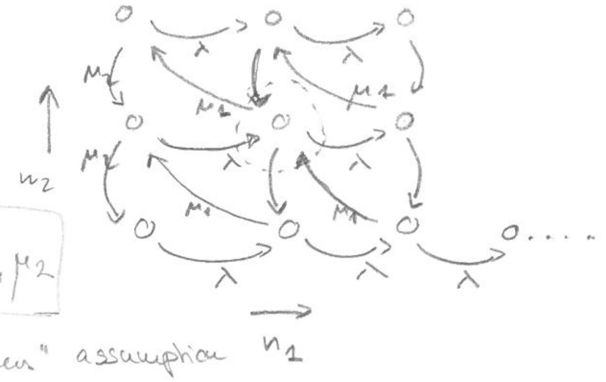
Queuing networks

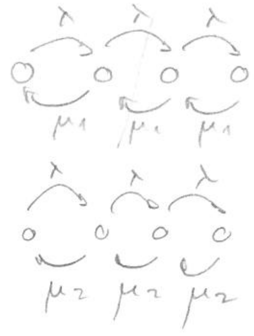
2017

Product form for tandem queues



$$\boxed{p_{ij}(\lambda+\mu_1+\mu_2) = p_{i-1,j}\lambda + p_{i+1,j-1}\mu_1 + p_{i,j+1}\mu_2}$$

We show that the "independent queu" assumption gives the same result.



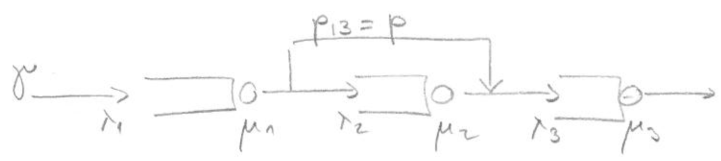
$$p_{i-1,j}\lambda = p_{i,j}\cdot\mu_1 \quad \text{or} \quad p_{i,j-1}\lambda = p_{i+1,j-1}\mu_1$$

$$p_{ij}\lambda = p_{i,j+1}\cdot\mu_2 \quad \text{or} \quad p_{i,j-1}\lambda = p_{ij}\mu_2$$

$$p_{ij}(\lambda+\mu_1+\mu_2) = p_{i,j+1}\mu_2 + p_{i-1,j}\lambda + p_{i,j-1}\lambda$$

$$= p_{i-1,j}\lambda + p_{i+1,j-1}\mu_1 + p_{i,j+1}\mu_2$$

Open queuing network.



$\lambda_1 = \gamma \quad \rho_1 = \frac{\gamma}{\mu_1} < 1$!

$\lambda_2 = (1-p)\lambda_1 = (1-p)\gamma \quad \rho_2 = \frac{(1-p)\gamma}{\mu_2} < 1$

$\lambda_3 = p\cdot\lambda_1 + \lambda_2 = (p+(1-p))\lambda_1 = \gamma$

$\rho_3 = \frac{\gamma}{\mu_3} < 1$

$p_{ij} = p_i(j) = P(j \text{ customers in queue } i)$

$p(\underline{n}) = \sum_{i=1}^{3} p_i(n_i)$

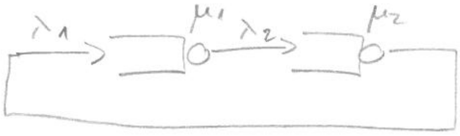
$$p(\text{empty network}) = P(0,0,0) = \prod_{i=1}^{3} p_{i0} \overset{M/M/1}{=} \prod_{i=1}^{3}(1-\rho_i)$$

$$N = \sum_{i=1}^{3} N_i = \sum_{i=1}^{3} \frac{\rho_i}{1-\rho_i}, \quad T = \frac{N}{\gamma}, \quad V = \frac{\sum_i \lambda_i}{\gamma} = \frac{\gamma + (1-p)\gamma + \gamma}{\gamma} = 3-p$$

2017.

Queuing networks

Closed queuing networks



$K=2$
$M=2$
$\lambda_1 = \lambda_2 \Rightarrow \underline{\lambda} = \{\lambda_1, \lambda_2\} = \alpha(1,1)$
$S = \{(2,0),(1,1),(0,2)\}$

$p_{10} = \left(1-\frac{\lambda}{\mu_1}\right), \quad p_{11} = \left(1-\frac{\lambda}{\mu_1}\right)\cdot\frac{\lambda}{\mu_1}, \quad p_{12} = \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^2$

$p_{20} = \left(1-\frac{\lambda}{\mu_2}\right) \quad \ldots.$

$P(2,0) = p_{12}\cdot p_{20} = \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^2\cdot\left(1-\frac{\lambda}{\mu_2}\right)$

$P(1,1) = p_{11}\cdot p_{21} = \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)\cdot\left(1-\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)$

$P(0,2) = p_{10}\, p_{22} = \left(1-\frac{\lambda}{\mu_1}\right)\cdot\left(1-\frac{\lambda}{\mu_2}\right)\cdot\left(\frac{\lambda}{\mu_2}\right)^2$

$P(2,0)+P(1,1)+P(0,2)=1$

e.g. $\mu_1=\mu_2$

$\Rightarrow p(2,0)=p(1,1)=p(0,2)=\frac{1}{3}$

$\Rightarrow \lambda$ (if needed)