Department of Mathematics

SF1626
Several Variable Calculus
Academic year 2016/2017, Period 2

Seminar 5

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a hand-in of one of the problems. Solve problems 1-4 below and write down your solutions on separate sheets. Write your name and personal number on the top of each page. When the seminar starts you will be informed about which problem to hand in. Before starting with the seminar problems you should solve the recommended exercises from the text book Calculus by Adams and Essex (8th edition). These exercises are:

Section	Recommended problems
15.1	3, 5, 17,
15.2	3, 5, 7, 21
15.3	7, 11
15.4	1, 5, 7, 15
15.5	1, 7, 13
15.6	5, 9, 13, 15

PROBLEMS

Problem 1. Let $\mathbf{F}$ and $\mathbf{G}$ be the vector fields given by

$$\mathbf{F}(x,y) = \nabla f(x,y) \quad \text{and} \quad \mathbf{G}(x,y) = \left(y^2, -x^2\right)$$

where $f(x,y) = x^4y^2 + xy$ for (x,y) in $\mathbb{R}^2$. Let C_1 be the curve given by the staight line segment from $(2,4)$ to $(-1,3)$ and let C_2 be the closed curve given by the unit circle centered at the origin oriented counter-clockwise.

(a) Compute the line integral $\displaystyle\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$.

(b) Compute the line integral $\displaystyle\int_{C_1} \mathbf{G}\cdot d\mathbf{r}$.

(c) Compute the line integral $\displaystyle\int_{C_2} \mathbf{F}\cdot d\mathbf{r}$.

(d) Compute the line integral $\displaystyle\int_{C_2} \mathbf{G}\cdot d\mathbf{r}$.

Problem 2. The function f is given by

$$f(x, y) = 2x + x^2 + 2y^2, \qquad \text{for all } (x, y) \text{ in } \mathbb{R}^2.$$

(a) Determine a vector field such that its field lines are level curves of f.
(b) Are there several such vector fields?

Problem 3. Let $\mathbf{F}$ be the vector field which is given by

$$\mathbf{F}(x, y) = \left(\frac{ax + by}{x^2 + y^2}, \frac{cx + dy}{x^2 + y^2}\right)$$

away from the origin. Let C be the curve given by

$$\mathbf{r}(t) = (1, 1) + e^{-t}(\cos t, \sin t),$$

for $t \geq 0$. Let $D_1 = \{(x, y) : \ y > 0\}$ adn $D_2 = \{(x, y) : \ y > -1\}$.

(a) For which values of a, b, c, d is $\mathbf{F}$ conservative in the region D_1?
(b) For which values of a, b, c, d is $\mathbf{F}$ conservative in the region D_2?
(c) Use the potential of $\mathbf{F}$ in order to compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

when $a = d = 1$, $b = c = 0$.
In order to compute the line integral for an infinite curve you can first restrict to a finite part $0 \leq t \leq T$ and then compute the limit as $T \to \infty$.

Problem 4. Let S be the surface which in spherical coordinates [1] is given by

$$r = 3, \qquad -\pi/4 \leq \theta \leq \pi/4.$$

The orientation of the surface is such that its normal vector is pointing away from the origin.

(a) Sketch the surface S and compute its area.
(b) Compute the flux of the vector field $\mathbf{F}(x, y, z) = (x^2, y^2, z^2)$ through S.

[1] In the text book they use R instead of r for the distance to the origin. Here the spherical coordinates are written as r, θ and ϕ where ϕ is the angle to the positive z-axis.