M/G/1 with vacations

Example

$$- W = \underbrace{\frac{\lambda E[x^2]}{2(1-\rho)}}_{\text{like in M/G/1} \Rightarrow \text{example on the M/G/1 lecture} \Rightarrow = 1\,\text{min}} + \frac{E[v^2]}{2E[v]}$$

- Deterministic vacation periods

$E[v] = 1\,\text{min}$
$E[v^2] = E[v]^2 = 1\,\text{min}^2$

$$\Rightarrow \frac{E[v^2]}{2E[v]} = \frac{1}{2} \Rightarrow W = 1.5\,\text{min}$$

M/G/1 with non-preemptive priority

Prio 1 (highest prio)

$$W_1 = R_s + E[x_1] \cdot N_{q,1} = \underbrace{E[x_1] \cdot \lambda_1}_{\rho_1} W_1 \Rightarrow W_1 = \frac{R_s}{1-\rho_1}$$

Prio 2

$$W_2 = R_s + E[x_2] N_{q,2} + E[x_1] N_{q,1} + E[x_1] \lambda_1 W_2 =$$

$$= R_s + \underbrace{E[x_2] \lambda_2}_{\rho_2} W_2 + \underbrace{E[x_1] \lambda_1}_{\rho_1} W_1 + \underbrace{E[x_1] \lambda_1}_{\rho_1} W_2 =$$

$$= R_s + \rho_2 W_2 + \rho_1 W_1 + \rho_1 W_2$$

$$= R_s + \rho_2 W_2 + \rho_1 \cdot \frac{R_s}{1-\rho_1} + \rho_1 W_2$$

$$W_2(1-\rho_1-\rho_2) = \frac{R_s - \rho_1 R_s + \rho_1 R_s}{1-\rho_1}$$

$$W_2 = \frac{R_s}{(1-\rho_1)(1-(\rho_1+\rho_2))}$$

M/G/1 with preemptive resume prio

Prio 1: $R_{s,1} = \frac{1}{2} \lambda_1 E[x_1^2]$

$$W_1 = \frac{\lambda_1 E[x_1^2]}{2(1-\rho_1)} \Rightarrow \text{M/G/1}$$

Average "extended" service time

$$E[x_1'] = E[x_1]$$

$$E[x_2'] = E[x_2] + \underbrace{\lambda_1 E[x_1]}_{\rho_1} E[x_2']$$

$$E[x_2'] = \frac{E[x_2]}{1-\rho_1}$$