Department of Mathematics

SF1626 Calculus in Several Variable
Exam
Thursday, August 18, 2016

Skrivtid: 08:00-13:00
Allowed aids: none
Examinator: Mats Boij

The exam consists of nine problems each worth a maximum of four points.

Part A of the exam consists of the first three problems. To the score from part A your bonus points are added. The total score from part A can never exeeed 12 points. The bonus points are added automatically and the number of bonus points can be seen from the result page.

The three following problems make up part B and the three last problems part C, which is primarily intended for the higher grades.

The thresholds for the grades will be given by

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
Score on part C	6	3	–	–	–	–

In order to acheive full credit on a problem the solution has to be well presented and easy to follow. This means in particular that all notation should be defined, the logical structure clearly described in words or symbols and that the arguments are well motivated and clearly explained. Solutions that suffer seriously regarding this can not acheive more than two points.

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DEL A

1. Let D be the region above the x-axis in the xy-plane that is bounded by the circle $x^2+y^2=1$ and the lines $y=-x$ and $y=\sqrt{3}x$. Compute the x-coordinate of the centre of mass of D which is given by
$$\frac{\iint_D x\,dxdy}{\iint_D \,dxdy}.$$
(4 p)

2. Compute the line integral
$$\int_C xy\,dx - y^2\,dy$$
where the curve C it the counter-clockwise oriented triangle with vertices in $(0,0)$, $(3,0)$ och $(0,4)$. **(4 p)**

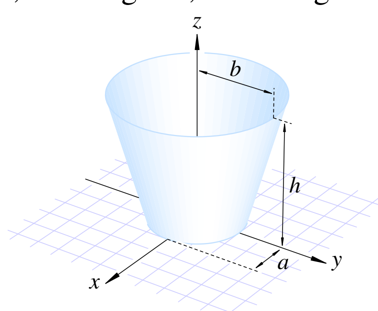
3. Let $f(x,y)=e^{x-y}-x+y+xy$.
   (a) Show that the origin is a critical point to the function f. **(2 p)**
   (b) Is the origin a local minimum, a local maximum or neither for the function f? **(2 p)**

DEL B

4. Let $f(x, y) = xg(x + 2y)$ where g is a twice continuously differentiable function. Show that
$$\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y \partial x} + \frac{1}{4}\frac{\partial^2 f}{\partial y^2} = 0.$$
(4 p)

5. A curve in the plane is parametrized by
$$\mathbf{r}(t) = (t^2 \cos t, t^2 \sin t),$$
where $t \geq 0$.
(a) Compute the velocity $\mathbf{r}'(t)$ and the speed $|\mathbf{r}'(t)|$ for $t \geq 0$. **(2 p)**
(b) Compute the arc length of the curve from the point $\mathbf{r}(0)$ to the point $\mathbf{r}(2)$. **(2 p)**

6. A filled water tank has the shape of a straight truncated circular cone with the lower radius a, upper radius b, where $b > a$, and height h, according to the figure.



The conical part S of the tank can be parametrized by
$$\begin{cases} x = \left(\dfrac{b-a}{h}s + a\right)\cos t \\ y = \left(\dfrac{b-a}{h}s + a\right)\sin t \\ z = s \end{cases}$$
where $0 \leq s \leq h$, $0 \leq t < 2\pi$. The force by which the water acts on the surface S has a z-component that is given by the flux of the vector field
$$\mathbf{P}(x, y, z) = \rho g(h - z)(0, 0, 1) = (0, 0, \rho g(h - z))$$
out through the surface S, where ρ is the desity of the water, and g is the gravitational acceleration. Compute this force component by means of computing the flux integral.
(4 p)

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DEL C

7. Show that if x, y and z satisfy that $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x + 2y + 3z \leq 3$ then $xyz \leq 1/6$. **(4 p)**

8. A solid K in space lies between the planes $z = 0$ and $z = 1$. IN addition, we know that the intersection by the plane $z = a$ and the solid consists of a cirdular disk of radius a^2 for every a in the interval $0 \leq a \leq 1$.
   (a) Sketch two different solids K with these properties. **(1 p)**
   (b) Do we know enough in order to determine the volume of the solid K? In that case, compute the volume, otherwise explain what is missing. **(3 p)**

9. Let $\mathbf{F}$ be the vector field
$$\mathbf{F}(x, y, z) = (x - 2, y + 1, z - 1)$$
and let C_P be an oriented smooth curve beginning at the origin and ending at the point $P = (x_0, y_0, z_0)$. Determine the point P that lies farthest away from the origin and which satisfies the condition
$$\int_{C_P} \mathbf{F} \cdot d\mathbf{r} = 9.$$
(4 p)