Lecture 4

2017

Little's result

$\tau(t)$

$$N(\tau) = A(\tau) - D(\tau)$$

$$\int_0^t N(\tau) = \underbrace{\sum_{k=1}^{D(t)} T_k \cdot 1}_{\text{already served}} + \underbrace{\sum_{k=D(t)+1}^{A(t)} (t - t_k) \cdot 1}_{\text{waiting or under service}} \qquad \cdot \frac{1}{t} \qquad \cdot \frac{A(t)}{A(t)}$$

$$\frac{1}{t} \int_0^t N(\tau) = \frac{A(t)}{t} \underbrace{\left[\sum \dots + \sum \dots \right]}_{\text{sum of all system times}} \cdot \frac{1}{A(t)}$$

$t \to \infty$

$$\boxed{N = \lambda \cdot T}$$

PASTA for M/*/* queuing systems

- For poisson arrival we know: $P[A(t, t+\Delta t) \mid N(t) = k] = P[A(t, t+\Delta t)]$

  Arrivals are independent from the system state.

$$a_k(t) = \lim_{\Delta t \to \emptyset} P[N(t) = k \mid A(t, t+\Delta t)]$$

$$= \lim_{\Delta t \to \emptyset} \frac{P[N(t) = k, A(t, t+\Delta t)]}{P[A(t, t+\Delta t)]} =$$

$$= \lim_{\Delta t \to \emptyset} \frac{P[A(t, t+\Delta t) \mid N(t) = k] \, P(N(t) = k)}{P[A(t, t+\Delta t)]} =$$

$$= P[N(t) = k] = p_k(t)$$

In stable state $p_k(t) \to p_k$

$$\Rightarrow \boxed{a_k = p_k}$$