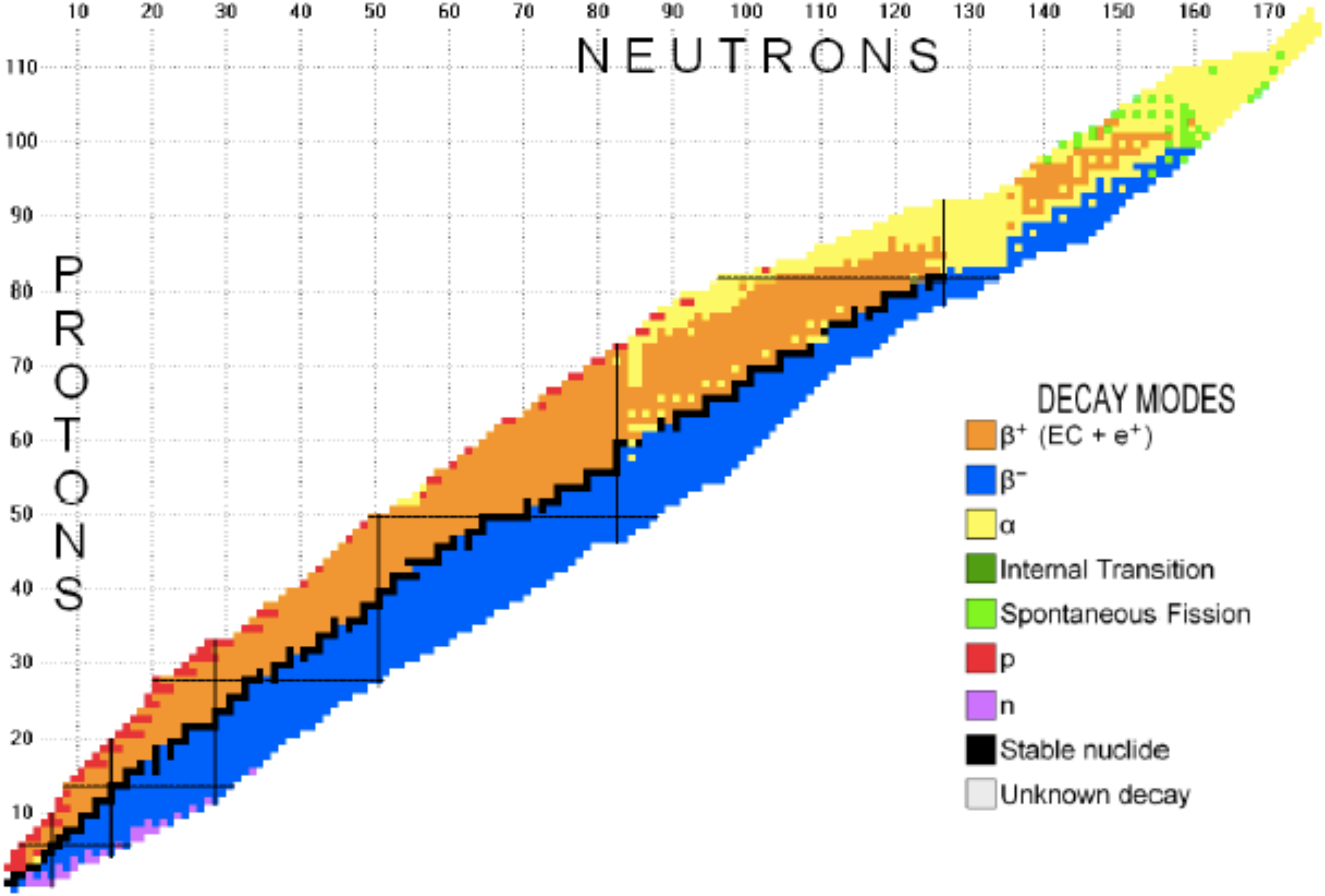
Beskow, PDC
1973 TeraFLOPS

Lindgren, 305 Teraflops

K computer, RIKEN, Japan, 10.51 Petaflops, power 1.27*10^4kW

- Theory of nuclear fission
- Stability of superheavy elements
- Nucleosysthesis (selected topics)

Audi et al., Nucl. Phys. A 729 (2003) 3-128

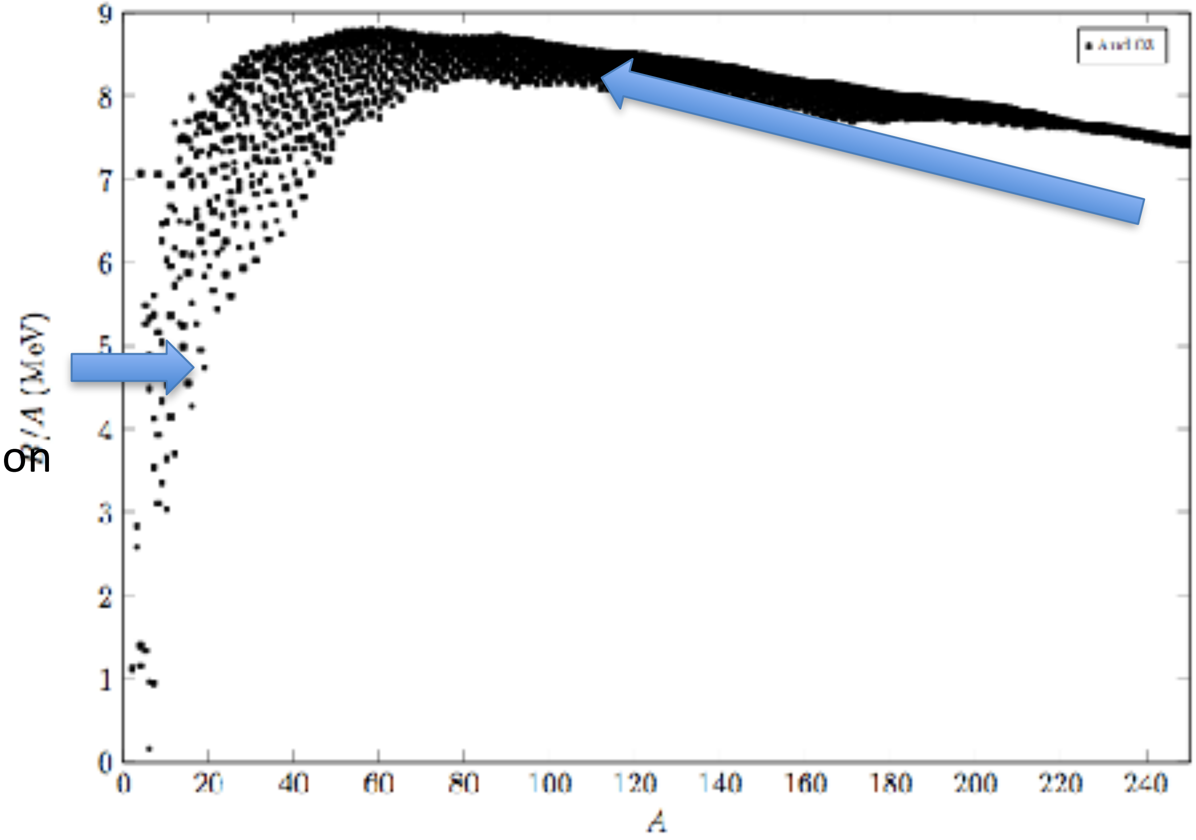


In the radioactive decay process

$$M \rightarrow D + C \tag{9.1}$$

where M is the mother nucleus, D the daughter and C the emitted cluster, energy conservation implies that the Q-value is

$$Q = B(D) + B(C) - B(M) \tag{9.2}$$



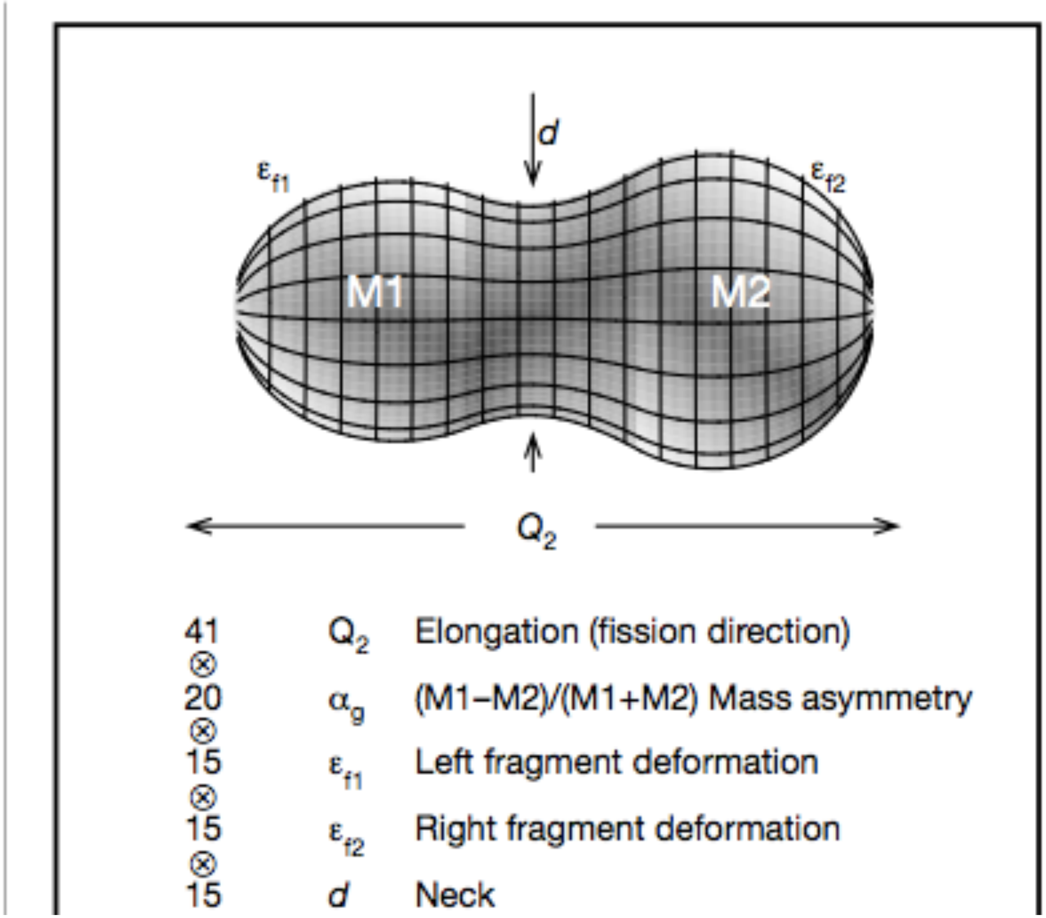
Fusion

Fission

igure 9.1: Experimental binding energies per nucleon (in MeV).

Theory of nuclear fission (liquid drop model)

the nucleus in an unstable state vibrates and changes form from spherical to a peanut-like shape. The Coulomb repulsion between the parts separates them, arriving to two well differentiated spheres which finally depart from each other, thus fissioning the mother nucleus.



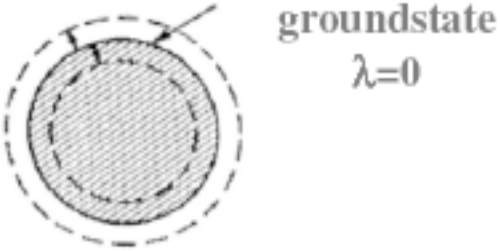
41 ⊗	Q_2	Elongation (fission direction)
20 ⊗	α_g	(M1–M2)/(M1+M2) Mass asymmetry
15 ⊗	ε_{f1}	Left fragment deformation
15 ⊗	ε_{f2}	Right fragment deformation
15	d	Neck

2.6M mesh points

Types of Multipole Deformatiions

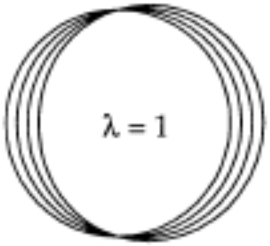
The monopole mode

$$Y_{00} = \frac{1}{4\pi} \Longrightarrow R = R(\theta, \varphi, t) = R_0\left(1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha^{*}_{\lambda\mu}(t) Y_{\lambda\mu}(\theta, \varphi)\right)$$



The associated excitation is the so-called breathing mode of the nucleus. A large amount of energy is needed for the compression of nuclear matter and this mode is far too high in energy.

The dipole mode

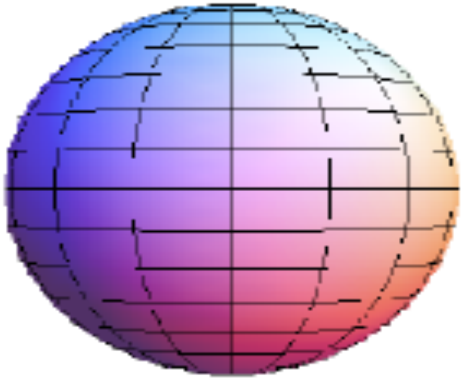


Dipole deformations, to lowest order, do not correspond to a deformation of the nucleus but rather to a shift of the center of mass, i.e. a translation of the nucleus, and should be ***disregarded*** for nuclear excitations since translational shifts are spurious.

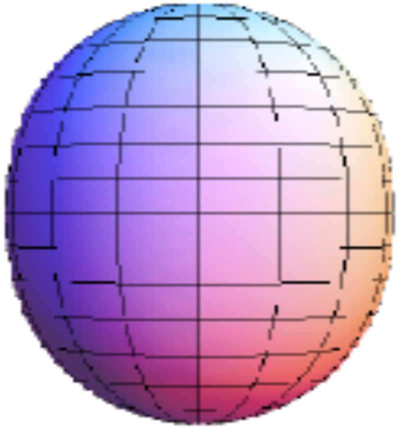
$$\vec{R}_{cm} = \frac{\int \vec{r}\rho(\vec{r})d^3r}{\int \rho(\vec{r})d^3r}$$

The quadrupole mode $\lambda = 2$

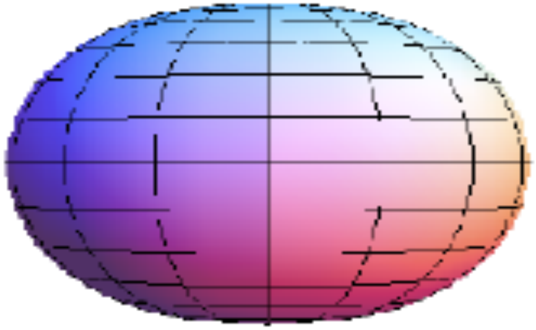
The most important nuclear shapes and collective low energy excitations of atomic nuclei.



Spherical



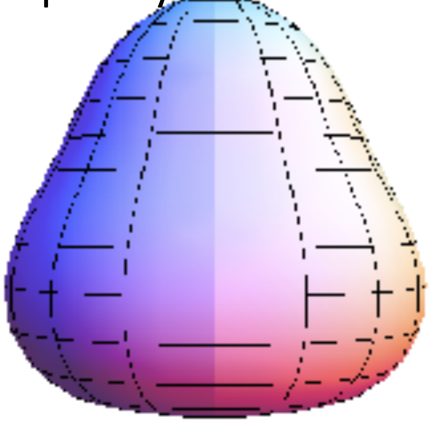
Prolate

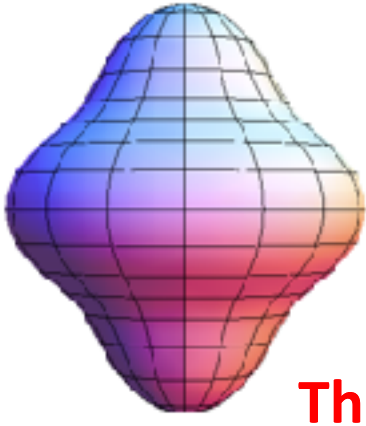


Obolate

The octupole mode $\lambda = 3$

The principal asymmetric modes of the nucleus associated with negative-parity bands.





The hexadecupole mode $\lambda = 4$

asymmetric & symmetric modes

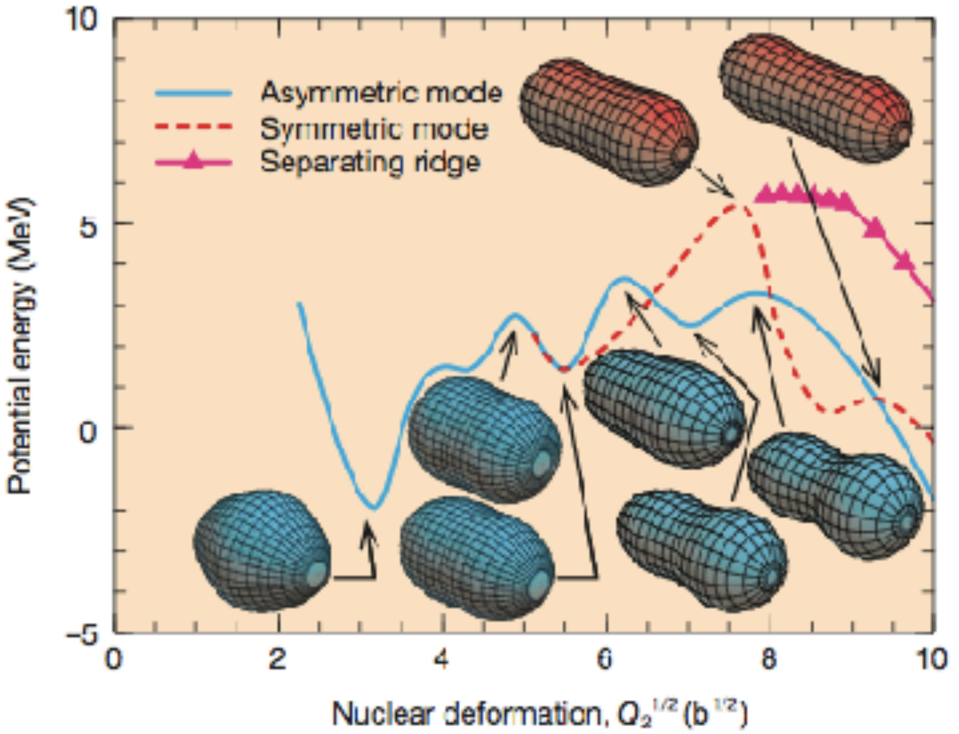


Figure 5 Calculated potential-energy valleys and ridges and corresponding nuclear shapes for ^{234}U. Two fission paths exist: one asymmetric path and one symmetric path. The symmetric path has a higher fission saddle point and the more elongated shapes in the valley beyond the saddle point indicate that total fragment kinetic energies in the symmetric mode are lower than in the asymmetric mode. The ridge separating the two valleys is certainly not high enough to permit two well-separated modes to evolve.

Exotic mode Mercury-180

^{180}Hg-> 90Zr + 90Zr ? no

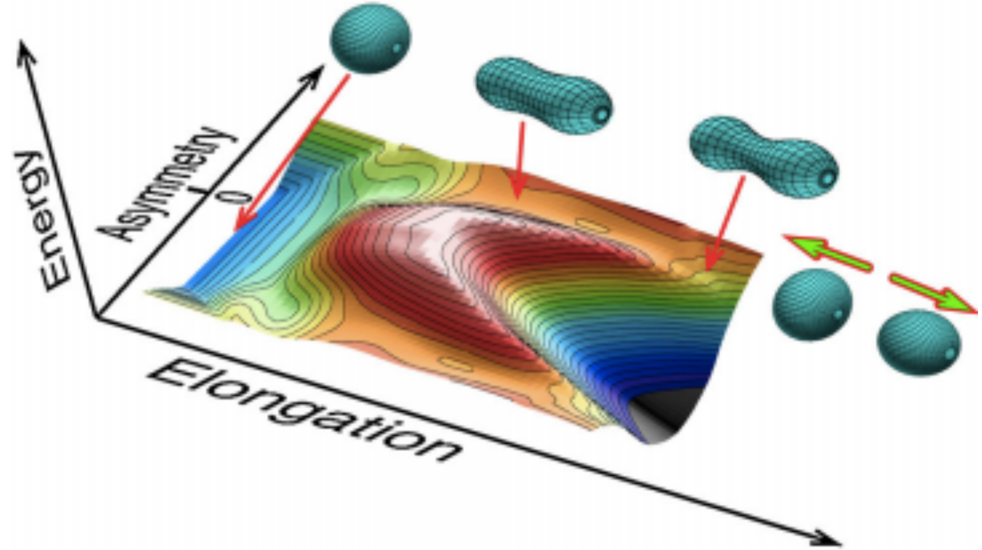
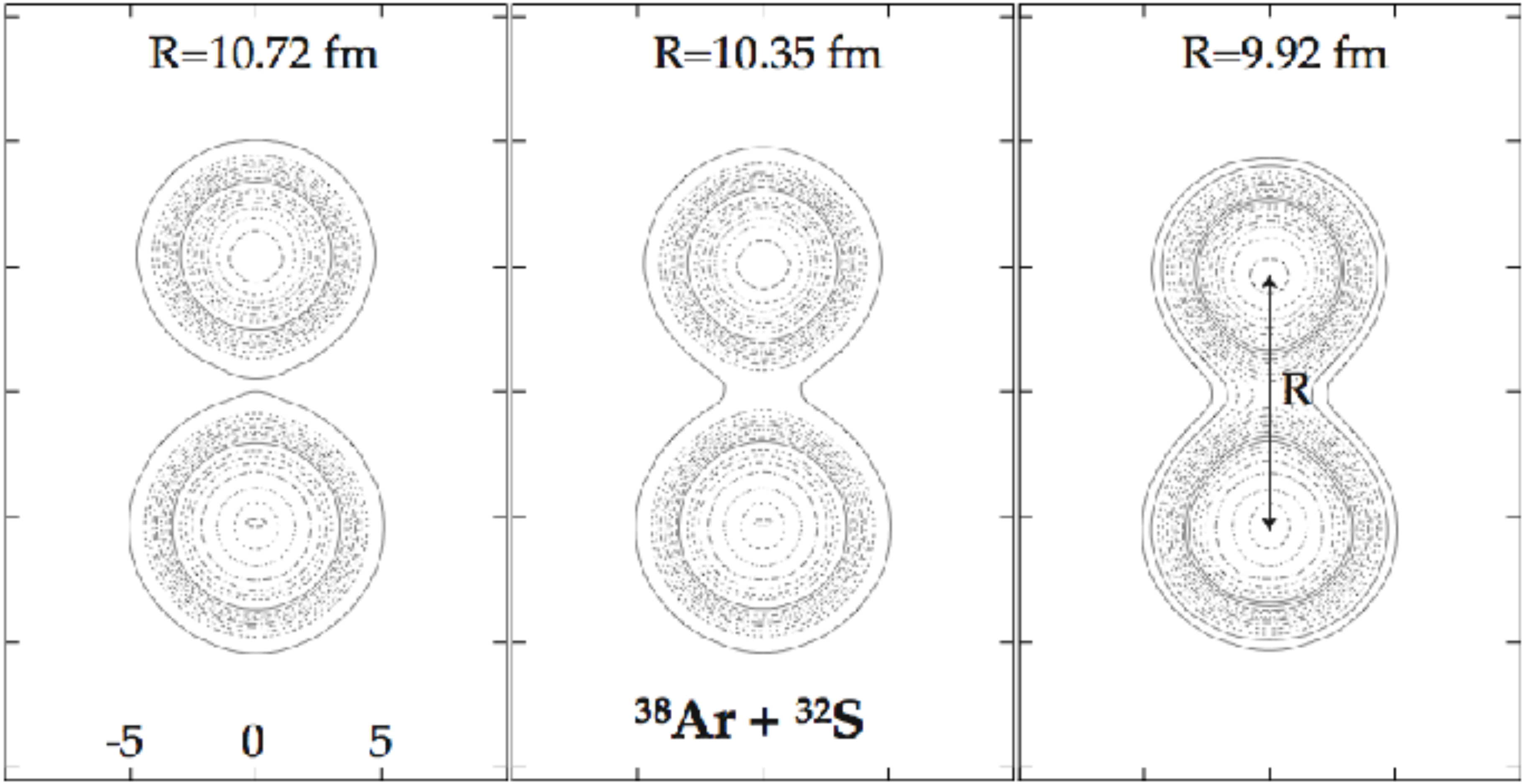


FIG. 5 (color online). A schematic representation of the potential-energy surface for ^{180}Hg in two dimensions (elongation and asymmetry) resulting from a five-dimensional analysis. The shapes shown, connected by arrows to their locations, are the ground state, the saddle point, and the point where the asymmetric valley disappears.

PRL 105, 252502 (2010)

Related problem: heavy-ion fusion

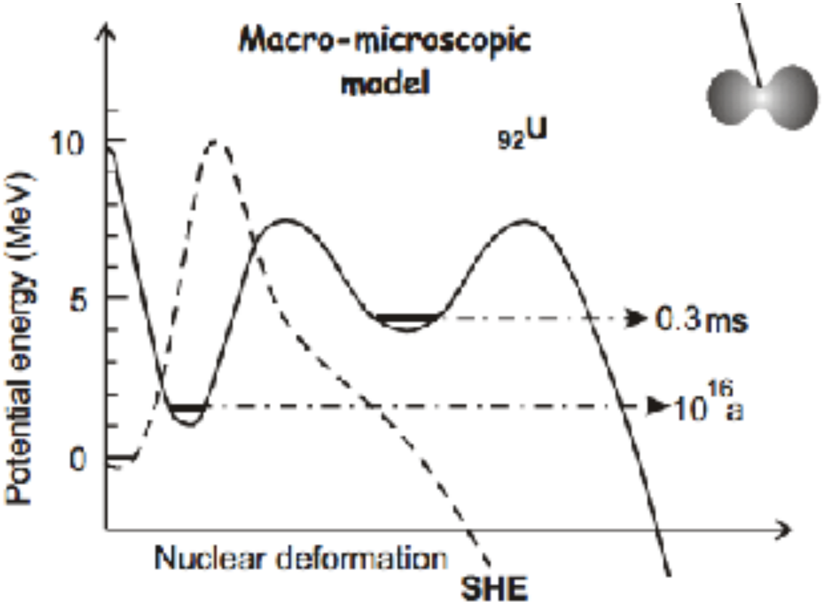
- nucleus-nucleus potentials
- adiabatic fusion barriers

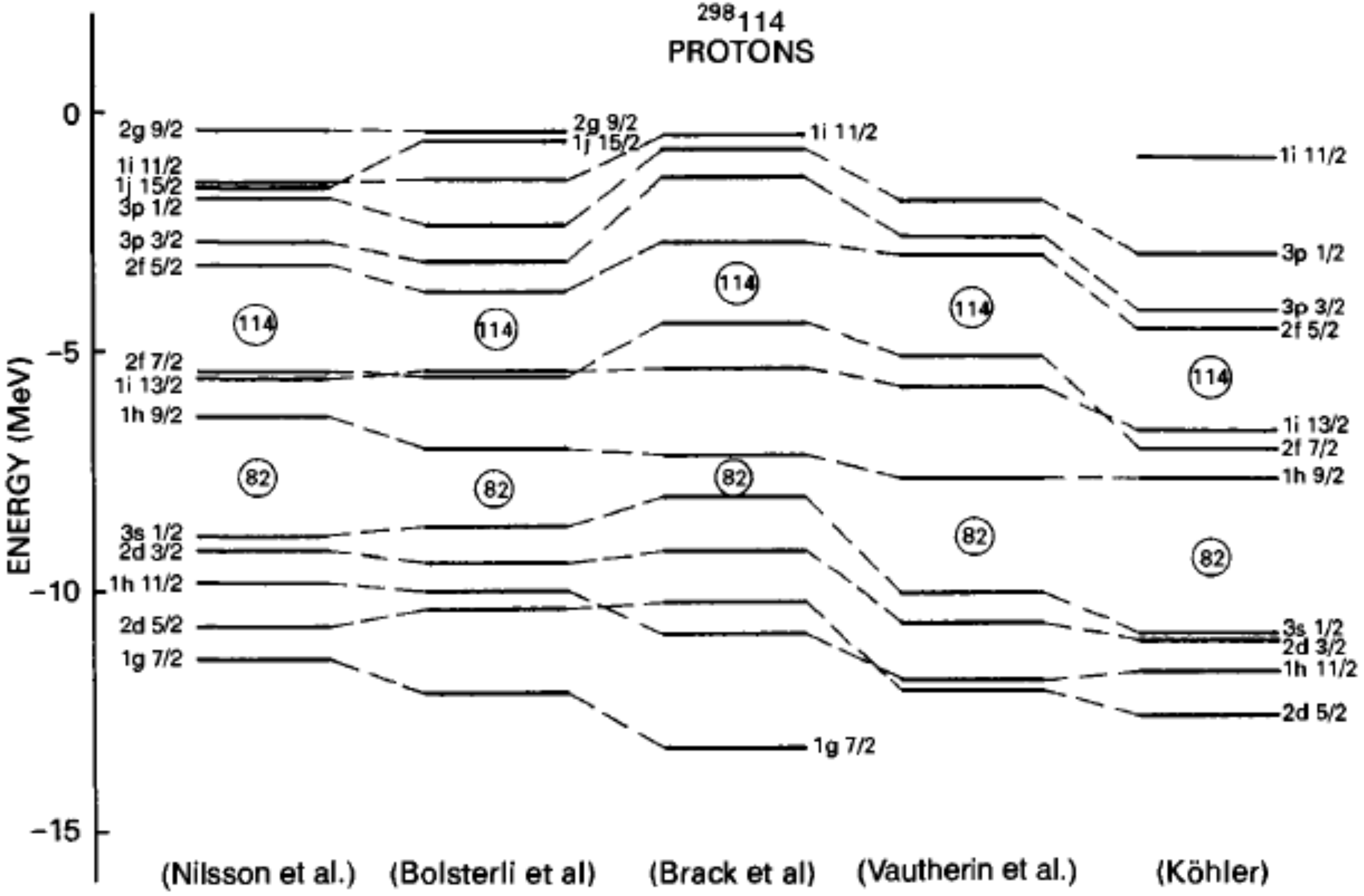


$^{38}\mathrm{Ar} + {}^{32}\mathrm{S}$

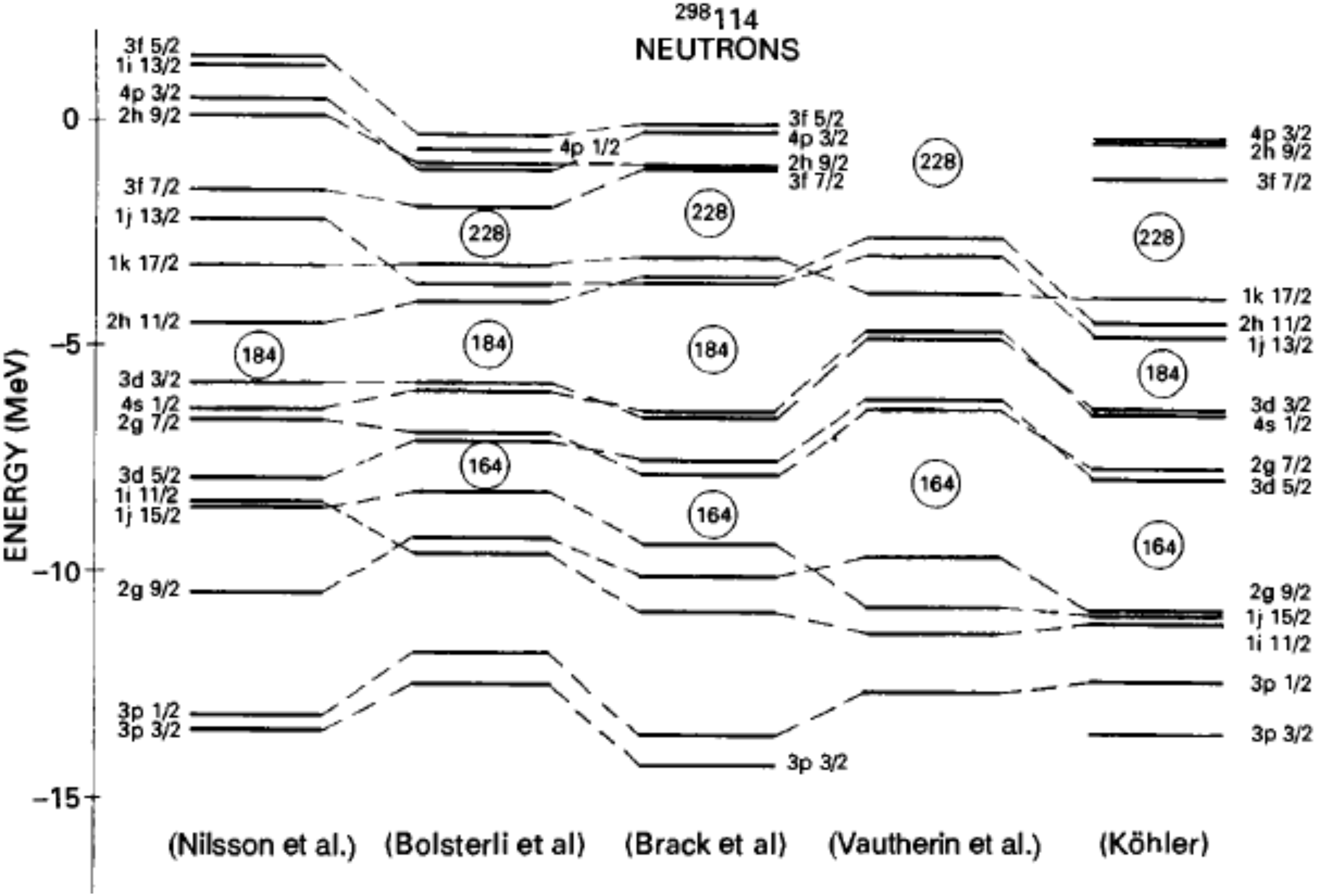
Island of stability

According to classical physics, elements with Z >104 should not exist due to the large Coulomb repulsion. The occurrence of superheavy elements with Z>104 is entirely due to nuclear shell effects.

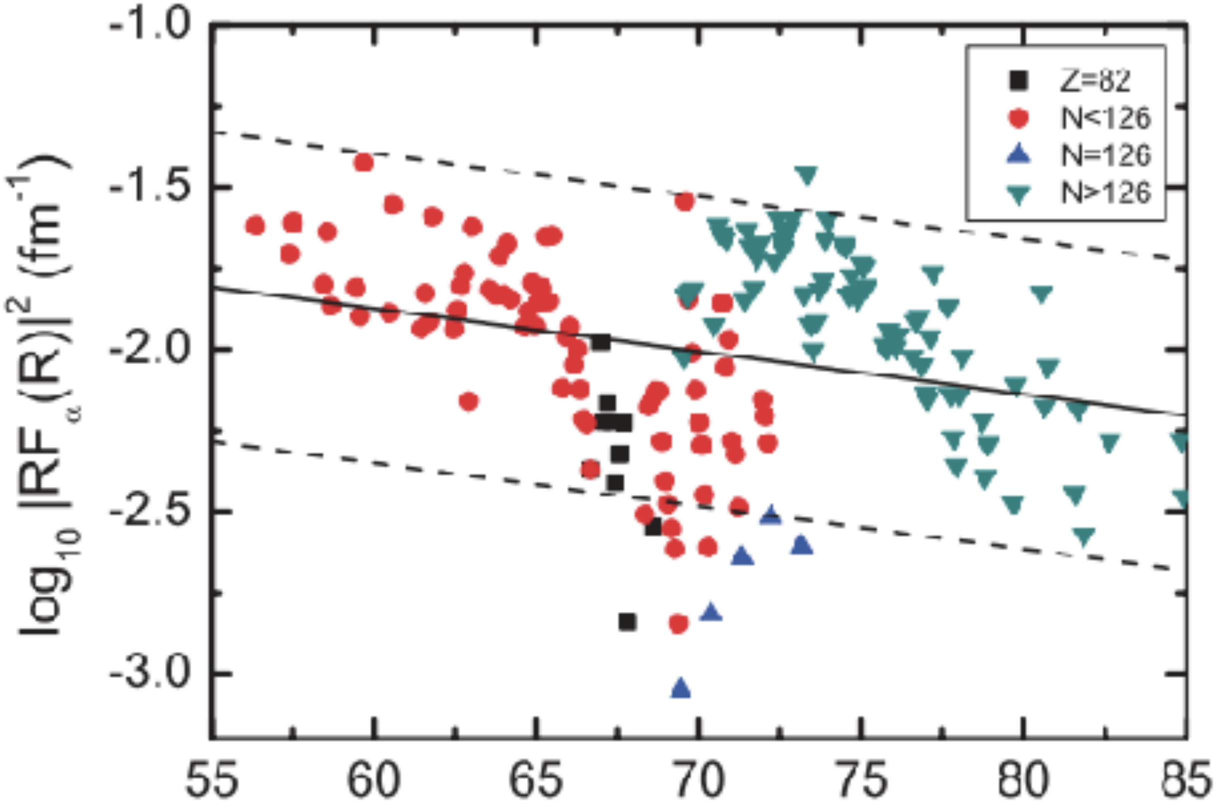




S.G. Nilsson and I. Ragnarsson: Shapes and Shells in Nuclear Structure, Cambridge Press, 1995



$^{298}114$
NEUTRONS
ENERGY (MeV)
0
−5
−10
−15
3f 5/2
1i 13/2
4p 3/2
2h 9/2
3f 7/2
1j 13/2
1k 17/2
2h 11/2
184
3d 3/2
4s 1/2
2g 7/2
3d 5/2
1i 11/2
1j 15/2
2g 9/2
3p 1/2
3p 3/2
4p 1/2
228
184
164
3f 5/2
4p 3/2
2h 9/2
3f 7/2
228
184
164
3p 3/2
228
164
4p 3/2
2h 9/2
3f 7/2
228
1k 17/2
2h 11/2
1j 13/2
184
3d 3/2
4s 1/2
2g 7/2
3d 5/2
164
2g 9/2
1j 15/2
1i 11/2
3p 1/2
3p 3/2
(Nilsson et al.)
(Bolsterli et al)
(Brack et al)
(Vautherin et al.)
(Köhler)



PHYSICAL REVIEW C **81**, 064319 (2010)

Abrupt changes in α-decay systematics as a manifestation of collective nuclear modes

C. Qi,[1,*] A. N. Andreyev,[2,3] M. Huyse,[2] R. J. Liotta,[1] P. Van Duppen,[2] and R. A. Wyss[1]

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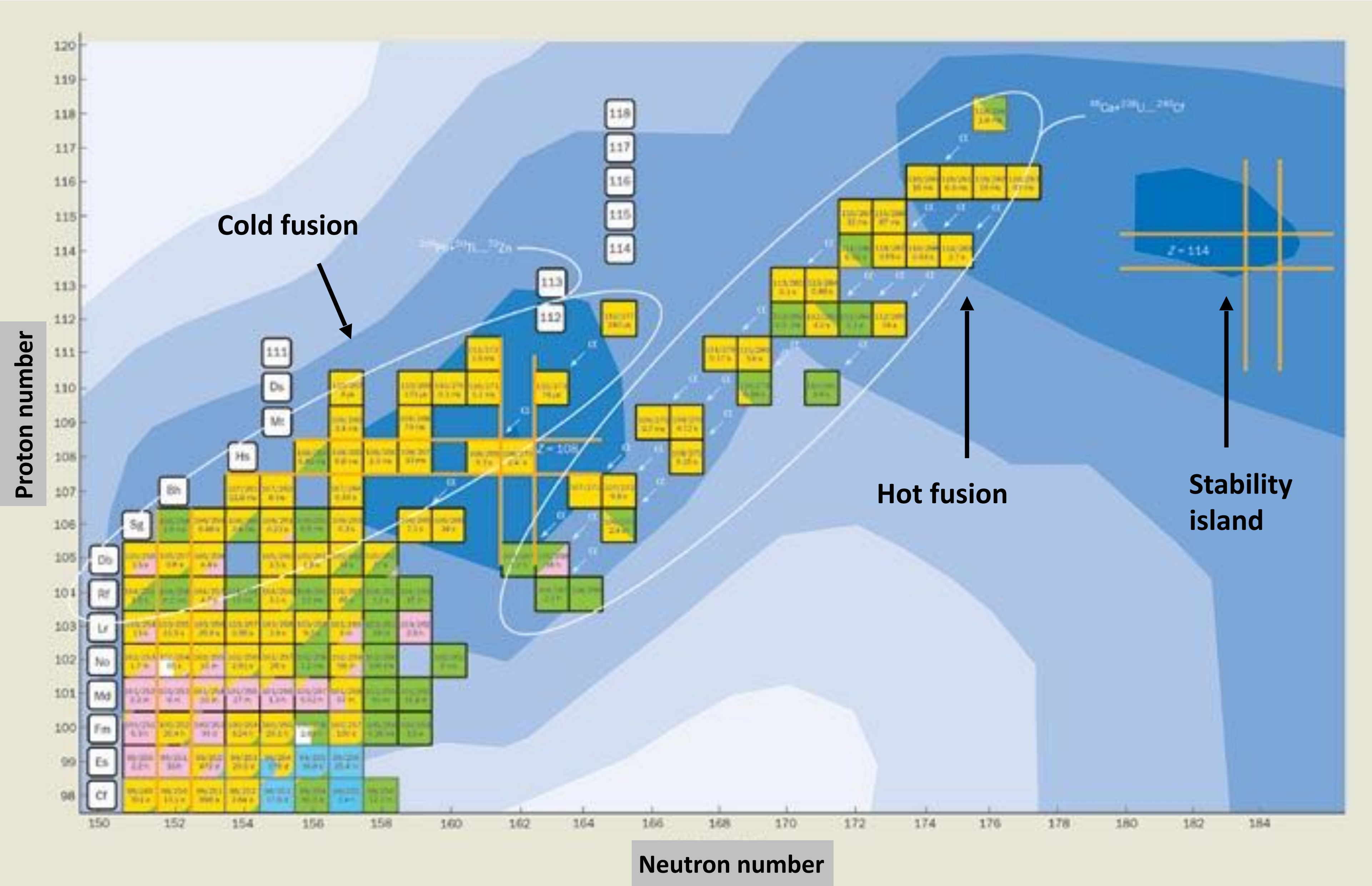
[3]*School of Engineering and Science, University of the West of Scotland, Paisley PA1 2BE, United Kingdom*

Map of superheavy elements



Cold fusion

Hot fusion

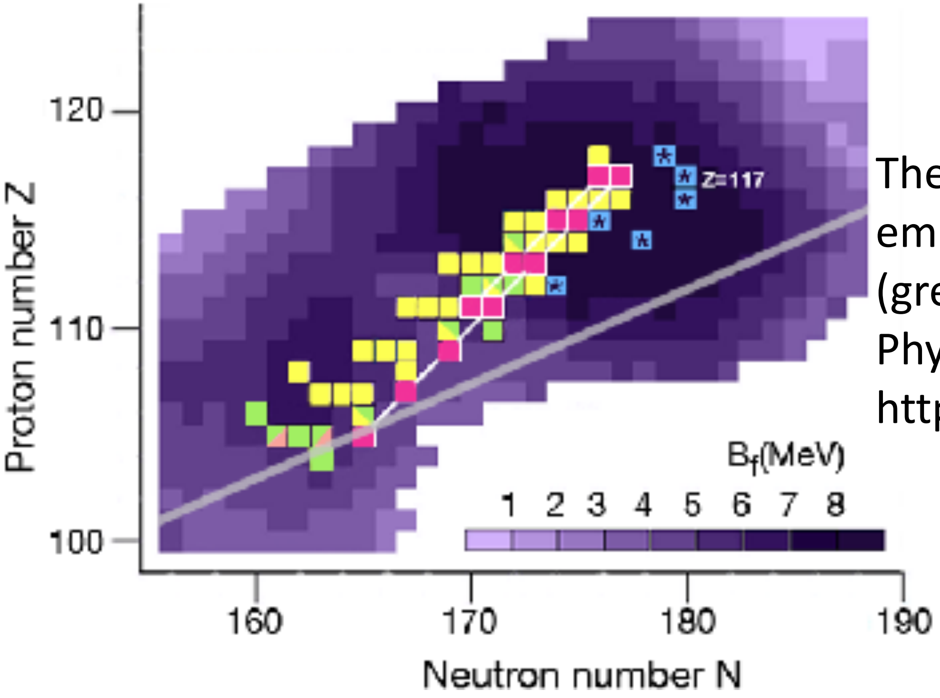
Stability island

Proton number

Neutron number

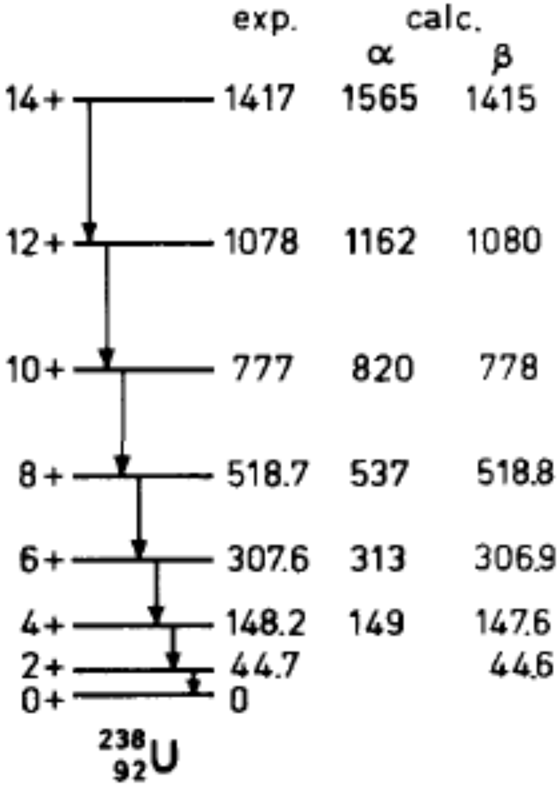
Shell closure in superheavy nuclei (an open problem)

Synthesis of a New Element with Atomic Number Z=117, PRL 104, 142502 (2010)



The nuclei in the chart decay by α emission (yellow), spontaneous fission (green), and β+ emission (pink).
Physics 3, 31 (2010)
http://physics.aps.org/articles/v3/31

However, the nucleus can not be viewed as a rigid body due the short range of strong-force interactions; typically measured moments of inertia for low-spin states are 30−50% relative to that of a rigid body.

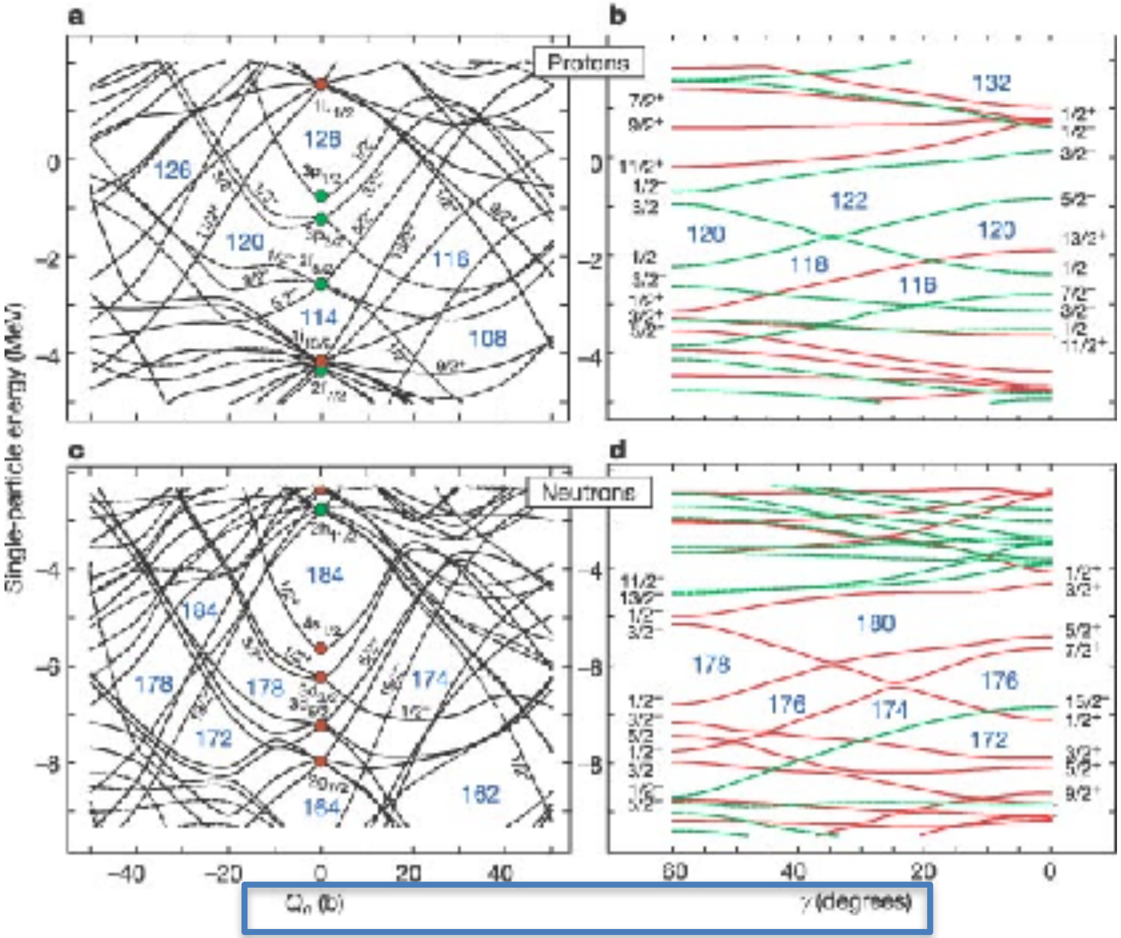


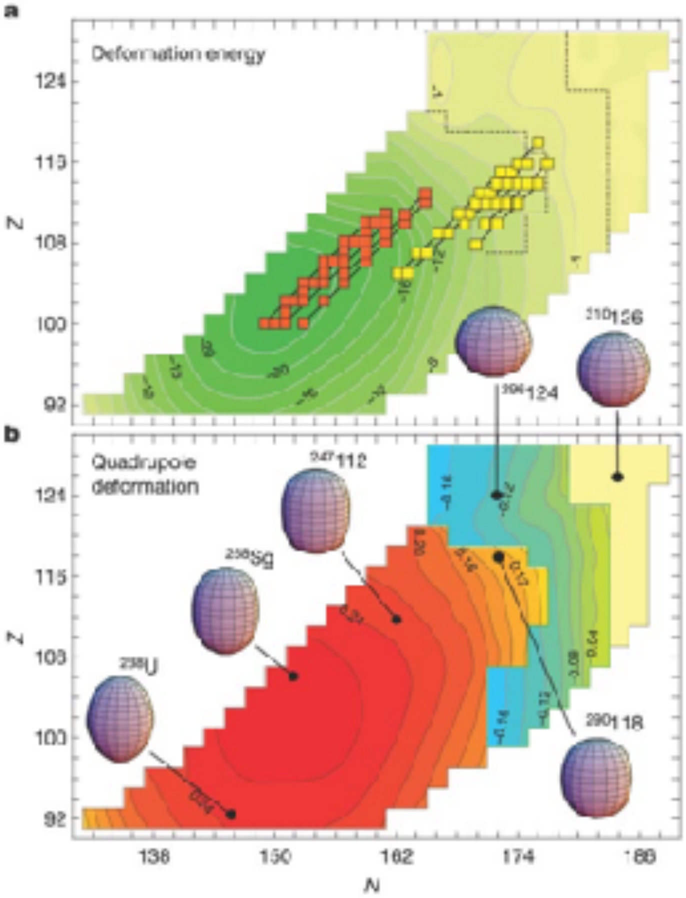
α: $E(\mathrm{keV}) = 7.45\, I(I+1)$

β: $E(\mathrm{keV}) = 7.45\, I(I+1) - 3.4\times10^{-3} I^2(I+1)^2$

Single-particle energy scheme as a function of deformation parameters.
What are they?

Superheavy nuclei may also be “deformed”.



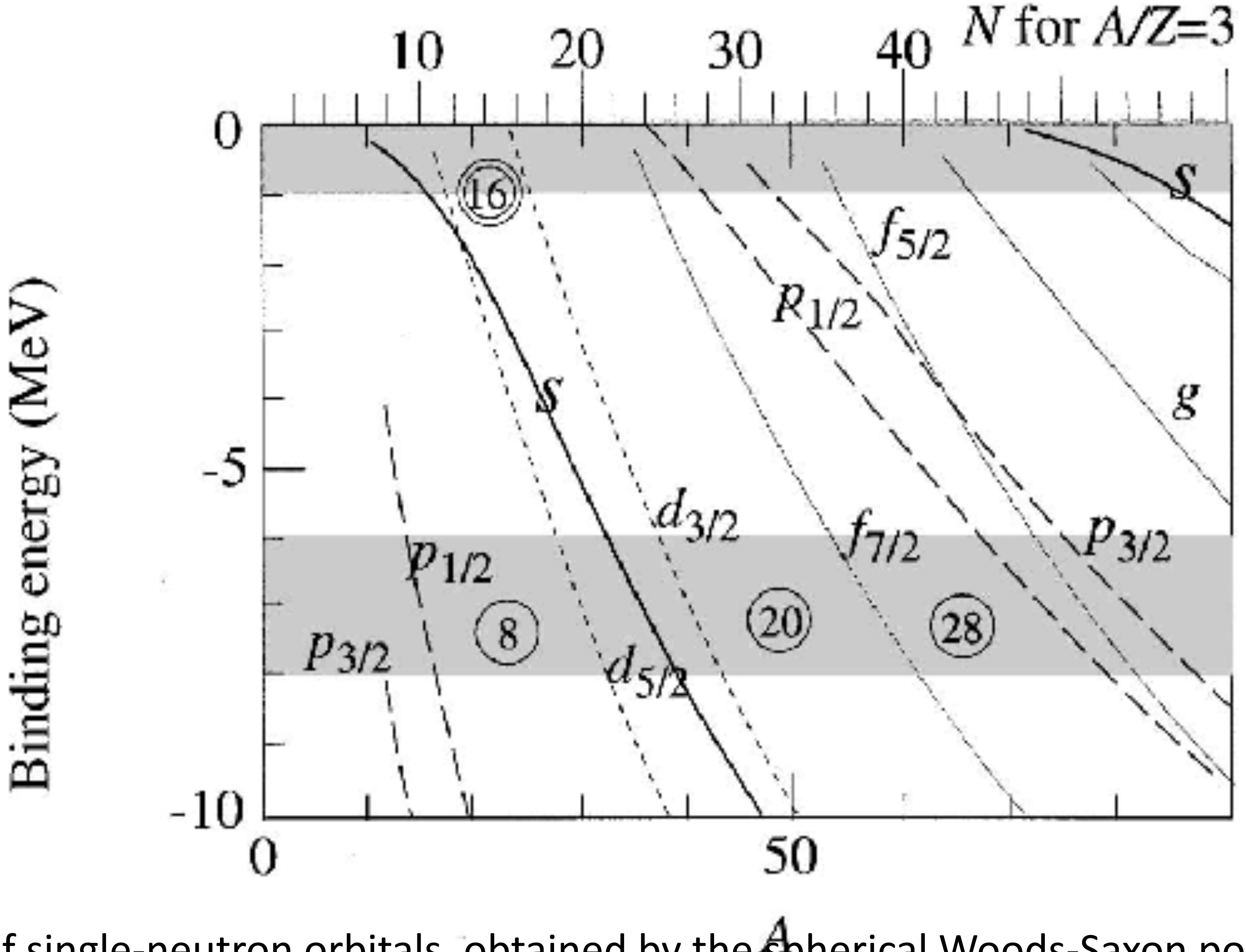


S. Ćwiok et al., Nature (London) 433, 705 (2005).

Prolate shapes are coloured red–orange, oblate shapes are blue–green, and spherical shapes are light yellow.

Nuclear Physics today: Shell evolution in neutron rich drip line nuclei

Intensive recent researches suggest that magic number or shell closure is a local concept.



Spectrum of single-neutron orbitals, obtained by the spherical Woods-Saxon potential, for *A/Z=3 nuclei. The lines* are labeled by the single-particle orbitals. The magic numbers are indicated by the numbers inside the circles.

Phys. Rev. Lett. 84, 5493–5495 (2000)

Recent application of spin-orbit coupling in other fields

For example,
Spin–orbit qubit in a semiconductor nanowire, S. Nadj-Perge, S.M. Frolov, E.P.A.M. Bakkers & L.P. Kouwenhoven, Nature 468, 1084 (2010).
http://www.nature.com/nature/journal/v468/n7327/full/nature09682.html

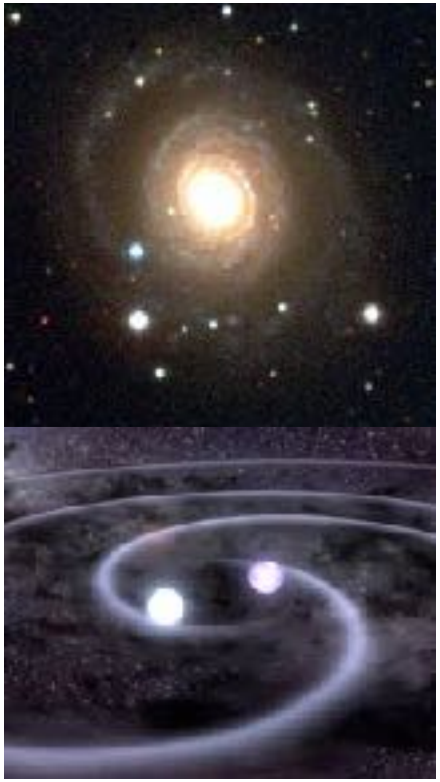
Spin–orbit-coupled Bose–Einstein condensates, Y.-J. Lin, K. Jiménez-García, I. B. Spielman, Nature 471, 83 (2011).
http://www.nature.com/nature/journal/v471/n7336/full/nature09887.html

Abundance of the element in the Universe

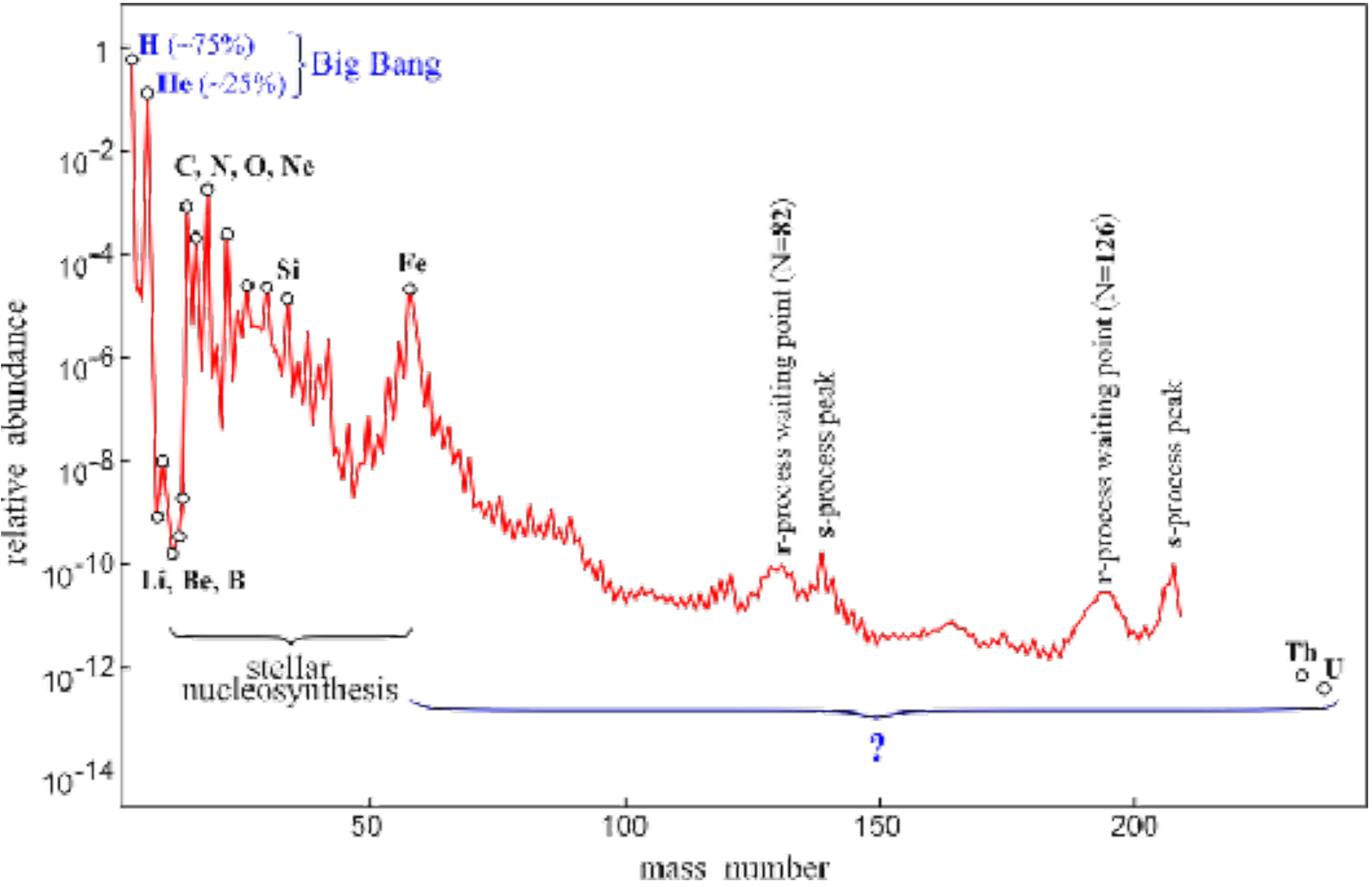
The 11 Greatest Unanswered Questions of Physics
(National Research Council, NAS, USA, 2002):

1. **What is dark matter?**
2. **What is dark energy?**
3. **How were the heavy elements from iron to uranium made?**
4. **Do neutrinos have mass?**

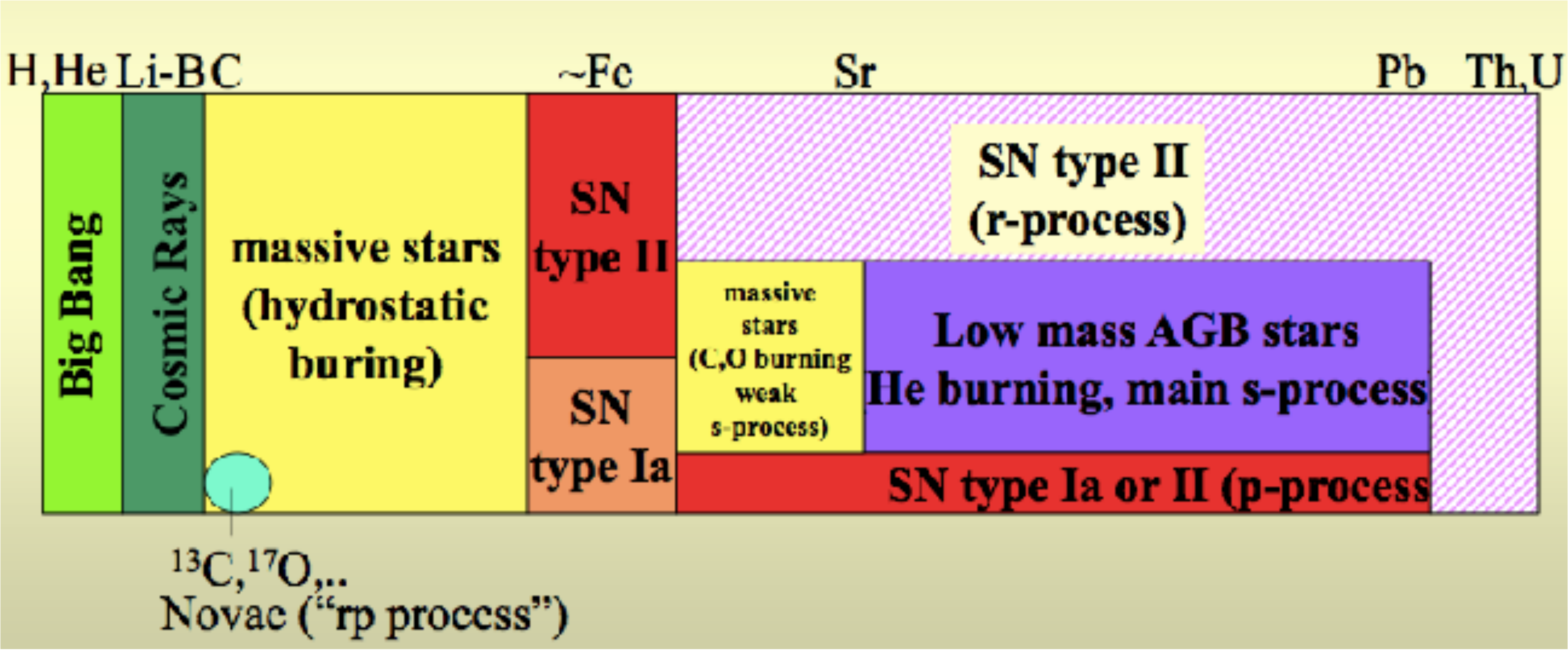
…



Strong neutron fluxes are expected in **core-collapse supernova explosions** or in the **mergers of neutron stars.**



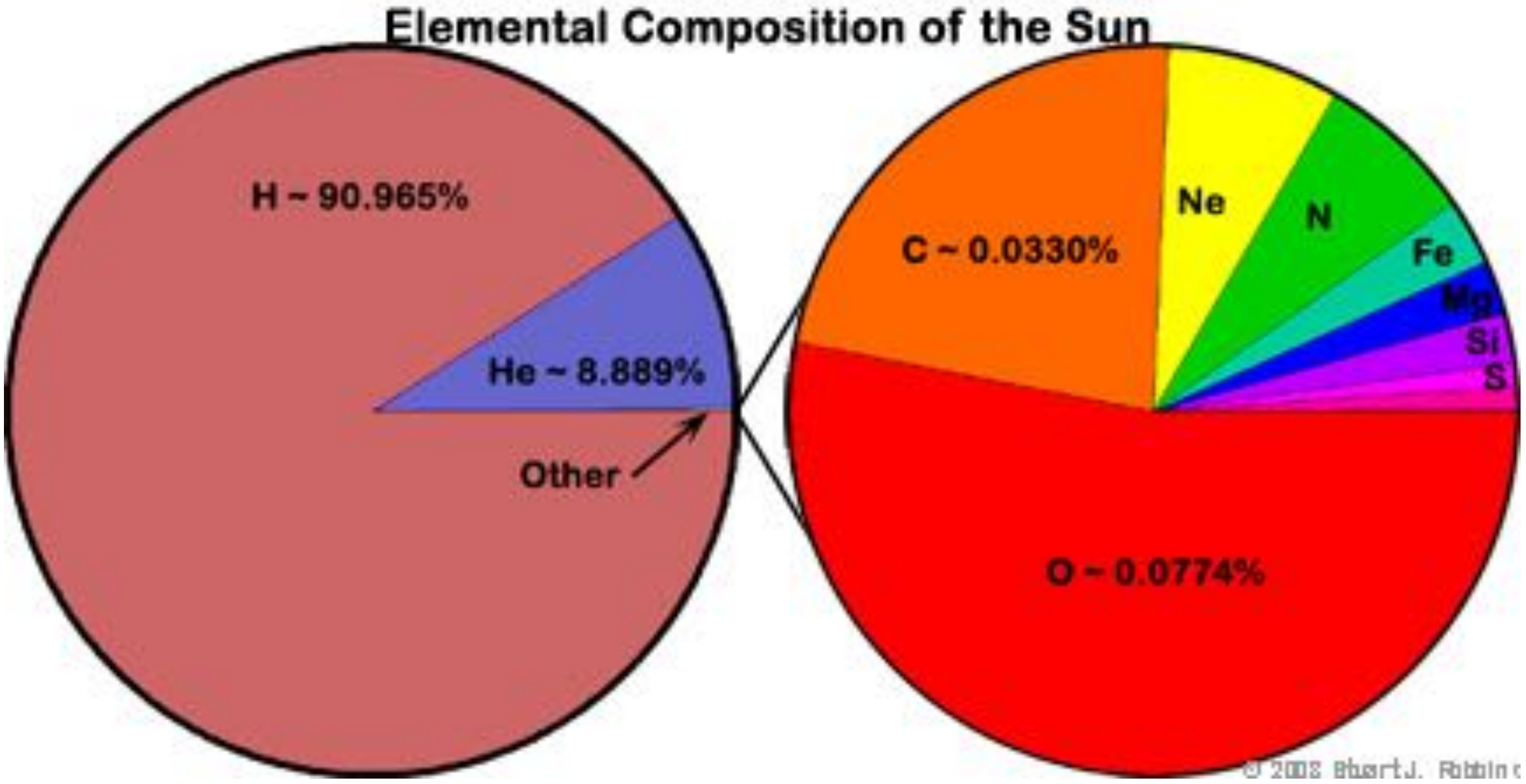
Brief Summary of nucleosynthesis



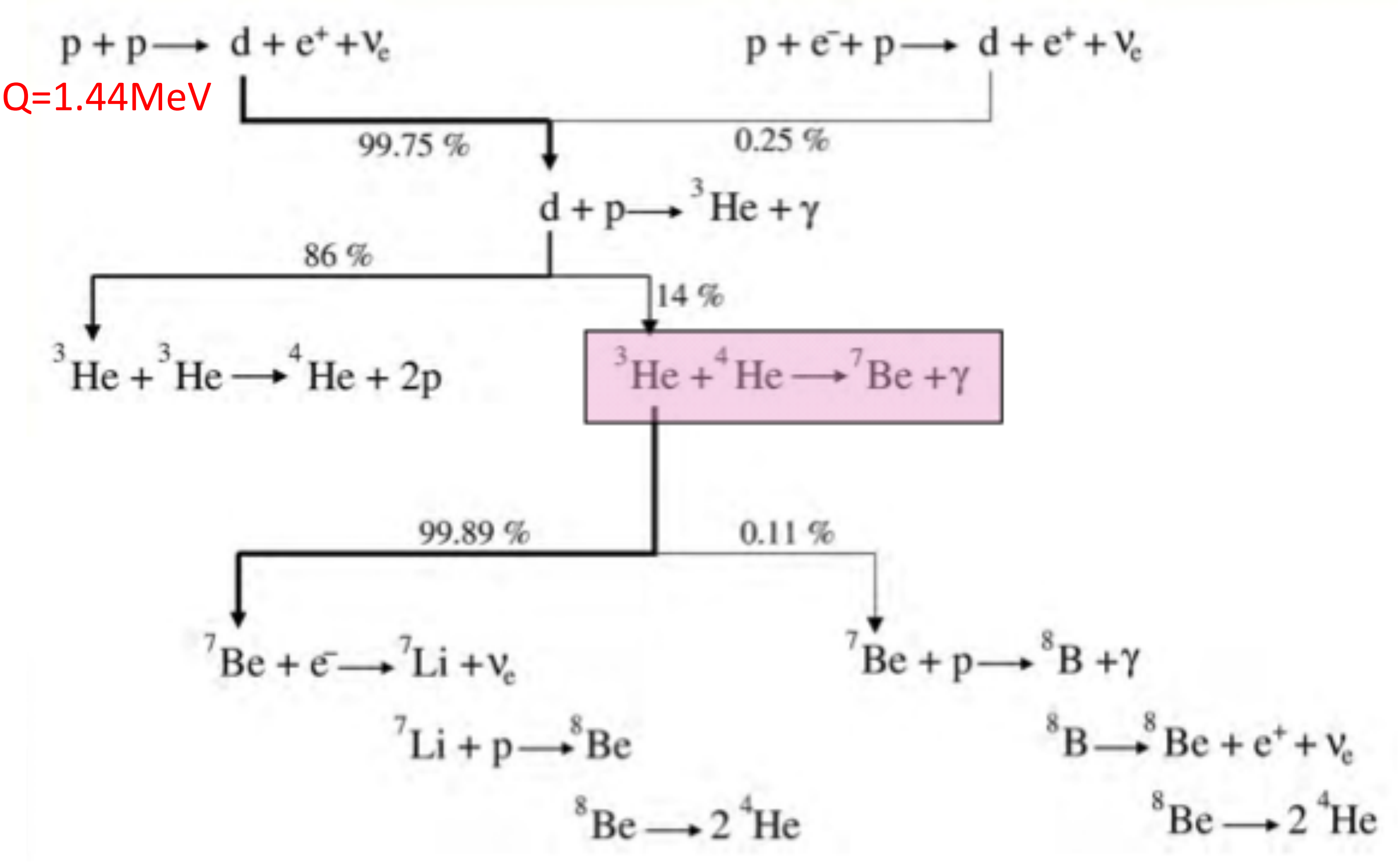
Processes that the structure of the nuclei involved may play a big role:
Production of 7Li in BBN
Hoyle state

Stars are formed through the presence of free protons in space which clump together under the influence of the gravitational field.

In the center of the stars thus formed the protons are concentrated in a high temperature and high density environment. In this environment the protons interact with each other to produce heavier isotopes in a process that goes on until the protons are depleted.



The pp chain



$p + p \rightarrow d + e^{+} + \nu_e$

Q=1.44MeV

$p + e^{-} + p \rightarrow d + e^{+} + \nu_e$

99.75 %

0.25 %

$d + p \rightarrow {}^{3}He + \gamma$

86 %

14 %

${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$

${}^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

99.89 %

0.11 %

${}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e$

${}^{7}Li + p \rightarrow {}^{8}Be$

${}^{8}Be \rightarrow 2\,{}^{4}He$

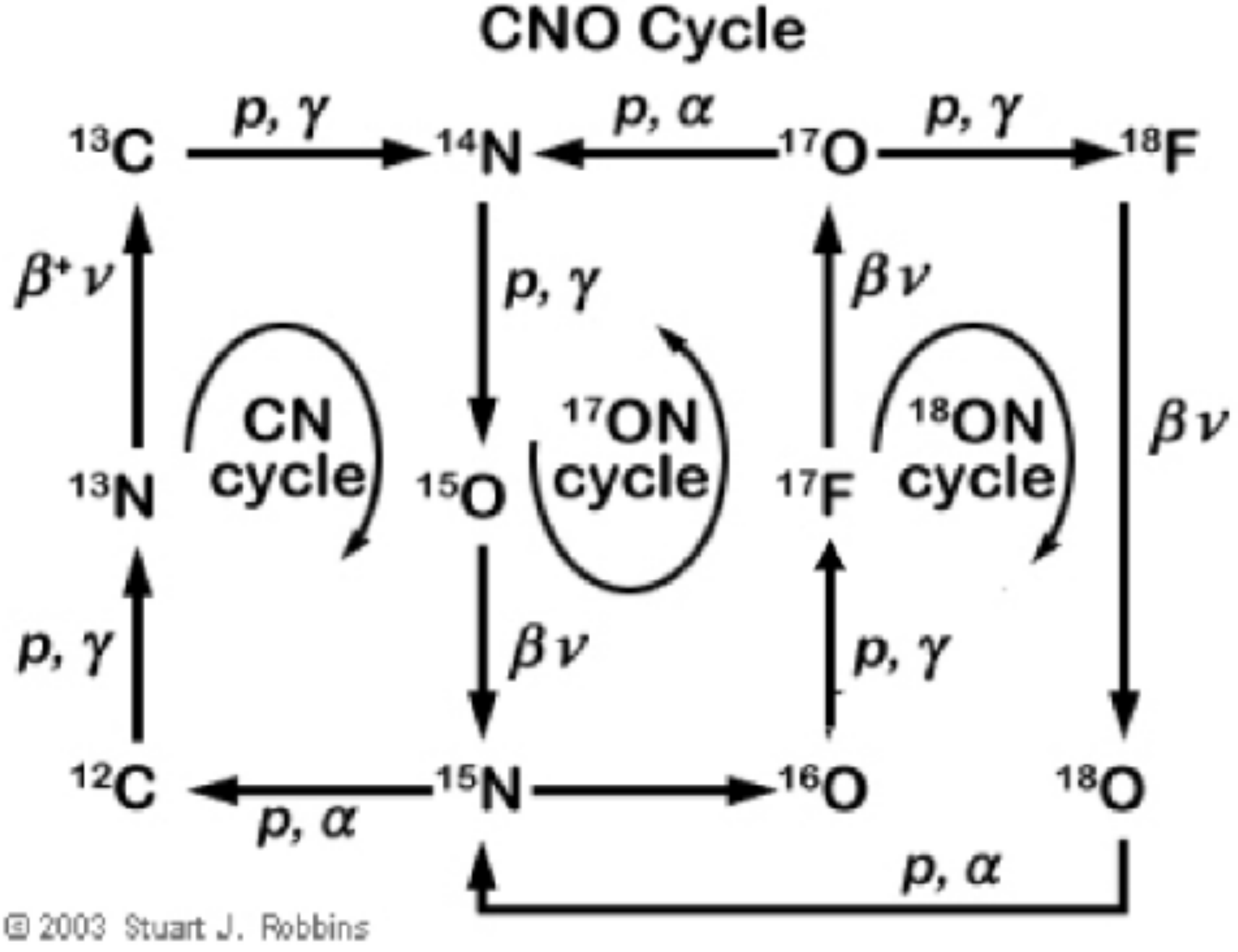
${}^{7}Be + p \rightarrow {}^{8}B + \gamma$

${}^{8}B \rightarrow {}^{8}Be + e^{+} + \nu_e$

${}^{8}Be \rightarrow 2\,{}^{4}He$

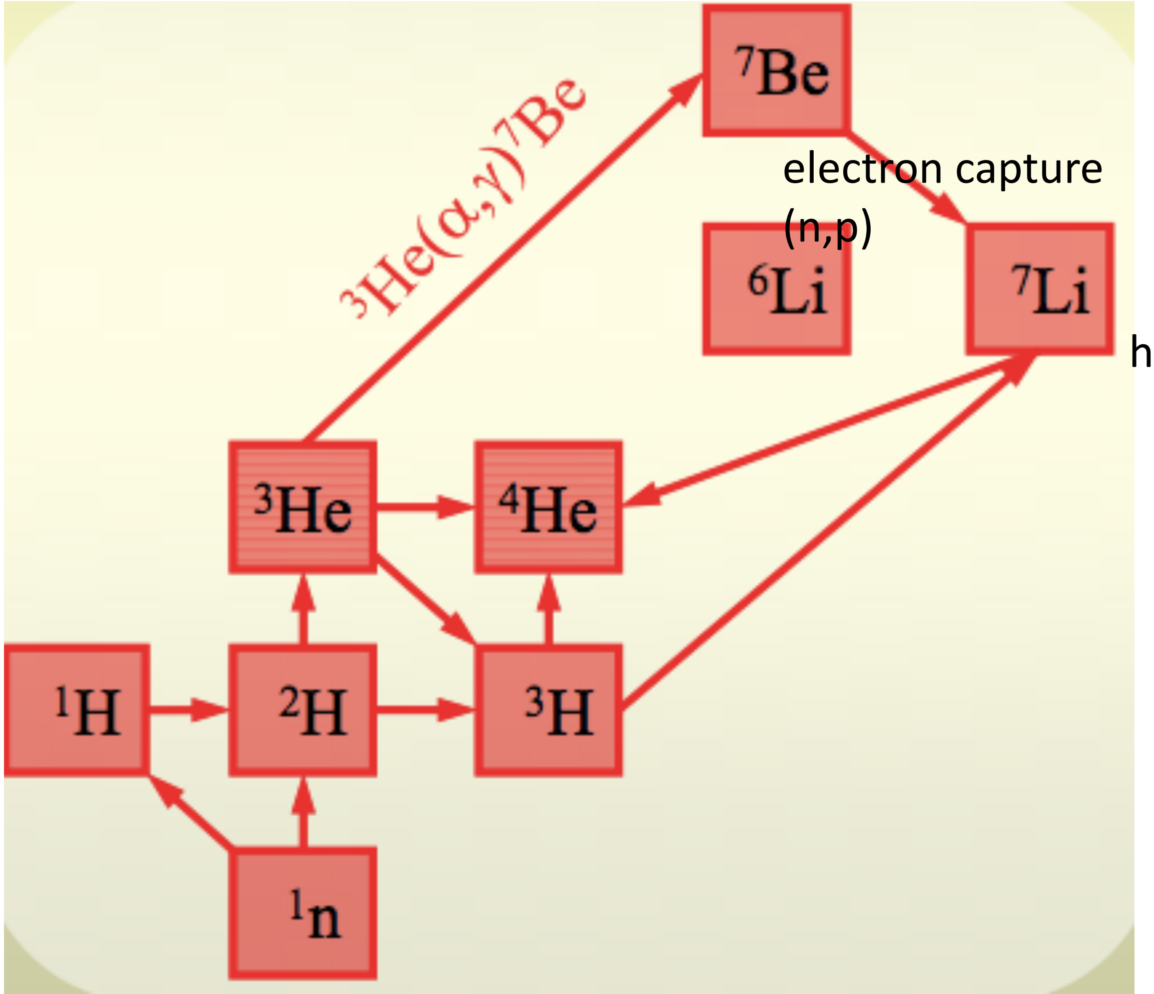
At the end of the reactions only alpha particles left.

Besides the pp chain there is another path through which Hydrogen is burned.



The hydrogen burning of the pp chain and the CNO cycle continues until the hydrogen fuel is nearly consumed.

Astrophysical motivation: Big bang nucleosynthesis



electron capture

(n,p)

The primordial abundance of 7Li as predicted by BBN is more than a factor 2 larger than what has been observed in metal-poor halo stars.

http://arxiv.org/pdf/1202.5232v2.pdf

Helium burning

When a star reaches the stage of a red giant its core consists mainly of helium.

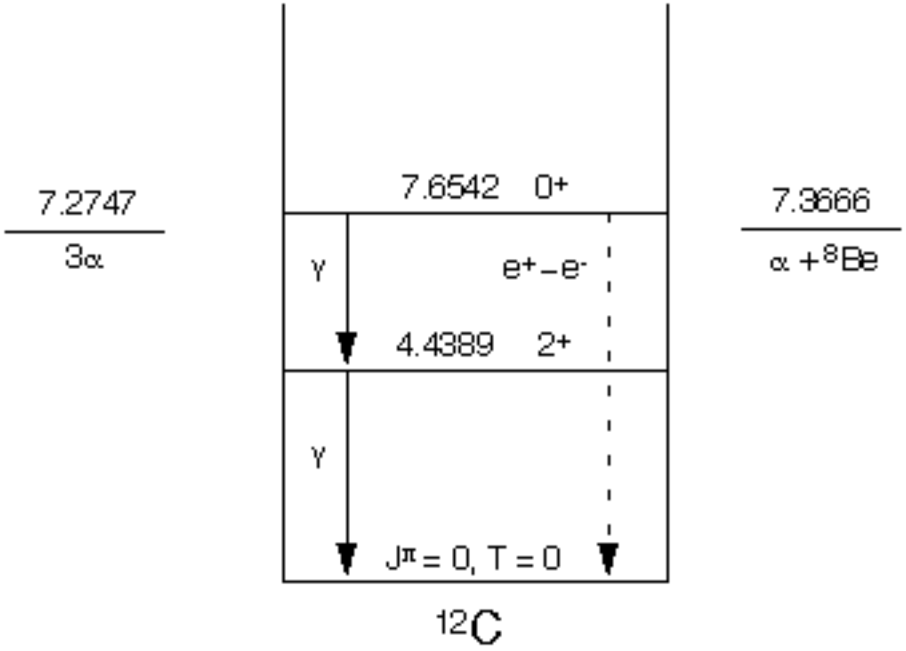
$$\alpha + \alpha \to {}^{8}\mathrm{Be}. \qquad (9.11)$$

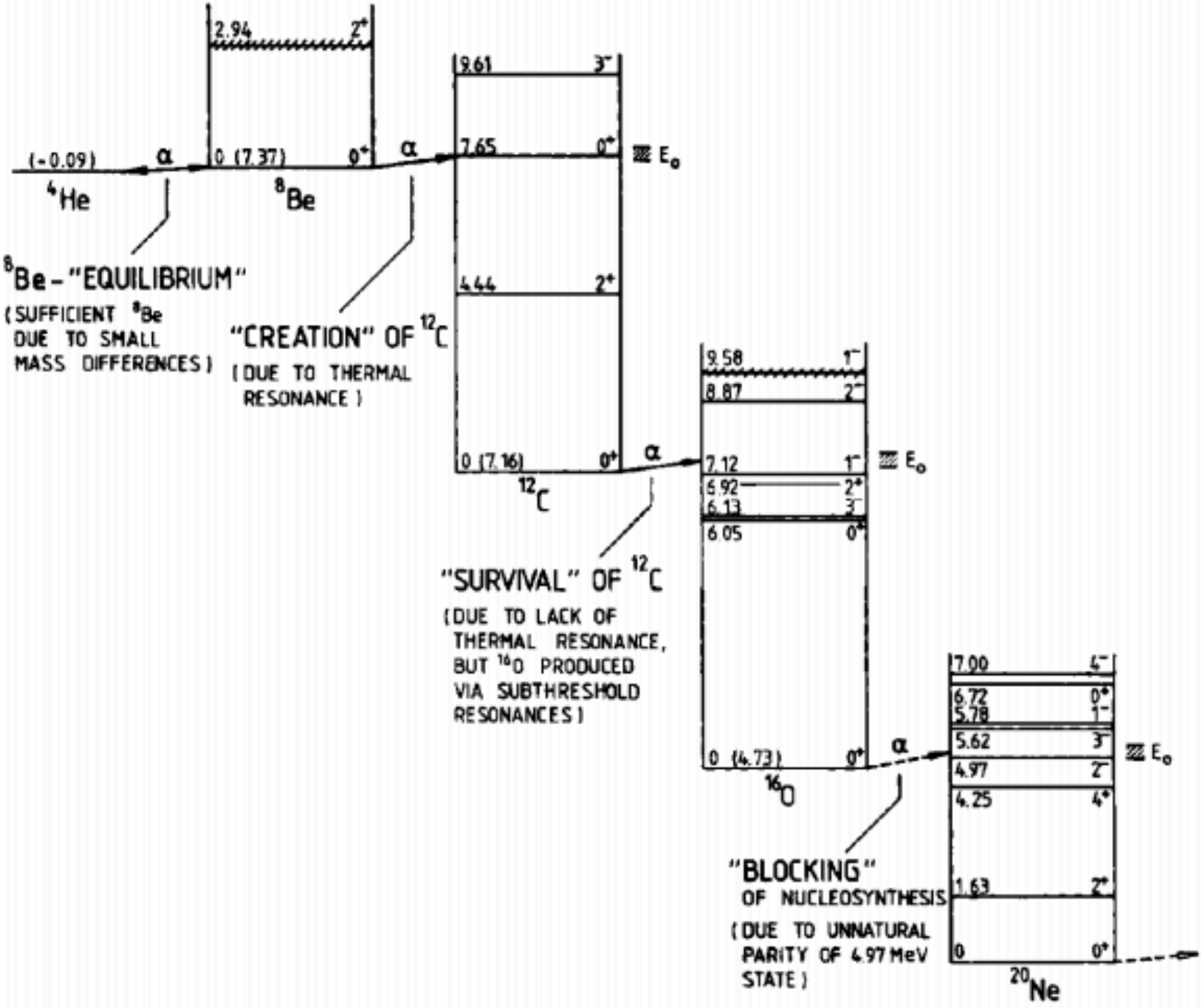
The ground state of ^{8}Be is unbound with respect to α decay with a Q_α value of 91.84keV and a decay width of 5.57eV. But from here no reasonable reaction was found to proceed

	0	1	2	3	4	5	6	7	8
6			8C 230 KeV P: 100.00% α	9C 126.5 MS ε: 100.00% εp: 61.6% [illegible]	10C 19.[illegible] S ε: 100.00% -5101	11C 20.334 M ε: 100.00% -7513.6	12C STABLE 98.93% -7366.55	13C STABLE 1.07% -[illegible]698.35	14C 5700 Y β-: 100.00% -12012.5[illegible]
5		6B 2P	7B 1.4 MeV P α -3.2E+3	8B 770 MS ε: 100.00% εα: 100.00% -4.83E-3	9B 0.54 KeV 2α: 100.00% P: 100.00% -1.69E+3	10B STABLE 19.9% -4461.[illegible]	11B STABLE 80.1% -8564.1	12B 20.20 MS β-: 100.00% β-α: 1.58% -[illegible]	13B 17.33 MS β-: 100.00% -10817.7
4		5Be P	6Be 92 KeV α: 100.00% P: 100.00%	7Be 53.24 D ε: 100.00% [illegible]	8Be 5.57 eV α: 100.00% 91.84	9Be STABLE 100% [illegible]	10Be 1.387E+6 Y β-: 100.00% [illegible]	11Be 13.81 S β-: 100.00% β-α: [illegible] [illegible]	12Be 21.49 MS β-: 100.00% β-n: [illegible] [illegible]
3	3Li P	4Li 6.03 MeV P: 100.00%	5Li 1.5 MeV α: 100.00% P: 100.00% 1.97E+3	6Li STABLE 7.59% [illegible]	7Li STABLE 92.41% -2467.53	8Li 839.9 MS β-α: 100.00% β-: 100.00% -6.10E+3	9Li 178.3 MS β-: 100.00% β-n: 50.80% [illegible]	10Li N: 100.00% [illegible]	11Li 8.75 MS β-: 100.00% β-n: [illegible] [illegible]
2		3He STABLE 0.000134%	4He STABLE 99.999866%	5He 0.60 MeV N: 100.00% α: 100.00% [illegible]	6He 801 MS β-: 100.00%	7He 150 KeV N	8He 119.1 MS β-: 100.00% β-n: 16.00%	9He N: 100.00%	10He 300 KeV N: 100.00%

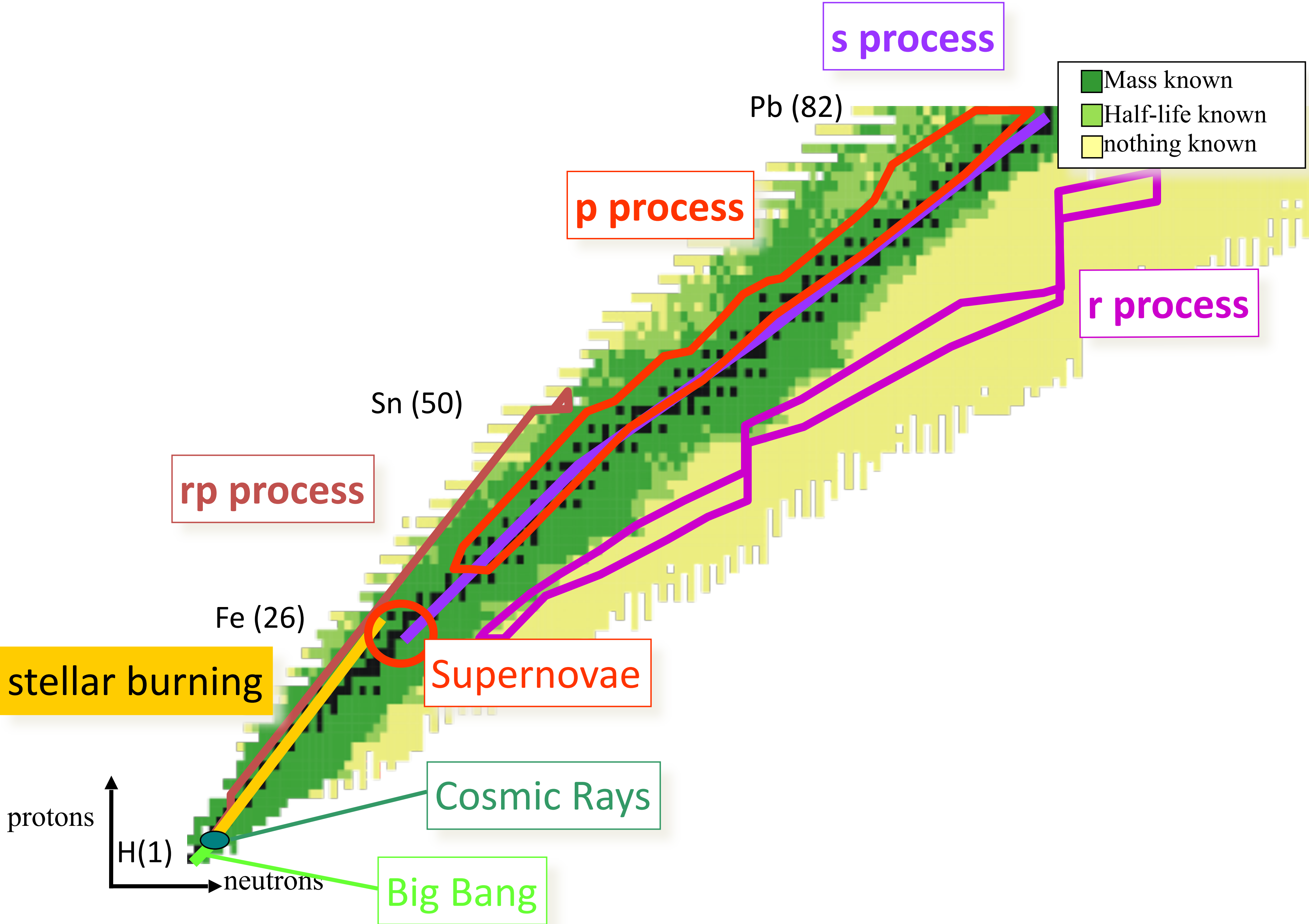
Hoyle state

$$\alpha + {}^{8}\mathrm{Be} \rightarrow {}^{12}\mathrm{C}^{*}$$





(-0.09)
α
^{4}He
2.94
2$^+$
0 (7.37)
0$^+$
^{8}Be
α
9.61
3$^-$
7.65
0$^+$
▨ E_0
4.44
2$^+$
0 (7.16)
0$^+$
^{12}C
^{8}Be - "EQUILIBRIUM"
(SUFFICIENT ^{8}Be DUE TO SMALL MASS DIFFERENCES)
"CREATION" OF ^{12}C
(DUE TO THERMAL RESONANCE)
α
9.58
1$^-$
8.87
2$^-$
7.12
1$^-$
▨ E_0
6.92
2$^+$
6.13
3$^-$
6.05
0$^+$
0 (4.73)
0$^+$
^{16}O
"SURVIVAL" OF ^{12}C
(DUE TO LACK OF THERMAL RESONANCE, BUT ^{16}O PRODUCED VIA SUBTHRESHOLD RESONANCES)
α
7.00
4$^-$
6.72
0$^+$
5.78
1$^-$
5.62
3$^-$
▨ E_0
4.97
2$^-$
4.25
4$^+$
1.63
2$^+$
0
0$^+$
^{20}Ne
"BLOCKING" OF NUCLEOSYNTHESIS
(DUE TO UNNATURAL PARITY OF 4.97 MeV STATE)



s process

🟩 Mass known

🟩 Half-life known

🟨 nothing known

Pb (82)

p process

r process

Sn (50)

rp process

Fe (26)

stellar burning

Supernovae

Cosmic Rays

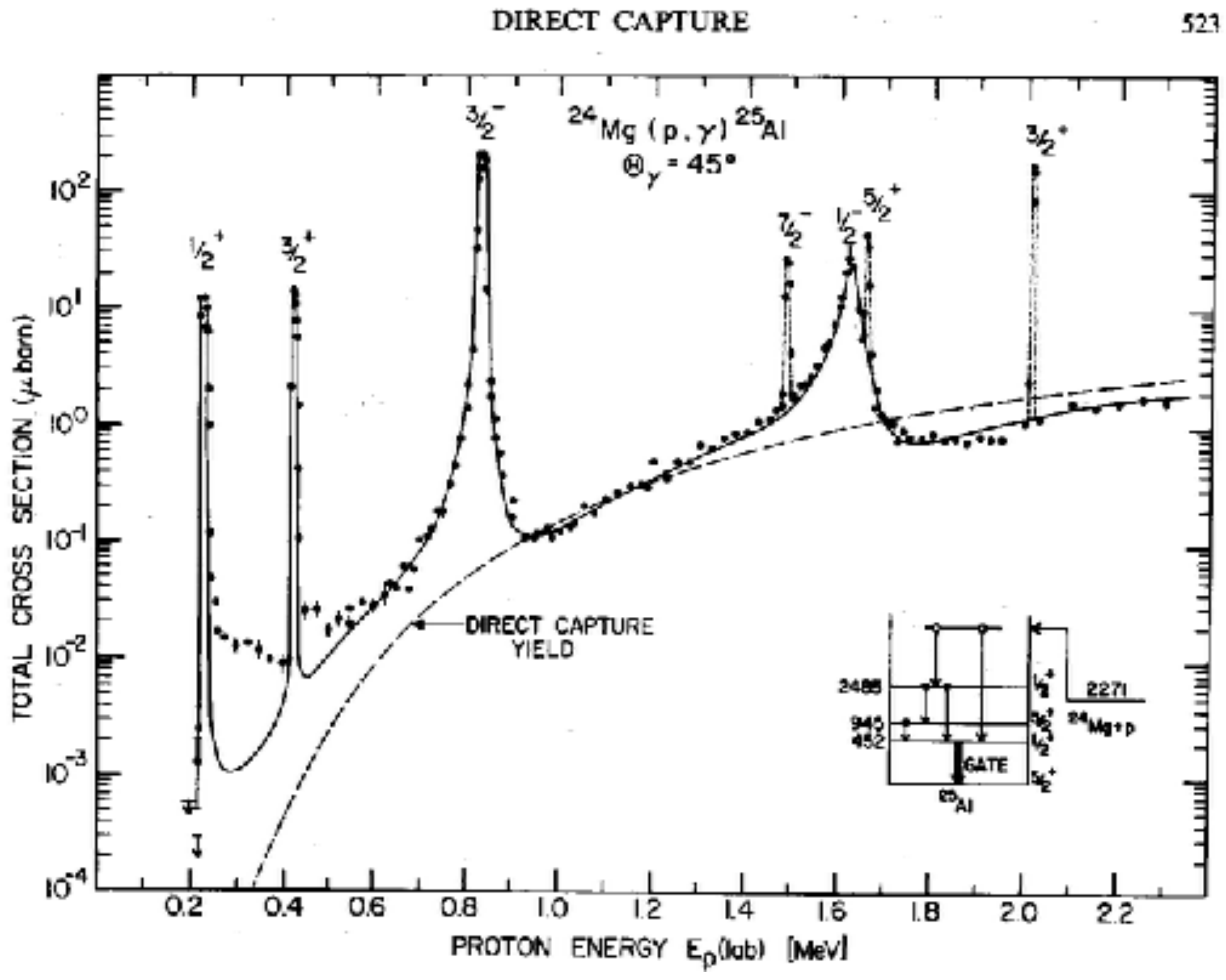
protons

H(1)

neutrons

Big Bang

Example:



Resonance contributions are on top of direct capture cross sections

Core-collapse supernovae (type II, Ib, Ic).

the $^{26}\mathrm{Si}(\mathrm{p},\gamma)^{27}\mathrm{P}$ reaction

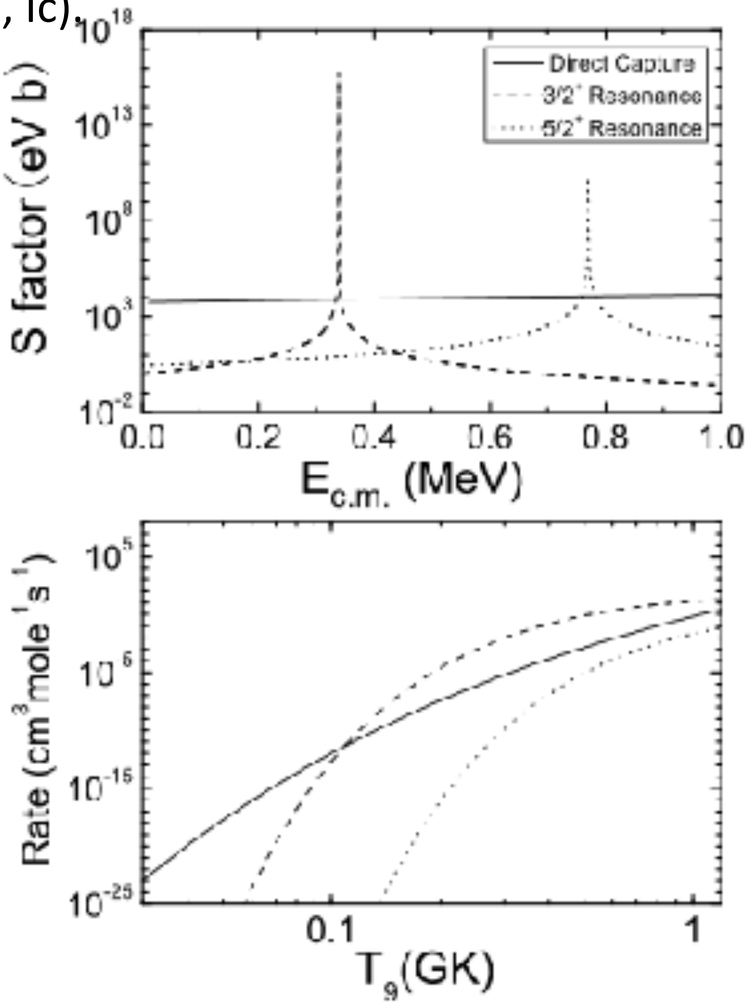
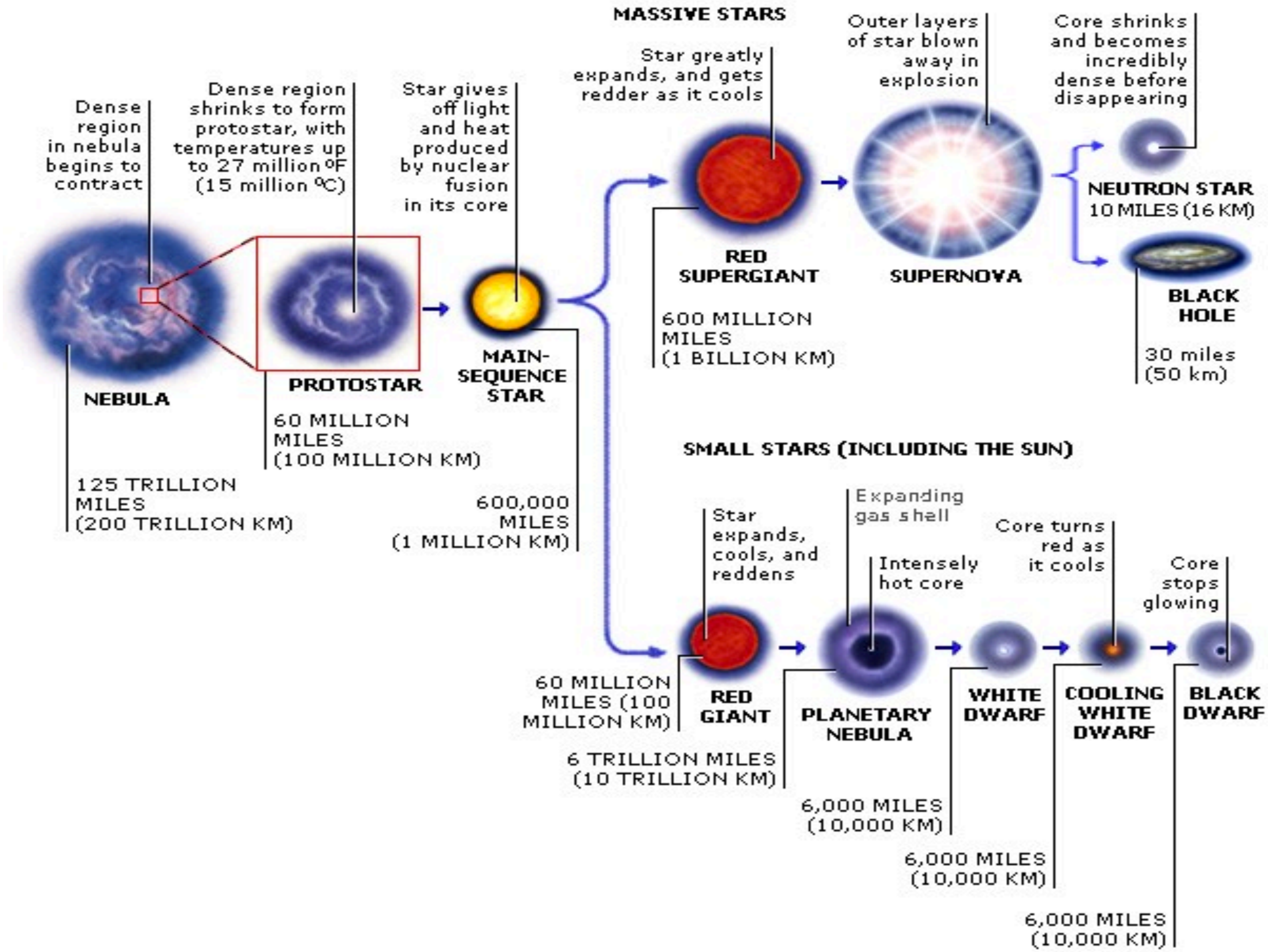
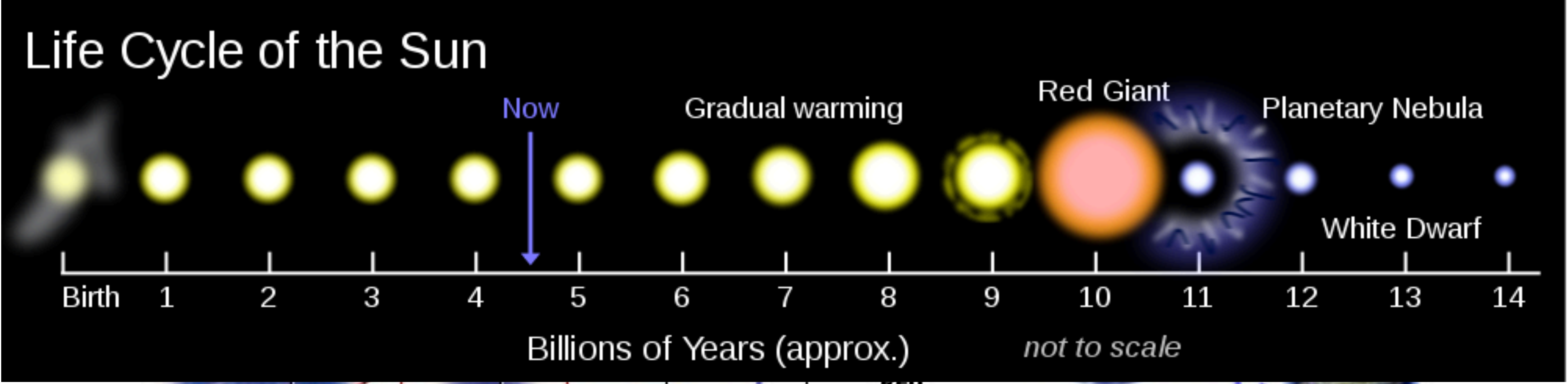
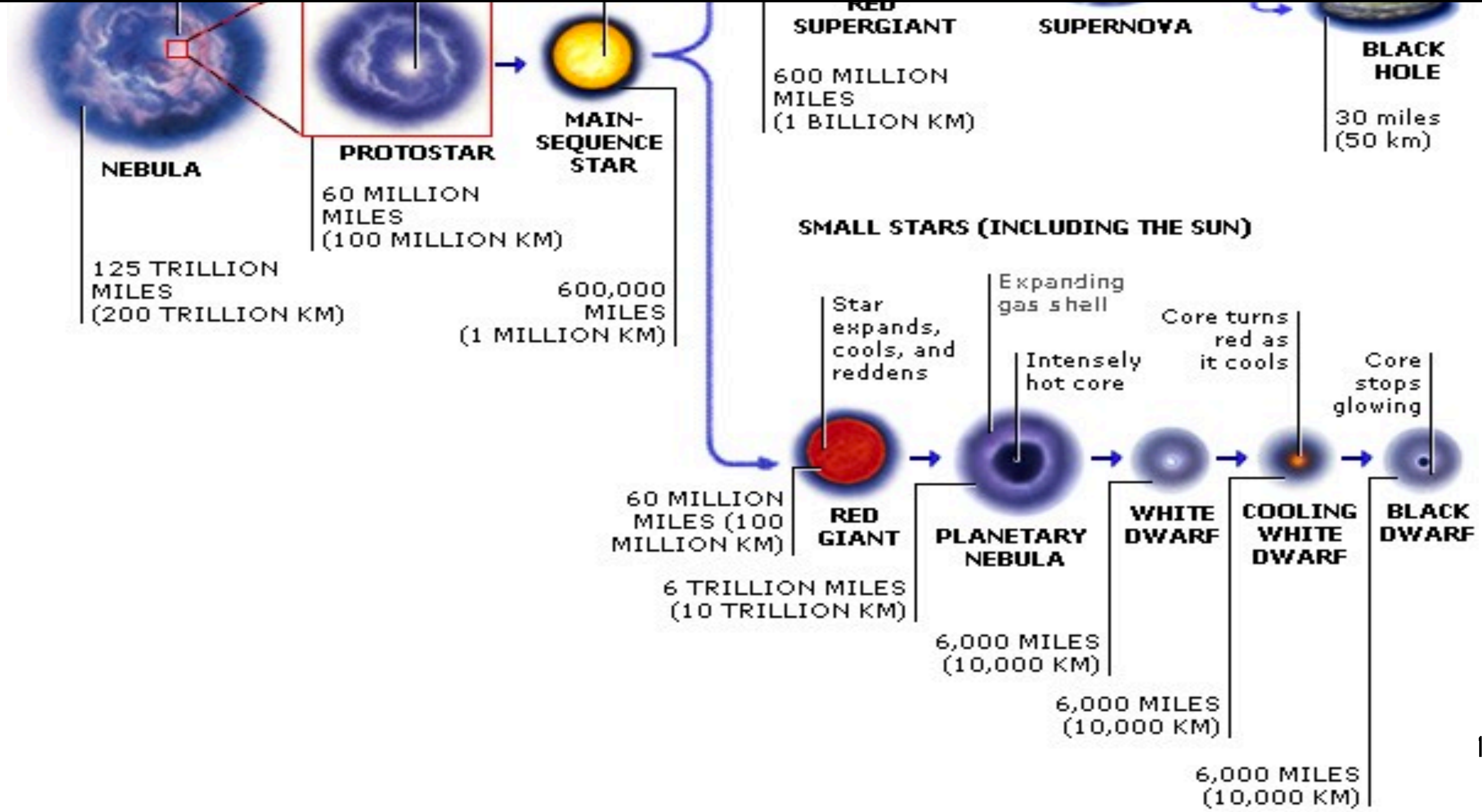


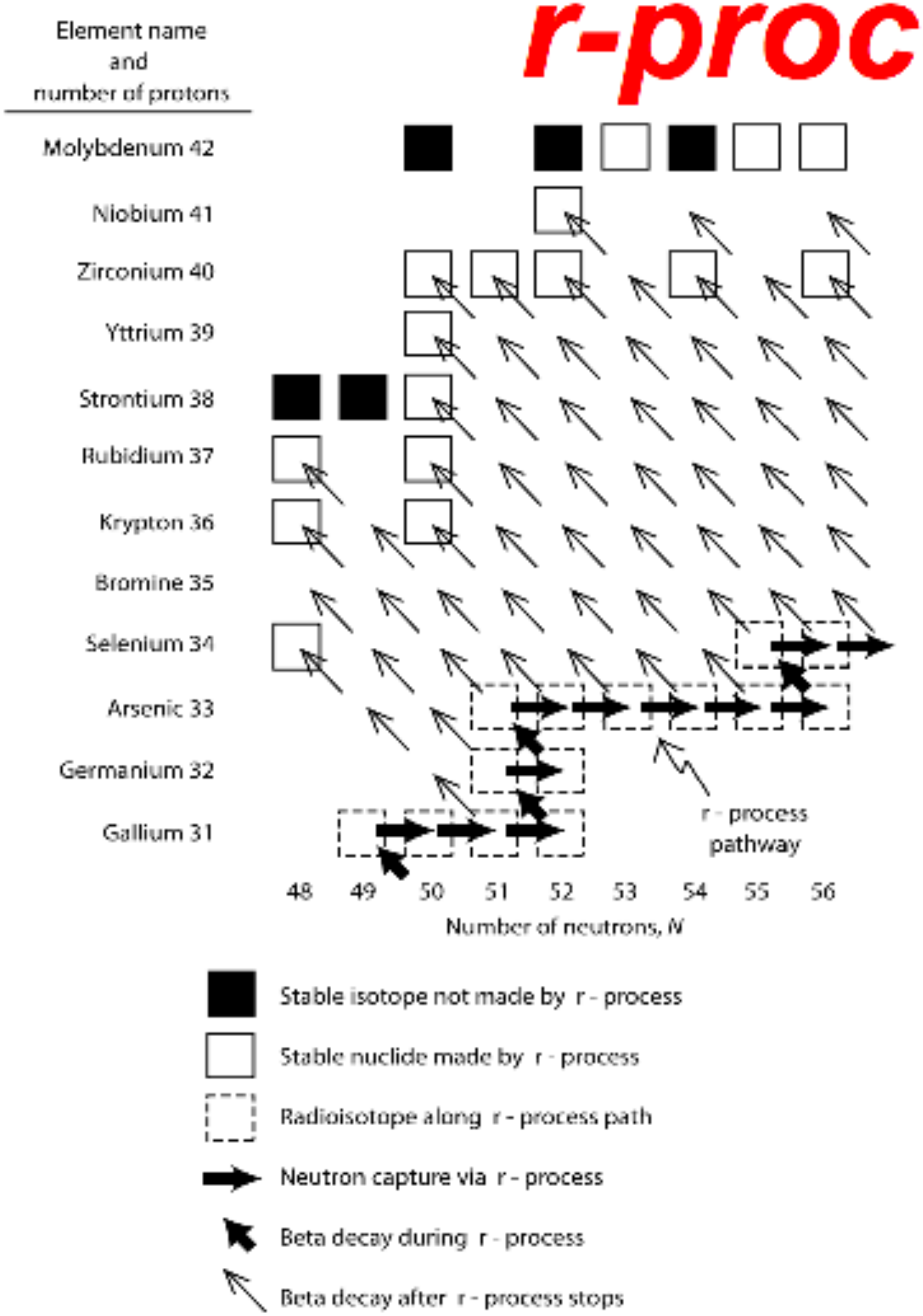
FIG. 5: Upper: Calculated (with the SkM parameter) S factors for the direct capture and resonant capture reactions; Lower: same as the upper panel but for the reaction rates.


MASSIVE STARS
Dense region in nebula begins to contract
Dense region shrinks to form protostar, with temperatures up to 27 million ºF (15 million ºC)
Star gives off light and heat produced by nuclear fusion in its core
Star greatly expands, and gets redder as it cools
Outer layers of star blown away in explosion
Core shrinks and becomes incredibly dense before disappearing
NEUTRON STAR
10 MILES (16 KM)
RED SUPERGIANT
SUPERNOVA
BLACK HOLE
600 MILLION MILES (1 BILLION KM)
30 miles (50 km)
NEBULA
PROTOSTAR
MAIN-SEQUENCE STAR
60 MILLION MILES (100 MILLION KM)
125 TRILLION MILES (200 TRILLION KM)
600,000 MILES (1 MILLION KM)
SMALL STARS (INCLUDING THE SUN)
Expanding gas shell
Star expands, cools, and reddens
Intensely hot core
Core turns red as it cools
Core stops glowing
60 MILLION MILES (100 MILLION KM)
RED GIANT
PLANETARY NEBULA
WHITE DWARF
COOLING WHITE DWARF
BLACK DWARF
6 TRILLION MILES (10 TRILLION KM)
6,000 MILES (10,000 KM)
6,000 MILES (10,000 KM)
6,000 MILES (10,000 KM)


Life Cycle of the Sun
Now
Gradual warming
Red Giant
Planetary Nebula
White Dwarf
Birth
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Billions of Years (approx.)
not to scale


SUPERGIANT
600 MILLION MILES (1 BILLION KM)
SUPERNOVA
BLACK HOLE
30 miles (50 km)
NEBULA
125 TRILLION MILES (200 TRILLION KM)
PROTOSTAR
60 MILLION MILES (100 MILLION KM)
MAIN-SEQUENCE STAR
600,000 MILES (1 MILLION KM)
SMALL STARS (INCLUDING THE SUN)
Star expands, cools, and reddens
Expanding gas shell
Intensely hot core
Core turns red as it cools
Core stops glowing
60 MILLION MILES (100 MILLION KM)
RED GIANT
6 TRILLION MILES (10 TRILLION KM)
PLANETARY NEBULA
6,000 MILES (10,000 KM)
WHITE DWARF
6,000 MILES (10,000 KM)
COOLING WHITE DWARF
6,000 MILES (10,000 KM)
BLACK DWARF

r-process



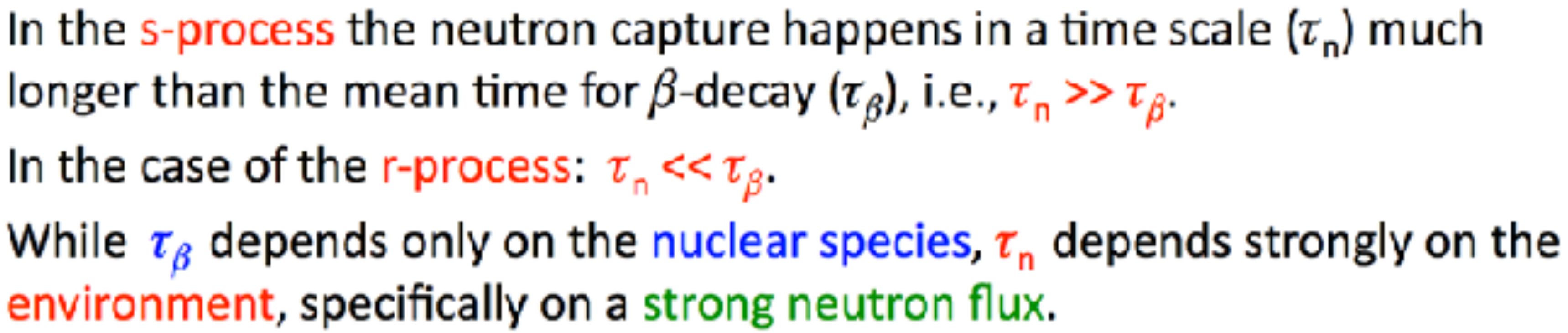
Series of neutron capture to saturation followed by a series of beta decay during supernova explosion.

Time scale:
0.01 to 10 sec

S-process (slow neutron capture)
It mainly operates in the red giant phase.
- R-process (rapid neutron capture)

The principal mechanism for building up the heavier nuclei.
Occurs during supernova explosion.
- P-process (proton capture)

Also occurs in supernovae, is responsible for the lightest isotopes of a given element.
The principal difference between the s-process and r-process is the rate of capture relative to the decay of unstable isotopes.



In the s-process the neutron capture happens in a time scale (τ_n) much longer than the mean time for β-decay (τ_β), i.e., $\tau_n >> \tau_\beta$.

In the case of the r-process: $\tau_n << \tau_\beta$.

While τ_β depends only on the nuclear species, τ_n depends strongly on the environment, specifically on a strong neutron flux.

- The s-process is relatively well understood.
- The nuclear properties of the involved species that are easier to measure in the lab than the ones of the r-process (longer τ_β).
- The site is also much better constrained: primarily low- and intermediate-mass stars (less than 8 solar masses).
- The r-process element formation is much more uncertain.
- The nuclear properties of the participating elements is much more difficult to measure.
- And the sites where the r-process take place are a mistery.
- R-process element formation requires large neutron fluxes that are associated to rather catastrophic events. The two main candidates are **type II (core-collapse) supernova explosions** and **neutron star mergers**. At present the astrophysical conditions of these two phenomena are not well understood (good review: Sneden et al. 2003).

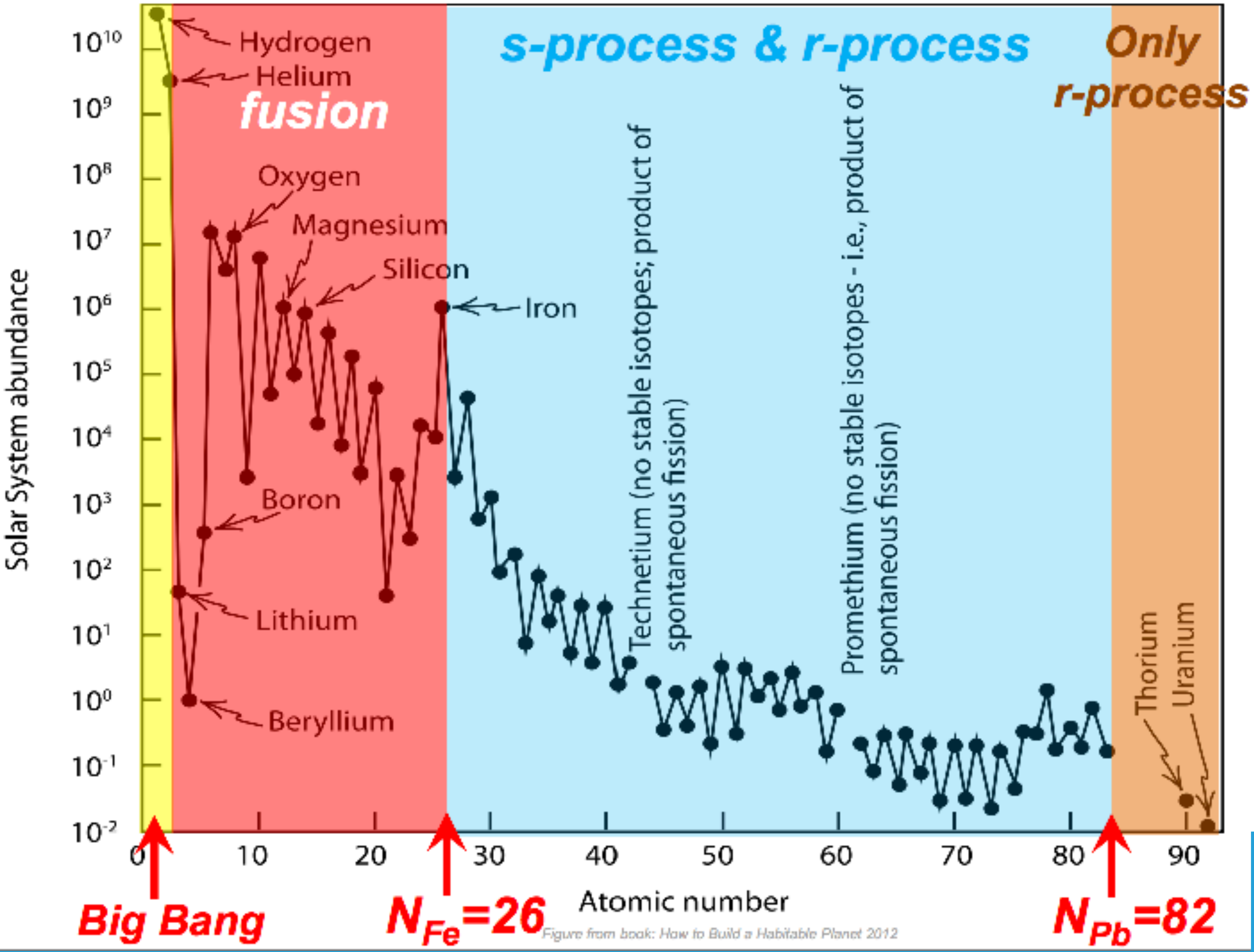


Figure from book: How to Build a Habitable Planet 2012

R-process during Supernova Explosion

The Crab Nebula, the remains of a supernova which exploded in 1054 AD. This picture was taken by the Hubble Space Telescope.