Department of Mathematics

SF1626 Calculus in Several Variable
Exam
Monday, March 21, 2016

Skrivtid: 08:00-13:00
Allowed aids: none
Examinator: Mats Boij

The exam consists of nine problems each worth a maximum of four points.

Part A of the exam consists of the first three problems. To the score from part A your bonus points are added. The total score from part A can never exeeed 12 points. The bonus points are added automatically and the number of bonus points can be seen from the result page.

The three following problems make up part B and the three last problems part C, which is primarily intended for the higher grades.

The thresholds for the grades will be given by

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
Score on part C	6	3	–	–	–	–

In order to acheive full credit on a problem the solution has to be well presented and easy to follow. This means in particular that all notation should be defined, the logical structure clearly described in words or symbols and that the arguments are well motivated and clearly explained. Solutions that suffer seriously regarding this can not acheive more than two points.

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DEL A

1. Let D be the quadrilateral with vertices in the points $(0,0)$, $(6,0)$, $(0,5)$ and $(4,5)$.
   (a) Sketch the quadrilateral D and compute its area. **(1 p)**
   (b) Determine the center of mass of the quadrilateral. **(3 p)**

2. The vector field $\mathbf{F}$ in the plane is given by $\mathbf{F}(x,y) = (y^2, 2xy+1)$.
   (a) Determine whether $\mathbf{F}$ is conservative. **(1 p)**
   (b) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve parametrized by
   $$\mathbf{r}(t) = \left(te^t, e^{t-1}\right), \qquad 0 \le t \le 1.$$
   (3 p)

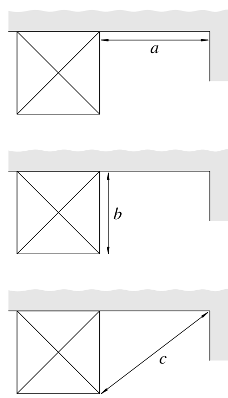
3. The plane curve C is parametrized by
   $$\mathbf{r}(t) = \left(\sin t, \sin t + \sqrt{2}\cos t\right), \qquad 0 \le t \le 2\pi.$$
   (a) Determine the highest speed for a particle travelling according to this parametrization. **(2 p)**
   (b) The curve C is closed and encloses a domain in the plane. The area of this domain is by Green's Theorem given by the line integral
   $$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C y\, dx,$$
   where $\mathbf{F}(x,y) = (y,0)$. Compute this area. **(2 p)**

DEL B

4. Let $f(x,y) = 2x - 20y + x^2 - xy^2 + 2y^3$.
   (a) Compute the Taylor polynomial of degree two of the function f around the point $(1,2)$. **(2 p)**
   (b) Use the Taylor polynomial to determine whether $f(x,y)$ has a local maximum, local minimum or neither at the point $(1,2)$. **(2 p)**

5. Let D be the quarter cylinder given by the inequalities
$$y^2 + z^2 \leq 9, \quad 0 \leq x \leq 2, \quad y \geq 0 \quad \text{and} \quad z \geq 0.$$
Compute the flux of the vector field $\mathbf{F}(x,y,z) = (x^2y, -y^2, yz^2)$ out through the boundary of D. **(4 p)**

6. Osquar will furnish a cozy corner in his apartment and he needs to measure the distance a between a cupboard and the opposite wall. Since the area in between is not accessible for the moment it is difficult to measure the distance a directly, so instead he measures the distances b and c, according to the picture



and he gets
$$b = 12 \pm 0{,}01 \text{ dm},$$
$$c = 20 \pm 0{,}01 \text{ dm}.$$
He uses the Pythagorean theorem in order to determine an expression for a in b and c. Use linearization (linear approximation) of this expression in order to determine a with error margin. **(4 p)**

Please turn page!

DEL C

7. In this problem we assume that all functions and vector fields are continuously differentiable.
   (a) Show that a conservative vector field $\mathbf{F}(x, y, z)$ in three dimensions is *irrotational*, that is, satisfies $\mathbf{curl}\,\mathbf{F} = \nabla \times \mathbf{F} = \mathbf{0}$. **(2 p)**
   (b) We say that a vector field $\mathbf{F}(x, y, z)$ in three dimensions has an *integrating factor* $f(x, y, z)$ if $f(x, y, z) \neq 0$ is a function such that the vector field $f\mathbf{F}$ is conservative. Show that if $\mathbf{F}$ has an integrating factor then $\mathbf{curl}\,\mathbf{F}$ is orthogonal to $\mathbf{F}$ everywhere. **(2 p)**

8. In computer science it is studied how random trees are updated at a random deletion of an element. In this study the triple integral

$$\iiint_{0 \leq x \leq y \leq z \leq 1} \left((y-x)^k - (z-y)^k\right) dx\, dy\, dz$$

needs to be computed where k is a positive integer. Compute this integral for all $k > 0$. (*Clue*: The complexity of the computations may depend on the order of integration.) **(4 p)**

9. Determine the highest point of the paraboloid $z = x^2 + 4y^2$ that is lit by a point light source in $(8, 3, 0)$. **(4 p)**