Department of Mathematics

SF1626
Several Variable Calculus
Academic year 2016/2017, Period 2

Seminar 3

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a hand-in of one of the problems. Solve problems 1-4 below and write down your solutions on separate sheets. Write your name and personal number on the top of each page. When the seminar starts you will be informed about which problem to hand in. Before starting with the seminar problems you should solve the recommended exercises from the text book Calculus by Adams and Essex (8th edition). These exercises are:

Section	**Recommended problems**
12.8	13, 17
12.9	1, 3, 5, 7, 11
13.1	5, 7, 9, 19, 23, 25
13.2	3, 5, 9, 15
13.3	3, 9, 11, 15
13.4	1, 3

PROBLEMS

Problem 1. Let f be the function given by $f(x, y) = ax^3y^2 + y^2 - 4x^3$ for all (x, y) in $\mathbb{R}^2$, where $a \in \mathbb{R}$.

(a) Dertermine all critical points for f, when $a = 1$.
(b) Show that the only critical point for f is the origin when $a < 0$.
(c) Determine the Taylor polynomial of the second order for f at one of the critical points of f away from the origin when $a = 1$.

Consider: For $a \geq 0$ what are the characters of the critical points?

Problem 2. The function f is given by

$$f(x,y) = (\sin 2x - \sin 2y)^2$$

for all (x,y) in $\mathbb{R}^2$.

(a) Determine all critical points of f.
(b) Draw the critical points of f together with the level curve $f(x,y) = 0$. Mark which are maxima, which are minima, and which are neither.

Problem 3. Consider the probelm of finding the maximal and minimal values of the function given by $f(x,y,z) = 2x - y + z$ under the conditions $x^2 + y^2 + z^2 \leq 1$ and $2y \leq 1$.

(a) Sketch the region D given by the conditions.
(b) Look for critical points to f in the interior of D.
(c) Look for possible extremal points at the boundary of D by means of parametrization. (Notice that the boundary is a surface consisting of two parts intersecting along a circle. Parametrize the two parts individually. The circle will be the common boundary of the two surfaces.)
(d) Look for possible extremal points on the boundary of D by means of Lagrange's method. (Exactly as in (c) the two parts of the boundary need to be treated separately as well as their intersection.)
(e) Draw a conclusion about the maximal and minimal valus of f. How can we be sure that the method leads to the correct answer?

Problem 4. Consider the function f given by

$$f(x,y) = \frac{2 - xy}{2x^2 + y^2 + 2}$$

for all (x,y) in the closed disk $C: \{2x^2 + y^2 \leq 4\}$.

(a) Determine all interior critical points of f.
(b) Use a parametrization fo the boundary of C to find all extremal points of f on the boundary.
(c) Use Lagrange's method to find candidates of extremal points on the boundary of C. (Note that the denominator is constant on the boundary.)
(d) Use the results from (a), (b) and (c) in order to determine the maximal and minimal values of f in C.