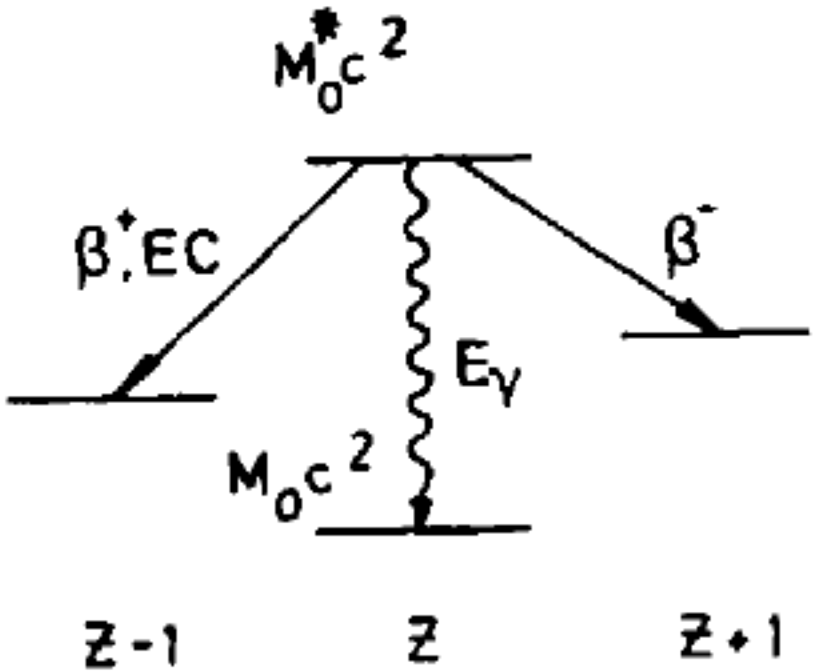
Electromagnetic transitions

May 05, 2017

Alpha, beta and gamma transitions



A large part of the knowledge of nuclei is obtained from the study of electromagnetic transitions. It is, for example, the main source of information about the spin assignments of nuclear states.

The basic one-photon emission process

$$\lambda_{if} = \frac{2\pi}{\hbar} |\langle f | \hat{H}_{\text{int}}^{\text{em}} | i \rangle|^2 \frac{dn}{dE}(E_\gamma).$$

where dn/dE is the density of final states

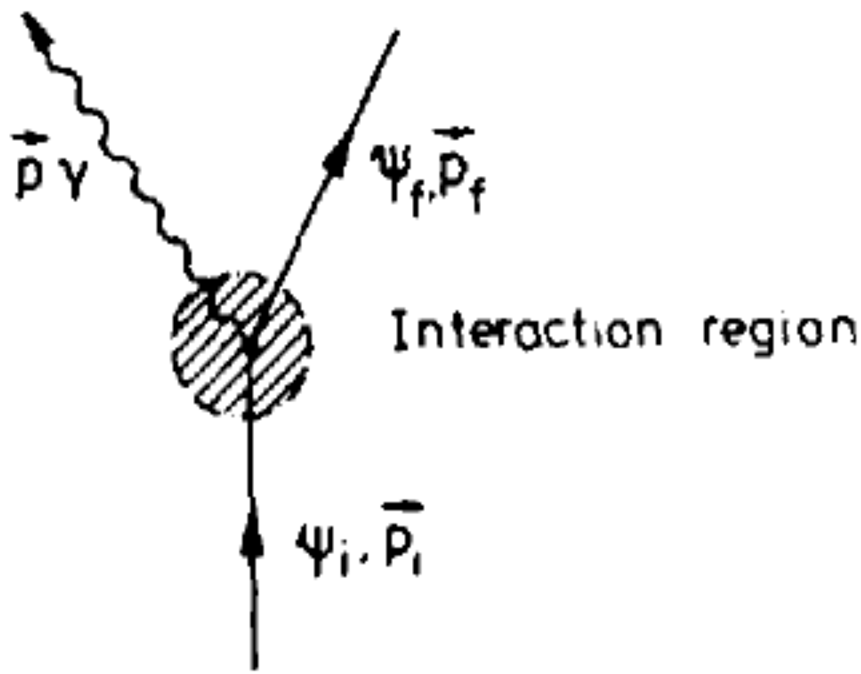
$$\hat{H}_{\text{int}}^{\text{em}} = \frac{e}{m} \vec{p} \cdot \vec{A},$$

with $\vec{p} \rightarrow -i\hbar\vec{\nabla}$

A is the vector potential

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{M} = \nabla \times \mathbf{A}$$



Emission process of a photon

Intrinsic spin of the photon
S=1

Name	Symbol	Antiparticle	Charge (e)	Spin	Mass (GeV/c^2)	Interaction mediated	Existence
Photon	γ	Self	0	1	0	Electromagnetism	Confirmed
W boson	W^-	W^+	−1	1	80.4	Weak interaction	Confirmed
Z boson	Z	Self	0	1	91.2	Weak interaction	Confirmed
Gluon	g	Self	0	1	0	Strong interaction	Confirmed
Higgs boson	H^0	Self	0	0	116 - 130	Mass	Unconfirmed
Graviton	G	Self	0	2	0	Gravitation	Unconfirmed

$$\vec{A} = a_0\vec{\epsilon}\cos(\vec{k}\cdot\vec{r} - \omega t),$$

It is convenient, though, to rewrite the expression of $\vec{A}$ as

$$\vec{A}(\text{one photon}) = \sqrt{\frac{\hbar^2}{2\epsilon_0 E_\gamma V}}\vec{\epsilon}(\exp[i(\vec{p}_\gamma\cdot\vec{r} - E_\gamma t)/\hbar] + \exp[-i(\vec{p}_\gamma\cdot\vec{r} - E_\gamma t)/\hbar]).$$

$$\mathbf{A}^E_{\lambda\mu}(\mathbf{r}) = \frac{-i}{k}\nabla\times(\mathbf{r}\times\nabla)(j_\lambda(kr)Y_{\lambda\mu}(\theta\phi)) \tag{1.7}$$

where $j_\lambda(kr)$ is a Bessel function and $Y_{\lambda\mu}(\theta\phi)$ is the spherical harmonics eigenfunctions of the angular momentum operator. This solution is called "electric" component of the electromagnetic field. The other solution is,

$$\mathbf{A}^M_{\lambda\mu}(\mathbf{r}) = (\mathbf{r}\times\nabla)(j_\lambda(kr)Y_{\lambda\mu}(\theta\phi)) \tag{1.8}$$

which is called the "magnetic" component. In both cases the photon carries angular

Multipole transitions

Integrating over the current density one obtains the electric component as,

$$\mathcal{O}(E\lambda\mu) = \frac{-i(2\lambda+1)}{ck^{\lambda+1}(\lambda+1}\int \mathbf{j}(\mathbf{r})\cdot\nabla\times(\mathbf{r}\times\nabla)\big(\mathrm{j}_\lambda(\mathrm{kr})\mathrm{Y}_{\lambda\mu}(\theta\phi)\big)\mathrm{d}\mathbf{r} \tag{7.15}$$

and the magnetic component becomes,

$$\mathcal{O}(M\lambda\mu) = \frac{-(2\lambda+1)}{ck^{\lambda}(\lambda+1}\int \mathbf{j}(\mathbf{r})\cdot(\mathbf{r}\times\nabla)\big(\mathrm{j}_\lambda(\mathrm{kr})\mathrm{Y}_{\lambda\mu}(\theta\phi)\big)\mathrm{d}\mathbf{r} \tag{7.16}$$

After some mathematics

$$\mathcal{O}(E\lambda\mu) = \int \rho(\mathbf{r})\mathrm{r}^\lambda\mathrm{Y}_{\lambda\mu}(\theta\phi)\mathrm{d}\mathbf{r}$$

$$\mathcal{O}(M\lambda\mu) = \frac{-1}{c(\lambda+1}\int \mathbf{j}(\mathbf{r})\cdot(\mathbf{r}\times\nabla)\mathrm{r}^\lambda\mathrm{Y}_{\lambda\mu}(\theta\phi)\mathrm{d}\mathbf{r}$$

Selection rules

When a nucleus emits (absorbs) a photon, the initial (final) total nuclear angular momentum should be equal to the sum of the final (initial) total nuclear angular momentum and the angular momentum carried by the radiation

$$J_i = J_f + L \qquad (J_f = J_i + L) .$$

This implies

$$|J_i - J_f| \leqslant L \leqslant J_i + J_f$$

Generally, for electric L-pole radiation the selection rules become *(E L)*

$$\pi_i = \pi_f (-1)^L , \qquad \vec{J}_i = \vec{J}_f + \vec{L} .$$

For magnetic L-pole radiation as ***(M L)***

$$\pi_i = \pi_f (-1)^{L+1} , \qquad \vec{J}_1 = \vec{J}_f + \vec{L} .$$

Another important selection rule results from the conservation of parity.

The electromagnetic interaction conserves parity and the transitions can be divided into two classes, the ones which do not change parity and the ones which do change parity change

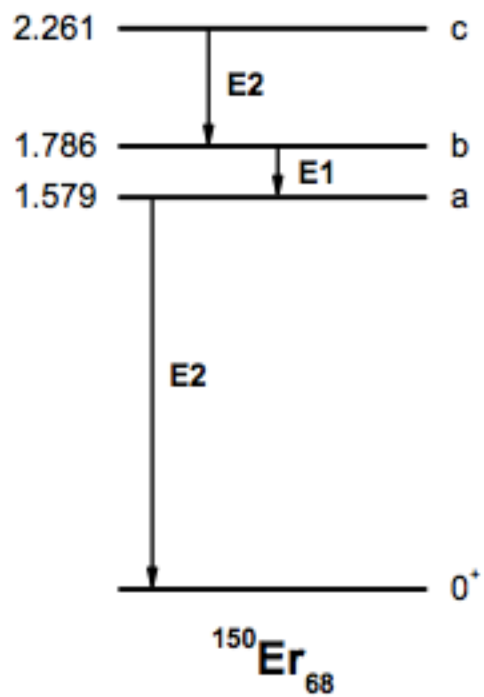
$$\pi_i \pi_f = +1 \text{ for } M1, E2, M3, E4 \ldots, \qquad \pi_O = +1$$

$$\pi_i \pi_f = -1 \text{ for } E1, M2, E3, M4 \qquad \pi_O = -1$$

the elements of the operators for $\pi_O = (-1)^\lambda$ an be classified according to their transformation under parity change:

$$POP^{-1} = \pi_O O.$$

$$\pi_O = (-1)^\lambda$$



2.261
c
E2
1.786
b
E1
1.579
a
E2
0+
150Er68

Transition probability

$$B(i \to f) = \sum_{\mu M_f} |\langle J_f M_f | \mathcal{O}(\sigma\lambda\mu) | J_i M_i \rangle|^2 = \frac{\left| \langle J_f || \mathcal{O}(\lambda) || J_i \rangle \right|^2}{2J_i + 1}$$

The total rate for a specific set of states and a given operator is given by:

$$T_{i,f,\lambda} = \left(\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \right) \frac{k^{2\lambda+1}}{\hbar} B(i \to f) \tag{7.25}$$

where k is the wave-number for the electromagnetic transition of energy E_γ given by:

$$k = \frac{E_\gamma}{\hbar c} = \frac{E_\gamma}{197\text{MeV fm}} \tag{7.26}$$

$$
\begin{aligned}
T(E1) &= 1.59 \times 10^{15} E^3 B(E1) \\
T(E2) &= 1.22 \times 10^{9} E^5 B(E2) \\
T(E3) &= 5.67 \times 10^{2} E^7 B(E3) \\
T(E4) &= 1.69 \times 10^{-4} E^9 B(E4)
\end{aligned}
$$

$$
\begin{aligned}
T(M1) &= 1.76 \times 10^{13} E^3 B(M1) \\
T(M2) &= 1.35 \times 10^{7} E^5 B(M2) \\
T(M3) &= 6.28 \times 10^{0} E^7 B(M3) \\
T(M4) &= 1.87 \times 10^{-6} E^9 B(M4)
\end{aligned}
$$

Operators and transition rates

The interaction of the electromagnetic field with the nucleons can be expressed in terms of a sum of electric and magnetic multipole operators with tensor rank λ

$$\mathcal{O} = \sum_{\lambda,\mu} [\mathcal{O}(E\lambda)_\mu + \mathcal{O}(M\lambda)_\mu].$$

The electric transition operator given by

$$O(E\lambda) = r^\lambda \, Y^\lambda_\mu(\hat{r}) \; e_{t_z} e,$$

The reduced transition probability is defined by:

$$B(i \to f) = \frac{|< J_f || \mathcal{O}(\lambda) || J_i >|^2}{(2J_i + 1)}.$$

$$|< J_f || \mathcal{O}(\lambda) || J_i >|^2 = |< J_i || \mathcal{O}(\lambda) || J_f >|^2 \; .$$

The most probable types of transitions are $E1$, $E2$

The $E1$ transition

$$O(E1) = r\ Y^{(1)}_{\mu}(\hat{r})\ e_{t_z} e = \sqrt{\frac{3}{4\pi}}\ \vec{r}\, e_{t_z} e,$$

The $E2$ transition operator

$$O(E2) = r^2\ Y^{(2)}_{\mu}(\hat{r})\ e_{t_z} e,$$

were Y^{λ}_{μ} are the spherical harmonics. Gamma transitions with λ=0 are forbidden because the photon must carry off at least one unit of angular momentum. The e_{t_z} are the electric charges for the proton and neutron in units of e. For the free-nucleon charge we would take $e_p = 1$ and $e_n = 0$, for the proton and neutron, respectively.

The magnetic transition operator is given by:

$$O(M\lambda) = \left[\vec{\ell}\,\frac{2g^{\ell}_{t_z}}{(\lambda+1)} + \vec{s}\,g^{s}_{t_z}\right]\,\vec{\nabla}[r^{\lambda}Y^{\lambda}_{\mu}(\hat{r})]\,\mu_N$$

$$= \sqrt{\lambda(2\lambda+1)}\left[[Y^{\lambda-1}\otimes\vec{\ell}]^{\lambda}_{\mu}\,\frac{2g^{\ell}_{t_z}}{(\lambda+1)} + [Y^{\lambda-1}\otimes\vec{s}]^{\lambda}_{\mu}\,g^{s}_{t_z}\right] r^{\lambda-1}\,\mu_N,$$

where μ_N is the nuclear magneton,

$$\mu_N = \frac{e\hbar}{2m_pc} = 0.105\,e\mathrm{fm},$$

and where m_p is the mass of the proton. The g-factors $g^{\ell}_{t_z}$ and $g^{s}_{t_z}$ are the orbital and spin g-factors for the proton and neutron, respectively. The free-nucleon values for the g-factors are $g^{\ell}_p = 1$, $g^{\ell}_n = 0$, $g^{s}_p = 5.586$ and $g^{s}_n = -3.826$. We may use effective

The $M1$ transition operator

$$O(M1) = \sqrt{\frac{3}{4\pi}}\,[\vec{\ell}\,g^{\ell}_{t_z} + \vec{s}\,g^{s}_{t_z}]\,\mu_N .$$

Electromagnetic moments

Electromagnetic moments have the general form:

$$\mathcal{M}_{em} = < \Psi, J, M = J \mid \sum_k T^{\lambda}_{\mu=0}(k) \mid \Psi, J, M = J >,$$

where T^{λ}_{μ} is a one-body tensor operator of rank λ associated with the interaction of the nucleus with the multipole components of the electromagnetic field. This is a diagonal matrix element, and by definition we take M to have its maximum value $M = J$. The matrix elements with other M values are related to $\mathcal{M}_{em}$ by the Wigner-Eckart theorem:

$$< \Psi, J, M \mid \sum_k T^{\lambda}_{\mu=0}(k) \mid \Psi, J, M >= \frac{< J, M, \lambda, \mu = 0 \mid J, M >}{< J, J, \lambda, \mu = 0 \mid J, J >} \mathcal{M}_{em}.$$

For a given J value, the allowed values of the λ are determined by the triangle condition, $\Delta(J, \lambda, J)$. In particular, $\lambda_{max} = 2J$, and for $J = 0$ only $\lambda = 0$ is allowed.

Nuclear matrix elements

Electromagnetic transitions and moments can be expressed as a sum over one-body transition densities (OBTD) times single-particle matrix elements

$$< f||\mathcal{O}(\lambda)||i >= \sum_{k_\alpha k_\beta} \mathrm{OBTD}(f i k_\alpha k_\beta \lambda) < k_\alpha||O(\lambda)||k_\beta >,$$

The labels i and f are a short-hand notation for the initial and final state quantum numbers $(n\omega_i J_i)$ and $(n\omega_f J_f)$, respectively. Thus the problem is divided into two parts, one involving the nuclear structure dependent one-body transition densities OBTD, and the other involving the reduced single-particle matrix elements (SPME).

The SPME for $E\lambda$ operator

$$< k_a||O(E\lambda)||k_b >= (-1)^{j_a+1/2}\frac{[1+(-1)^{\ell_a+\lambda+\ell_b}]}{2}$$

$$\times\sqrt{\frac{(2j_a+1)(2\lambda+1)(2j_b+1)}{4\pi}}\begin{pmatrix} j_a & \lambda & j_b \\ 1/2 & 0 & -1/2 \end{pmatrix} < k_a|r^\lambda \mid k_b > e_{t_z} e.$$

The SPME for the spin part of the magnetic operator

$$< k_a||O(M\lambda, s)||k_b > =$$

$$= \sqrt{\lambda(2\lambda+1)} \; < j_a||[Y^{\lambda-1} \otimes \vec{s}\,]^{\lambda}||j_b >< k_a|r^{\lambda-1} \mid k_b > \; g^s_{t_z} \; \mu_N$$

$$= \sqrt{\lambda(2\lambda+1)} \sqrt{(2j_a+1)(2j_b+1)(2\lambda+1)} \left\{ \begin{array}{ccc} \ell_a & 1/2 & j_a \\ \ell_b & 1/2 & j_b \\ \lambda-1 & 1 & \lambda \end{array} \right\}$$

$$\times \; < \ell_a||Y^{\lambda-1}||\ell_b >< s||\vec{s}||s >< k_a|r^{\lambda-1} \mid k_b > \; g^s_{t_z} \; \mu_N \, ,$$

where

$$< s||\vec{s}||s > = \sqrt{3/2} \, .$$

The SPME for the orbital part of the magnetic operator

$$< k_a||O(M\lambda,\ell)||k_b >=$$

$$= \frac{\sqrt{\lambda(2\lambda+1)}}{\lambda+1} < j_a||[Y^{\lambda-1}\otimes\vec{\ell}]^\lambda||j_b >< k_a|r^{\lambda-1} \mid k_b > \; g^\ell_{t_z} \; \mu_N$$

$$= \frac{\sqrt{\lambda(2\lambda+1)}}{\lambda+1}(-1)^{\ell_a+1/2+j_b+\lambda}\sqrt{(2j_a+1)(2j_b+1)}$$

$$\times \left\{ \begin{array}{ccc} \ell_a & \ell_b & \lambda \\ j_b & j_a & 1/2 \end{array} \right\} < \ell_a||[Y^{\lambda-1}\otimes\vec{\ell}]^\lambda||\ell_b >< k_a|r^{\lambda-1} \mid k_b > \; g^\ell_{t_z} \; \mu_N,$$

where

$$< \ell_a||[Y^{\lambda-1}\otimes\vec{\ell}]^\lambda||\ell_b >= (-1)^{\lambda+\ell_a+\ell_b}\sqrt{(2\lambda+1)\ell_b(\ell_b+1)(2\ell_b+1)}$$

$$\times \left\{ \begin{array}{ccc} \lambda-1 & 1 & \lambda \\ \ell_b & \ell_a & \ell_b \end{array} \right\} < \ell_a||Y^{\lambda-1}||\ell_b >,$$

with

$$< \ell_a||Y^{\lambda-1}||\ell_b >= (-1)^{\ell_a}\sqrt{\frac{(2\ell_a+1)(2\ell_b+1)(2\lambda-1)}{4\pi}} \left(\begin{array}{ccc} \ell_a & \lambda-1 & \ell_b \\ 0 & 0 & 0 \end{array} \right).$$

For the $M1$ operator

$$< k_a||O(M1,s)||k_b > = \sqrt{\frac{3}{4\pi}} < j_a||\vec{s}||j_b > \delta_{n_a,n_b}\, g^s_{t_z}\, \mu_N$$

$$= \sqrt{\frac{3}{4\pi}} (-1)^{\ell_a+j_a+3/2} \sqrt{(2j_a+1)(2j_b+1)} \left\{ \begin{array}{ccc} 1/2 & 1/2 & 1 \\ j_b & j_a & \ell_a \end{array} \right\}$$

$$\times < s||\vec{s}||s > \delta_{\ell_a,\ell_b} \delta_{n_a,n_b}\, g^s_{t_z}\, \mu_N,$$

where

$$< s||\vec{s}||s > = \sqrt{3/2},$$

and

$$< k_a||O(M1,\ell)||k_b > = \sqrt{\frac{3}{4\pi}} < j_a||\vec{\ell}||j_b > \delta_{n_a,n_b}\, g^\ell_{t_z}\, \mu_N$$

$$= \sqrt{\frac{3}{4\pi}} (-1)^{\ell_a+j_b+3/2} \sqrt{(2j_a+1)(2j_b+1)} \left\{ \begin{array}{ccc} \ell_a & \ell_b & 1 \\ j_b & j_a & 1/2 \end{array} \right\}$$

$$\times < \ell_a||\vec{\ell}||\ell_b > \delta_{n_a,n_b}\, g^\ell_{t_z}\, \mu_N,$$

where

$$< \ell_a||\vec{\ell}||\ell_b > = \delta_{\ell_a,\ell_b} \sqrt{\ell_a(\ell_a+1)(2\ell_a+1)}.$$

Thus the M1 operator can connect only a very limited set of orbits, namely those which have the same n and ℓ values.

Applications to Closed shell plus one particle

single-particle matrix elements

For a closed shell plus one particle, the only term contributing to the sum (for λ >0) comes from the transition between two specific particle states

$$< J_f = j_f || O(\lambda) || J_i = j_i > = < k_f || O(\lambda) || k_i >$$

The reduced nuclear matrix elements can be expressed as a sum over one-body transition densities times single-particle matrix elements. For a closed shell plus one particle one finds that OBTD=1.

The reduced transition probability for this cases is:

$$B(\lambda) = \frac{|< k_f || O(\lambda) || k_i >|^2}{(2j_i + 1)}.$$

Many nucleons in a single shell

For a closed shell plus n particles in a single state k these expressions (for λ >0) reduce to:

$$< k^n, \omega_f, J_f || \mathcal{O}(\lambda) || k^n, \omega_i, J_i >= \mathrm{OBTD}(fik\lambda) < k || O(\lambda) || k >,$$

$$\mathrm{OBTD}(fik\lambda) = n\sqrt{(2J_f+1)(2J_i+1)} \sum_{\omega J} (-1)^{J_f+\lambda+J+j}$$

$$\times \left\{ \begin{array}{ccc} J_i & f & \lambda \\ j & j & J \end{array} \right\} < j^n \omega_f J_f \, |\} j^{n-1} \omega J >< j^n \omega_i J_i \, |\} j^{n-1} \omega J > .$$

For $n = 2$,

$$\mathrm{OBTD}(fik\lambda) = (-1)^{J_f+\lambda+1} n\sqrt{(2J_f+1)(2J_i+1)} \left\{ \begin{array}{ccc} J_i & J_f & \lambda \\ j & j & j \end{array} \right\}$$

The reduced transition rate becomes:

$$B(\lambda) = n^2(2J_f+1)\left\{\begin{array}{ccc} J_i & J_f & \lambda \\ j & j & j \end{array}\right\}^2 |< k||O(\lambda)||k >|^2 .$$

$$= n^2(2J_f+1)(2j+1)\left\{\begin{array}{ccc} J_i & J_f & \lambda \\ j & j & j \end{array}\right\}^2 \frac{|< k||O(\lambda)||k >|^2}{(2j+1)}$$

$$= C(J_i, J_f, j)\frac{|< k||O(\lambda)||k >|^2}{(2j+1)},$$

transition	$(3/2)^2$	$(5/2)^2$	$(7/2)^2$	$(9/2)^2$	$(11/2)^2$	$(13/2)^2$
$2 \rightarrow 0$	0.800	0.914	0.952	0.970	0.979	0.985
$4 \rightarrow 2$		0.630	0.950	1.114	1.207	1.265
$6 \rightarrow 4$			0.433	0.771	0.990	1.132
$8 \rightarrow 6$				0.308	0.612	0.841
$10 \rightarrow 8$					0.229	0.491
$12 \rightarrow 10$						0.177

Electric quadrupole moments

The electric quadrupole operator is defined to be

$$T^{\lambda=2}_{\mu=0} = \hat{Q} = (3z^2 - r^2) e_{t_z} e = \sqrt{\frac{16\pi}{5}}\; r^2\, Y^2_0(\hat{r})\; e_{t_z} e,$$

For one-particle outside of a closed-shell configuration,

$$\mathcal{M}(Ci)_{em} = < i \mid T^{\lambda}_{\mu=0} \mid i > = < j, m = j \mid T^{\lambda}_{\mu=0} \mid j, m = j >$$

the single-particle quadrupole moment in the state $j = \ell + \frac{1}{2}$ is:

$$\frac{Q}{e} = \sqrt{\frac{16\pi}{5}} < Y^{\ell}_{\ell} \mid Y^2_0 \mid Y^{\ell}_{\ell} > < r^2 > e_{t_z}.$$

where

$$< r^2 > = \int f^2(r)\, r^4 dr = \int R^2_{n_r,\ell_j}\, r^2 dr.$$

$$< Y^{\ell}_{\ell} \mid Y^2_0 \mid Y^{\ell}_{\ell} > = (-1)^{\ell} \begin{pmatrix} \ell & 2 & \ell \\ -\ell & 0 & \ell \end{pmatrix} (2\ell + 1) \sqrt{\frac{5}{4\pi}} \begin{pmatrix} \ell & 2 & \ell \\ 0 & 0 & 0 \end{pmatrix}$$

$$= -\sqrt{\frac{5}{4\pi}} \left(\frac{\ell}{2\ell + 3} \right).$$

Thus, the single-particle quadrupole moment in the state $j = \ell + \frac{1}{2}$ simplifies to:

$$\frac{Q(i)}{e} = -\left(\frac{2\ell}{2\ell + 3}\right) < r^2 > e_{t_z} .$$

The radial integrals can be evaluated with the chosen radial wave functions such as harmonic-oscillator or Woods-Saxon.

For the closed shell we have Jπ = 0+ and the EM moment is zero unless λ=0.

For the single-particle configuration $\mid Ci >$, the extra nucleon will go into one of the empty states (n_r, ℓ, j, m) above the fermi surface. There are $(2j+1)$ allowed M values from $-j$ to j. Thus the total angular momentum is $J = j$, and the parity is $(-1)^{\ell}$. For the moments we need the $M-$state with $M = m = j$.

For the single-hole configuration, $\mid Ci^{-1} >$, the nucleon will be removed from one of the filled states (n_r, ℓ, j, m') below the fermi surface. The M value of the state is:

$$M = \sum_{m \neq m'} m = -m'.$$

There are $(2j+1)$ values for M from $-j$ to j. Thus the total angular momentum is $J = j$, and the parity parity is $(-1)^{\ell}$. If we want to have a many-body state with $M = J = j$, then the nucleon must be removed from the single-particle state with $m' = -j$.

For the one-hole configuration

$$\mathcal{M}(Ci^{-1})_{em} = - < i \mid T^{\lambda}_{\mu=0} \mid i > = - < j, m = -j \mid T^{\lambda}_{\mu=0} \mid j, m = -j >$$

$$= (-1)^{\lambda+1} < j, m = j \mid T^{\lambda}_{\mu=0} \mid j, m = j > .$$

This is because we have (from the Wigner-Eckart theorem)

$$< j, m = -j \mid T^{\lambda}_{\mu=0} \mid j, m = -j > =$$

$$= \frac{< j, -j, \lambda, \mu = 0 \mid j, -j >}{< j, j, \lambda, \mu = 0 \mid j, j >} < j, m = j \mid T^{\lambda}_{\mu=0} \mid j, m = j >$$

$$= (-1)^{\lambda} < j, m = j \mid T^{\lambda}_{\mu=0} \mid j, m = j > .$$

The single-hole quadrupole moment in the state j = ℓ + 1/2 is:

$$\frac{Q(i^{-1})}{e} = \left(\frac{2\ell}{2\ell + 3} \right) < r^2 > e_{t_z} .$$

For the general case which includes both j = ℓ + 1/2 and j = ℓ− 1/2 one obtains

$$\frac{Q(i)}{e} = -\left(\frac{2j-1}{2j+2}\right) < r^2 > e_{t_z}.$$

$$\frac{Q(i^{-1})}{e} = \left(\frac{2j-1}{2j+2}\right) < r^2 > e_{t_z}.$$

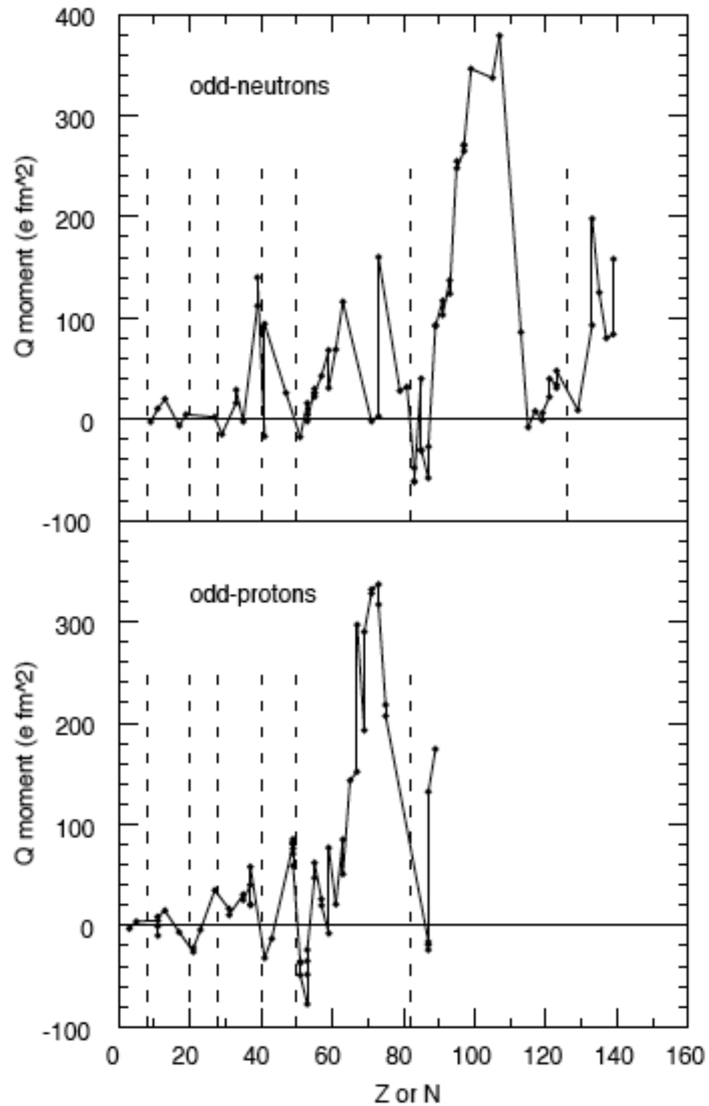


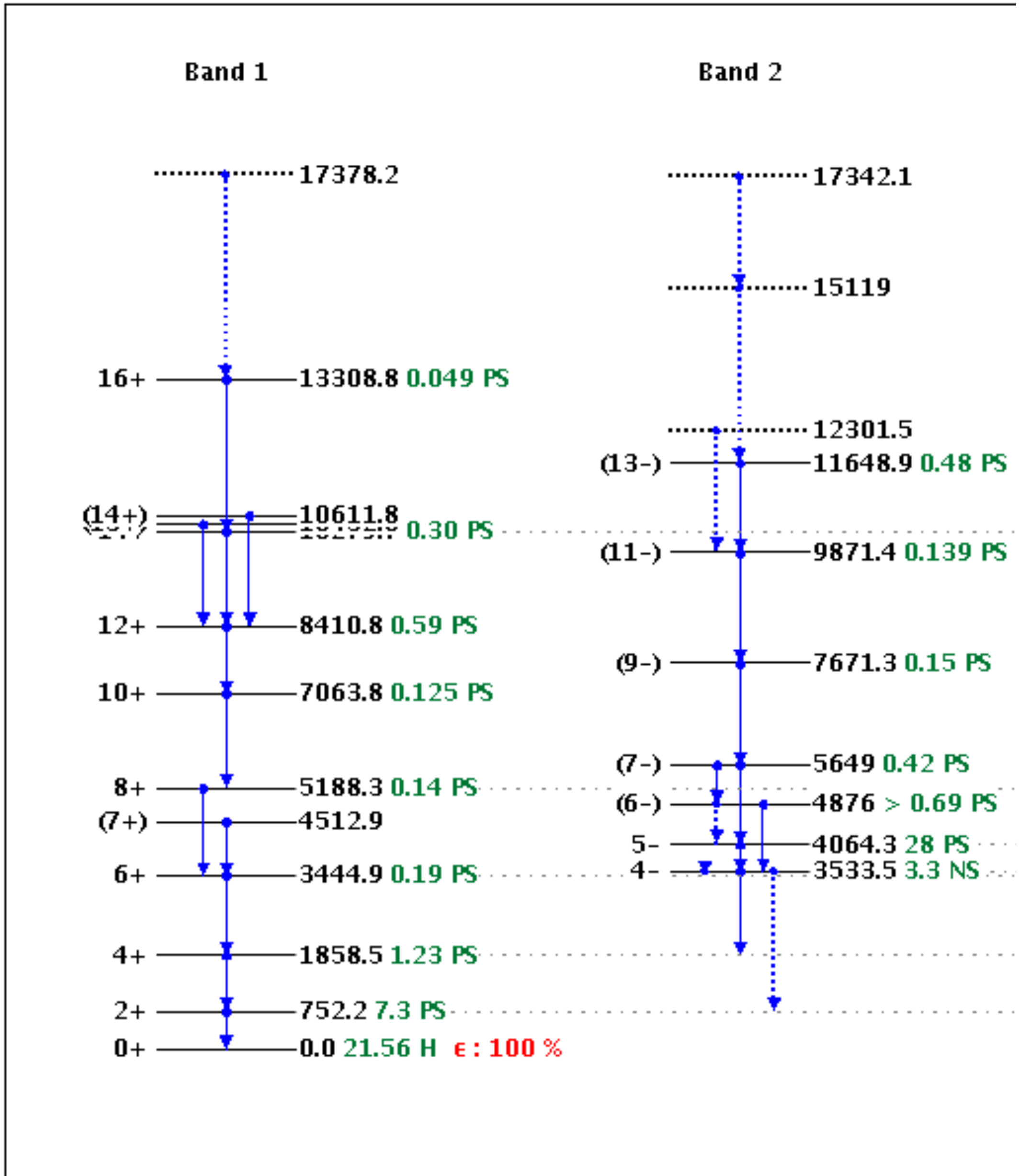
Figure 2: Experimental quadrupole moments for the ground states of odd-even nuclei. The dashed lines show the magic numbers 8, 20, 28, 40, 50, 82 and 126.

In the single-particle model we would expect the **Q moments for the neutrons to be zero** or at least small compared to protons. **But one observes that the Q moments for odd-proton and odd-neutron nuclei are about the same.** These deviations from the single-particle model are signatures of configuration mixing.

✧Near the magic numbers this can be understood in terms of the interaction between the valence nucleon and the core nucleons producing a "core-polatization" that can be modeled in terms of an effective change for protons and neutrons.

✧Away from the closed shell the interaction between valence nucleons results in a collective (coherent) motion between many nucleons that is qualitatively understood in the deformed model for nuclei.

^{48}Cr


Band 1
Band 2
17378.2
17342.1
15119
16+
13308.8 0.049 PS
12301.5
(13-)
11648.9 0.48 PS
(14+)
10611.8
0.30 PS
(11-)
9871.4 0.139 PS
12+
8410.8 0.59 PS
10+
7063.8 0.125 PS
(9-)
7671.3 0.15 PS
(7-)
5649 0.42 PS
8+
5188.3 0.14 PS
(6-)
4876 > 0.69 PS
(7+)
4512.9
5-
4064.3 28 PS
4-
3533.5 3.3 NS
6+
3444.9 0.19 PS
4+
1858.5 1.23 PS
2+
752.2 7.3 PS
0+
0.0 21.56 H ϵ : 100 %

Magnetic moments

The magnetic moment operator is defined to be

$$T^{\lambda=1}_{\mu=0} = \hat{\mu}_z[\ell_z\, g^{\ell}_{t_z} + s_z\, g^{s}_{t_z}]\mu_N,$$

where $g^{\ell}_{t_z}$ and $g^{s}_{t_z}$ are the orbital and spin g-factors for the proton ($t_z = -\frac{1}{2}$) and neutron ($t_z = \frac{1}{2}$). The free-nucleon values for the g-factors are $g^{\ell}_p = 1$, $g^{\ell}_n = 0$, $g^{s}_p = 5.586$ and $g^{s}_n = -3.826$. The values of the magnetic moments are conventionally taken to be in units of the nuclear magneton,

$$\mu_N = \frac{e\hbar}{2m_p c} = 0.105\, e\,\mathrm{fm}$$

where m_p is the mass of the proton.

for $m = j$ and $j = \ell + \frac{1}{2}$ we have

$$\frac{\mu(j = \ell + \frac{1}{2})}{\mu_N} = \ell\, g^{\ell}_{t_z} + \frac{1}{2}\, g^{s}_{t_z},$$

for $m = j$ and $j = \ell - \frac{1}{2}$ we have

$$\frac{\mu(j=\ell-\frac{1}{2})}{\mu_N} = \left[\frac{1}{(2\ell+1)}(\ell-1) + \frac{2\ell}{(2\ell+1)}\ell\right] g^{\ell}_{t_z} + \left[\frac{1}{2\ell+1}\left(\frac{1}{2}\right) + \frac{2\ell}{2\ell+1}\left(-\frac{1}{2}\right)\right] g^{s}_{t_z}$$
$$= \frac{(2\ell-1)(\ell+1)}{(2\ell+1)} g^{\ell}_{t_z} - \frac{(2\ell-1)}{(4\ell+2)} g^{s}_{t_z}.$$

The expressions can be written in a compact way as

$$\frac{\mu}{\mu_N} = j\left[g^{\ell}_{t_z} \pm \frac{g^{s}_{t_z} - g^{\ell}_{t_z}}{2\ell+1}\right],$$

there the $\pm$ sign goes with $j \pm \frac{1}{2}$. The g-factor is defined as $\mu/(\mu_N J)$ which for the single-particle case gives:

$$g = \left[g^{\ell}_{t_z} \pm \frac{g^{s}_{t_z} - g^{\ell}_{t_z}}{2\ell+1}\right]$$

Moments in terms of electromagnetic operators

The operator for electromagnetic moment can be expressed in terms of the electromagnetic transition operators.

$$\mu = \sqrt{\frac{4\pi}{3}} < J, M = J \mid \mathcal{O}(M1) \mid J, M = J >$$

$$= \sqrt{\frac{4\pi}{3}} \left(\begin{array}{ccc} J & 1 & J \\ -J & 0 & J \end{array} \right) < J||\mathcal{O}(M1)||J >,$$

$$Q = \sqrt{\frac{16\pi}{5}} < J, M = J \mid \mathcal{O}(E2) \mid J, M = J >$$

$$= \sqrt{\frac{16\pi}{5}} \left(\begin{array}{ccc} J & 2 & J \\ -J & 0 & J \end{array} \right) < J||\mathcal{O}(E2)||J > .$$

Single-Particle Transition (Weisskopf Estimate)

Electromagnetic transition rates show a rather strong dependence on the transition energy. This dependence increases with the multiplicity of the transition. The Weisskopf estimates give a rough-idea about the expected magnitudes of the radiation widths. These estimates for radiation of multipolarity 2L are based on a very simple model with the assumptions

(i) The nucleus consists of an inert core plus one active particle.

(ii) The transition takes place between states $j_i = L \pm \frac{1}{2}$ and $j_f = \frac{1}{2}$.

(iii) The radial parts of the initial- and final-state wave functions are both given by $u(r)$ = constant for $r \leqslant R$ and $u(r) = 0$ for $r > R$, where R denotes the nuclear radius.

$$B(E\lambda; I_i \to I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} \;\; e^2 (fm)^{2\lambda}$$

$$B(M\lambda; I_i \to I_{gs}) = \frac{10}{\pi}(1.2)^{2\lambda-2}\left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)/3} \;\; \mu_N^2 (fm)^{2\lambda-2}$$

For the first few values of λ, the Weisskopf estimates are

$$B(E1; I_i \rightarrow I_{gs}) = 6.446\ 10^{-4}\ A^{2/3}\ \ e^2(barn)$$

$$B(E2; I_i \rightarrow I_{gs}) = 5.940\ 10^{-6}\ A^{4/3}\ \ e^2(barn)^2$$

$$B(E3; I_i \rightarrow I_{gs}) = 5.940\ 10^{-8}\ A^{2}\ \ e^2(barn)^3$$

$$B(E4; I_i \rightarrow I_{gs}) = 6.285\ 10^{-10}\ A^{8/3}\ \ e^2(barn)^4$$

$$B(M1; I_i \rightarrow I_{gs}) = 1.790\ \ (\frac{e\hbar}{2Mc})^2$$

A barn is defined as 10^{-28} m^2 (100 fm^2)

The lowest allowed multipolarity in the decay rate dominates over the next higher one (when more than one is allowed) by several orders of magnitude. The most common types of transitions are electric dipole (E1), magnetic dipole (M1), and electric quadrupole (E2)

10.7 Deuteron Structure

Other important information on the structure of the deuteron comes from the values of the magnetic moment μ and quadrupole moment Q:

$$\mu = 0.8574\mu_N; \quad Q = 0.2857e\ \mathrm{fm}^2 \tag{6.88}$$

Since $Q \neq 0$, the deuteron cannot be pure $l = 0$. But generally $l = 0$ is energetically favored for a central potential. Therefore, we write the deuteron wave function as a linear combination of S- and D- waves

$$\begin{aligned} \psi(\boldsymbol{r} &= a\psi_{^3S_1}(\boldsymbol{r}) + b\psi_{^3D_1}(\boldsymbol{r}) \\ &= \left[aR_0(r)\mathcal{Y}^1_{011} + bR_2(r)\mathcal{Y}^1_{211}\right]\psi^T_{00} \end{aligned} \tag{6.89}$$

$$\begin{aligned} \mathcal{Y}^1_{011} &= Y_{00}\chi_{00} \\ \mathcal{Y}^1_{211} &= \sum_M \langle 1(1-M)2M|11\rangle Y_{2M}\chi_{1,1-M} \\ &= \sqrt{\frac{1}{10}}Y_{20}\chi_{1,1} - \sqrt{\frac{3}{10}}Y_{21}\chi_{1,0} + \sqrt{\frac{3}{5}}Y_{22}\chi_{1,-1} \end{aligned}$$

The magnetic moment

As mentioned before, the free-nucleon values for the g-factors are $g_p^l = 1$, $g_n^l = 0$, $g_p^s = 5.586$ and $g_n^s = -3.826$. The magnetic moment operator can be rewritten as

$$\mu = \mu_N \sum_i (g_s s_{zi} + g_l l_{zi}) \tag{7.66}$$

where $g_s = 4.706\tau_i + 0.88$, where the first term is isovector, and the second term is isoscalar. $g_l = (\tau_i + 1)/2$. Since the deuteron is an isoscalar particle with $T = 0$, only the isoscalar magnetic moment operator contributes to μ. Then, the above equation becomes,

$$\begin{aligned} \mu &= \mu_N \sum_i (0.88 s_{zi} + \frac{1}{2} l_{zi}) \\ &= \mu_N \sum_{i=1}^{2} \left[0.88 \langle s_i^z \rangle_{M=1} + \frac{1}{2} \langle l_i^z \rangle_{M=1} \right] \\ &= \mu_N \left[0.88 \langle S^z \rangle + \frac{1}{2} \langle L^z \rangle \right] \end{aligned}$$

Let us now calculate the matrix element of S_z

$$\begin{aligned}
\langle \mathcal{Y}^1_{011}|S_z|\mathcal{Y}^1_{011}\rangle &= 1 \\
\langle \mathcal{Y}^1_{211}|S_z|\mathcal{Y}^1_{011}\rangle &= 0 \\
\langle \mathcal{Y}^1_{211}|S_z|\mathcal{Y}^1_{211}\rangle &= \sum_{M_S} \left|\langle 2(1-M_S)1M_S|11\rangle\right|^2 = -1/2
\end{aligned}$$

Thus, for pure $l=0$ or $l=2$ states we would have the values $\mu = 0.88\mu_N$, $0.31\mu_N$. More generally we obtain the relation

$$\mu = \left[a^2(0.88) + b^2(0.31)\right]\mu_0 = (0.88 - 0.57b^2)\mu_0$$

$$\mu_D = 0.857\mu_N$$

$b^2 = 0.04$

Quadrupole Moment

$$Q = \sqrt{\frac{16\pi}{5}} \langle J(M = J) | \hat{Q}_{20} | J(M = J) \rangle$$

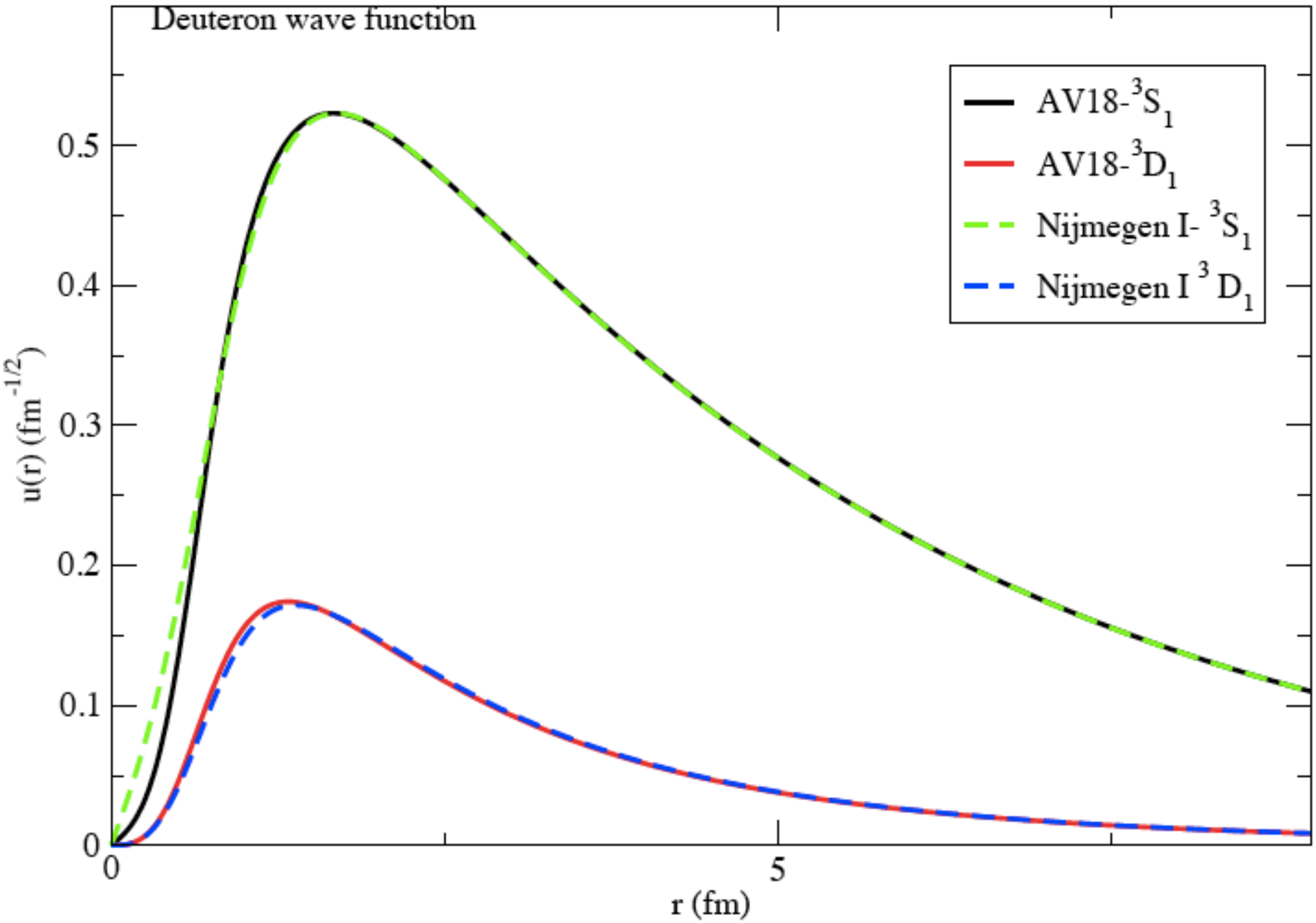
The quadrupole moment of the deuteron is calculated to be

$$\begin{aligned} Q =& e\sqrt{\frac{16\pi}{5}} \int \psi^*_{J=M=1}(\boldsymbol{r}) \left[\sum_{i=1}^{2} e_{t_z} r_i^2 Y_{20}(\hat{r}_i) \right] \psi_{J=M=1}(\boldsymbol{r}) \mathrm{d}^3\boldsymbol{r} \\ =& e\sqrt{\frac{16\pi}{5}} \int \psi^*_{J=M=1}(\boldsymbol{r}) \frac{r^2}{4} Y_{20}(\hat{r}) \psi_{J=M=1}(\boldsymbol{r}) \mathrm{d}^3\boldsymbol{r}, \end{aligned} \tag{7.71}$$

where we have used the fact that for each nucleon the distance from the center of mass is only half the distance between them, $r_i = r/2$. Inserting the wave function

$$\begin{aligned} Q =& e\sqrt{\frac{\pi}{5}} \Big\{ |a^2| \int r^2 R_0(r)^2 \mathrm{d}r \int Y^*_{00} Y_{20} Y_{00} \mathrm{d}\Omega + 2Re(ab^*) \int r^2 R_0(r) R_2(r) \mathrm{d}r \\ & \times \sum_M \langle 1(1-M) 2M | 11 \rangle \int Y^*_{00} Y_{20} Y_{2M} \mathrm{d}\Omega \\ & + |b|^2 \int r^2 R_2^2(r) \mathrm{d}r \times \sum_M \left| \langle 1(1-M) 2M | 11 \rangle \right|^2 \int Y^*_{2M} Y_{20} Y_{2M} \mathrm{d}\Omega \Big\} \end{aligned} \tag{7.72}$$

R0 (r) and R2 (r)



Deuteron wave function
AV18-$^3S_1$
AV18-$^3D_1$
Nijmegen I- $^3S_1$
Nijmegen I $^3D_1$
u(r) (fm$^{-1/2}$)
r (fm)
0.5
0.4
0.3
0.2
0.1
0
0
5

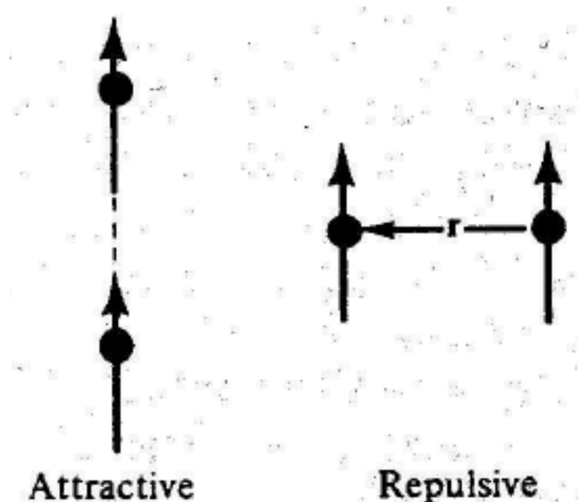
After evaluating the angular integrals and putting in the CG coefficients

$$Q = e\left\{\frac{\sqrt{2}}{10}Re(ab^*)\int r^4 R_0 R_2 - \frac{|b|^2}{20}\int r^4 R_2^2\right\}$$

In case (a)$\boldsymbol{\sigma}_1 \cdot \hat{r} = \boldsymbol{\sigma}_2 \cdot \hat{r} = 1$, so we have $S_{12} = +2$ for this geometrical arrangement. This is a prolate configuration so we expect $Q > 0$ for case (a). In case (b) we have $\boldsymbol{\sigma}_1 \cdot \hat{r} = \boldsymbol{\sigma}_2 \cdot \hat{r} = 0$ so $S_{12} = -1$ and the oblate shape relative to the $\hat{z}$ axis would imply $Q < 0$.

$$Q \simeq e\frac{0.2\sqrt{2}}{10}\int r^4 R_0 R_2 \mathrm{d}r = 0.286e\ \mathrm{fm}^2$$

$$Q = 0.2857e\ \mathrm{fm}^2$$



Tensor force