KTH Teknikvetenskap

SF1626 Calculus in Several Variable
Solutions to the exam 2016-01-12

DEL A

1. Let D be the quadrilateral with vertices in the points $(0,0)$, $(6,0)$, $(1,4)$ and $(5,6)$.
   (a) Sketch the quadrilateral D and compute its area. **(1 p)**
   (b) Determine the center of mass of the quadrilateral. **(3 p)**

Solutions.

(a)

2. The vector field $\mathbf{F}$ in the plane is given by $\mathbf{F}(x, y) = (y^2, 2xy + 1)$.
   (a) Determine whether $\mathbf{F}$ is conservative. **(1 p)**
   (b) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve parametrized by

$$\mathbf{r}(t) = \left(te^t, e^{t-1}\right), \qquad 0 \leq t \leq 1.$$

(3 p)

Solutions.

(a)

3. The plane curve C is parametrized by

$$\mathbf{r}(t) = \left(\sin t, \sin t + \sqrt{2}\cos t\right), \qquad 0 \leq t \leq 2\pi.$$

(a) Determine where a paticle travelling according to this parametrization has the highest speed. **(2 p)**

(b) The curve is closed and encloses a domain D with area given by the line integral

$$\int_C y\,dx.$$

Compute this area. **(2 p)**

Solutions.

(a)

$$\sin^2 t + \sin^2 t + 2\sin t\cos t + \cos^2 t = 1 + \frac{1-\cos 2t}{2} + \sin 2t = \frac{3}{2} + \sin 2t - \frac{1}{2}\cos 2t = \frac{3}{2} +$$

DEL B

4. Let $f(x, y) = 2x - 20y + x^2 - xy^2 + 2y^3$.
   (a) Compute the Taylor polynomial of degree two of the function f around the point $(1, 2)$. **(2 p)**
   (b) Use the Taylor polynomial to determine whether $(1, 2)$ as a critical point to f is a local maximum, local minimum or neither. **(2 p)**

Solutions.

(a)

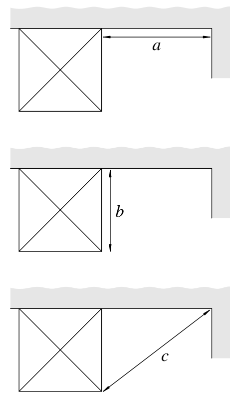
5. Compute the flux of the vector field $\mathbf{F}(x, y, z) = (x^2y, -y^2, yz^2)$ out through the boundary surface of the domain that is enclosed by the surface $y^2 + z^2 = 9$, the plane $x = 2$ and the coordinate planes. **(4 p)**

Solutions.

(a)

6. Osquar will furnish a cozy corner in his apartment and he needs to measure the distance a between a cupboard and the opposite wall.

   Since the area in between is not accessible for the moment it is difficult to measure the distance a directly, so instead he measures the distances b and c, according to the picture



   and he gets

$$b = 12 \pm 0{,}01 \text{ dm},$$
$$c = 20 \pm 0{,}01 \text{ dm}.$$

   Then he uses the Pythagorean theorem in order to determine an expression for a in b and c. Use linearization (linear approximation) of this expression in order to determine a with error margin. **(4 p)**

Solutions.

(a)

DEL C

7. (a) Show that a conservative vector field $\mathbf{F}(x,y,z)$ in three dimensions is *irrotational*, det vill säga uppfyller $\mathbf{curl}\,\mathbf{F} = \nabla \times \mathbf{F} = 0$. **(2 p)**

   (b) We say that a vector field $\mathbf{F}(x,y,z)$ in three dimensions has an *integrating factor* $f(x,y,z)$ if $f(x,y,z) \neq 0$ is a function such that the vector field $f\mathbf{F}$ is conservative. Show that if $\mathbf{F}$ has an integrating factor then $\mathbf{rot}\,\mathbf{F}$ is orthogonal to $\mathbf{F}$ everywhere. **(2 p)**

Solutions.

(a)

8. In computer science it is studied Inom datalogin studeras bland annat hur slumpträd uppdateras vid slumpmässigt borttagande av element. Då behöver trippelintegralen

$$\iiint\limits_{0 \le x \le y \le z \le 1} \left((y-x)^k - (z-y)^k\right) dx\,dy\,dz$$

beräknas där k är ett positivt heltal. Beräkna denna trippelintegral för alla $k > 0$.

(4 p)

Solutions.

(a)

9. Determine the highest point of the paraboloid $z = x^2 + 4y^2$ that is lit by a point light source in $(a, b, 0)$. **(4 p)**

Solutions.

(a)
