

# EP2200 Queueing theory and teletraffic systems

## Summary

Viktorija Fodor  
KTH EES/LCN

# Reminders

- Project (contact Viktoria with questions)
  - Submit through the course web: VT2017 / Assignments
- Registration for the exam (if question: [stex@kth.se](mailto:stex@kth.se))
- Office hours before the exam – check the web
- Exam March 14
- Course evaluation form on the web – please fill it in!

# Course content

- Markov-processes – tool to analyze queuing systems
- Markovian queuing systems (M/M/\*/\*/\*)
- Semi-Markovian queuing systems (M\*/1)
- Queuing networks
  
- Knowledge on different levels, e.g.,
  - M/M/1
    - derive the waiting time distribution
    - analyze similar systems
  - M/G/1
    - apply the P-K transform equations for different service time distributions

# Markov-process

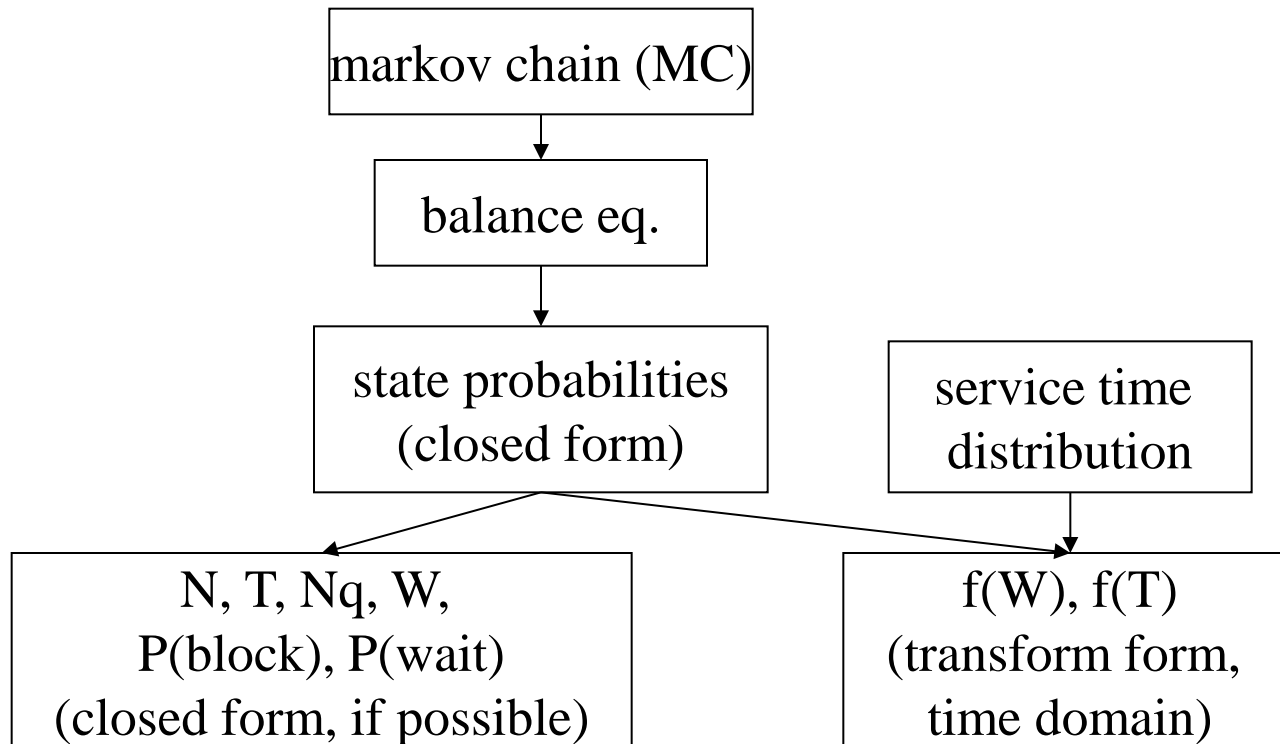
- Definition of continuous time Markov chain and the memoryless property
- Continuous time Markov-chains
  - state probability distribution in steady state – matrix equation
  - balance equations – derivation from the matrix equation
  - application for continuous time stochastic systems
- What “state probability in steady state” means (for ergodic systems) ?
  - statistical average: consider the process at arbitrary point of time, what is the probability that the process is in state  $k$
  - time average: consider one process for a long time, what fraction of time the process is in state  $k$
- Poisson process and B-D process as special cases

# Queuing systems

- General results
  - Kendall notation – application
  - Little's result – no proof – but application
  - Definitions of offered load and utilization
- Markovian queuing systems
- Semi-markovian queuing systems
- Queuing networks

# Markovian queuing systems – M/M/\*/\*/\*

- Can be represented with continuous time MC
  - state: number of customers in the system
- Performance in steady state



# Markovian queuing systems – M/M/\*/\*/\*

| server = m<br>(m=1 spec. case) | System capacity  |  |  |
|--------------------------------|--|--|--|
|                                | infinite   | S  | = servers  |
| Infinite population            | M/M/m<br>•MC, $p_k$<br>•P(wait) -Erlang-C<br>–Erlang table<br>•L( $f_w(t)$ ) - derive<br>•F <sub>w</sub> (t) - apply | M/M/m/S<br>(M/M/1/S)<br>•MC, $p_k$<br>•P(blocking)     | M/M/m/m<br>•MC, $p_k$<br>•P(block) -Erlang-B<br>–Erlang table<br>–general result!                    |
| Finite population              | Not covered, you have to be able to do it on your own.   | Not covered, you have to be able to do it on your own. | M/M/m/m/C<br>Engset loss system<br>•MC, $p_k$<br>•time blocking and call blocking<br>•effective load |

Time blocking ≠ call blocking

# Markovian queuing systems – M/M/\*/\*/\*

- Time blocking: fraction of time the system spends in blocking state =  $P(\text{the system is in blocking state})$
- Call blocking: ratio of calls arriving when the system is in blocking state
  - Equal to time blocking for Poisson arrivals with state independent intensity – due to the PASTA property
  - Not equal to time blocking in other cases – e.g., in the case of finite population, when the arrival intensity is state dependent.



# Semi-Markovian queuing systems M/Er/1, M/Hr/1, M/G/1, vacation, priority

## M/G/1 – priority, vac.

- derive, apply mean forms

## M/G/1

- derive, apply mean forms
- apply transform eq.

## M/Er/1, M/Hr/1

- Er, Hr –  $E[x]$ ,  $C_x^2$
- MC,  $p_k$  – for simple cases

## M/M/1

- MC,  $p_k$
- $L(fw(t))$  - derive
- $Fw(t)$  - apply

# Markovian queuing networks

- Tandem queues
  - output process of M/M/1 - proof
  - product form solution – reasoning
- Open queuing networks
  - independence of queues - reasoning
  - Application

## Continuation

- EP2210 Performance evaluation of communication networks (P1 2017)
- EO 3330 Advanced PhD Course on Network Calculus
- Master thesis
- PhD