Department of Mathematics

SF1626
Several Variable Calculus
Academic year 2016/2017, Period 2

Seminar 2

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text bookCalculus by Adams and Essex (8th edition) which are marked by boldface in the following list:

Avsnitt	Rekommenderade uppgifter
12.3	**5**, 7, **13**, 23
12.4	**5**, 7, **11**, 15, 17
12.5	**7**, 11, **17**, 21
12.6	3, **5**, **17**, 19
12.7	**3**, 5, 13, **17**, 25

In the seminar the following problems will be discussed.

PROBLEMS

Problem 1. When looking at the intersection between two graphs of functions $f(x, y)$ and $g(x, y)$ it could define a curve in three-space. For example the intersection of the graphs of the functions

$$f(x, y) = \sqrt{x^2 + y^2} \qquad \text{och} \qquad g(x, y) = 20 + x - y$$

gives a *conic section*, C, i.e., a degree two curve in the plane $x - y - z + 20 = 0$.

(a) Determine the tangent planes of the two graphs in the example at the point $(x, y, z) = (5, 12, 13)$.
(b) Compute normal vectors of the two graphs at the same point.
(c) How can we know that the tangent line to the conic section is orthogonal two the normal vectors of both graphs?
(d) Determine the tangent line to the curve C at the point $(x, y, z) = (5, 12, 13)$, for example by means of the cross product.

Problem 2. The vector values function $\mathbf{f}$ is given by $\mathbf{f}(\mathbf{r}) = \mathbf{v} \times \mathbf{r}$ where $\mathbf{v}$ is a constant vector $\mathbf{v} = (2, -1, 2)$.

(a) Compute the Jacobi matrix $D\mathbf{f}$.
(b) Show that $\mathbf{f}$ is a *linear* function.
(c) What relation does the matrix of $\mathbf{f}$ as a linear map have to the Jacobi matrix $D\mathbf{f}$?

Problem 3. A function $f(x, y)$ is called *harmonic* if it satisfies the Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

(a) Use the chain rule to show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (a^2 + b^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$$

if

$$\begin{cases} u = ax - by, \\ v = bx + ay, \end{cases}$$

where a and b are constants.

(b) Use (a) to show that the function $e^{2x+3y}\sin(3x - 2y)$ is harmonic.
(c) Show that the product of two harmonic functions is harmonic if their gradients are orthogonal to each other at every point.

Problem 4. The curve C with equation

$$y^2 = x^3 + x^2$$

can be parametrized by $\mathbf{r}(t) = (x(t), y(t)) = (t^2 - 1,\ t^3 - t)$, where t is a real parameter.

(a) Sketch the curve C or use a suitable software to plot it.
(b) Check that $\mathbf{r}(t)$ lies on the curve C for all t. How can we be sure that the parametrization reaches all points of C?
(c) Check that $\mathbf{r}'(t)$ is orthogonal to the gradient $\nabla f(\mathbf{r}(t))$ where

$$f(x, y) = y^2 - x^3 - x^2.$$

(d) Discuss why the vectors in (c) are orthogonal to each other.