KTH Teknikvetenskap

SF1626 Calculus in Several Variable
Solutions to the exam 2016-01-12

DEL A

1. Let D be the region above the x-axis in the xy-plane that is bounded by the circle $x^2+y^2 = 1$ and the lines $y = -x$ and $y = \sqrt{3}x$. Compute the x-coordinate of the centre of mass of D which is given by
$$\frac{\iint_D x\,dxdy}{\iint_D dxdy}.$$
(4 p)

Solutions.

Answer.

2. Compute the line integral
$$\int_C xy\,dx - y^2\,dy$$
where the curve C it the counter-clockwise oriented triangle with vertices in $(0,0)$, $(3,0)$ och $(0,4)$. **(4 p)**

Solutions.

Answer.

3. Let $f(x, y) = e^{x-y} - x + y + xy$.
   (a) Show that the origin is a critical point to the function f. **(2 p)**
   (b) Is the origin a local minimum, a local maximum or neither for the function f? **(2 p)**

Solutions.

Answer.

DEL B

4. Let $f(x, y) = xg(x + 2y)$ where g is a twice continuously differentiable function. Show that
$$\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y \partial x} + \frac{1}{4}\frac{\partial^2 f}{\partial y^2} = 0.$$
(4 p)

Solutions.

Answer.

5. A curve in the plane is parametrized by

$$\mathbf{r}(t) = (t^2 \cos t, t^2 \sin t),$$

where $t \geq 0$.

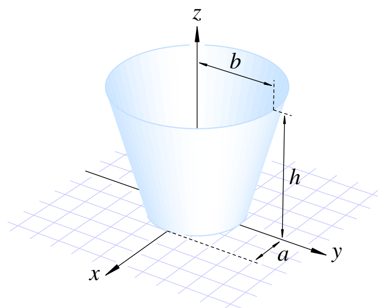
(a) Compute the velocity $\mathbf{r}'(t)$ and the speed $|\mathbf{r}'(t)|$ for $t \geq 0$. **(2 p)**

(b) Compute the arc length of the curve from the point $\mathbf{r}(0)$ to the point $\mathbf{r}(2)$. **(2 p)**

Solutions.

Answer.

6. A filled water tank has the shape of a straight truncated circular cone with the lower radius a, upper radius b, where $b > a$, and height h, according to the figure.



The conical part S of the tank can be parametrized by

$$\begin{cases} x = \left(\dfrac{b-a}{h}s + a\right)\cos t \\ y = \left(\dfrac{b-a}{h}s + a\right)\sin t \\ z = s \end{cases}$$

where $0 \le s \le h$, $0 \le t < 2\pi$. The force by which the water acts on the surface S has a z-component that is given by the flux of the vector field

$$\mathbf{P}(x,y,z) = \rho g(h-z)(0,0,1) = (0,0,\rho g(h-z))$$

out through the surface S, where ρ is the density of the water, and g is the gravitational acceleration. Compute this force component by means of computing the flux integral.

(4 p)

Solutions.

DEL C

7. Show that if x, y and z satisfy that $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x + 2y + 3z \leq 3$ then $xyz \leq 1/6$. **(4 p)**

Solutions.

Answer.

8. A solid K in space lies between the planes $z = 0$ and $z = 1$. In addition, we know that the intersection of the plane $z = a$ and the solid consists of a cirdular disk of radius a^2 for every a in the interval $0 \leq a \leq 1$.
   (a) Sketch two different such solids K. **(1 p)**
   (b) Do we know enough in order to determine the volume of the solid K? In that case, compute the volume, otherwise explain what is missing. **(3 p)**

Solutions.

Answer.

9. Let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y, z) = (x - 2, y + 1, z - 1)$$

and let C_P be an oriented smooth curve beginning at the origin and ending at the point $P = (x_0, y_0, z_0)$. Determine the point P that lies farthest away from the origin and which satisfies the condition

$$\int_{C_P} \mathbf{F} \cdot d\mathbf{r} = 9.$$

(4 p)

Solutions.