Department of Mathematics

**SF1626 Calculus in Several Variable**
**Exam**
**Tuesday, January 12, 2016**

Skrivtid: 08:00-13:00
Allowed aids: none
Examinator: Mats Boij

The exam consists of nine problems each worth a maximum of four points.

Part A of the exam consists of the first three problems. To the score from part A your bonus points are added. The total score from part A can never exeeed 12 points. The bonus points are added automatically and the number of bonus points can be seen from the result page.

The three following problems make up part B and the three last problems part C, which is primarily intended for the higher grades.

The thresholds for the grades will be given by

| Grade | A | B | C | D | E | Fx |
|---|---|---|---|---|---|---|
| Total score | 27 | 24 | 21 | 18 | 16 | 15 |
| Score on part C | 6 | 3 | – | – | – | – |

In order to acheive full credit on a problem the solution has to be well presented and easy to follow. This means in particular that all notation should be defined, the logical structure clearly described in words or symbols and that the arguments are well motivated and clearly explained. Solutions that suffer seriously regarding this can not acheive more than two points.

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## DEL A

1. Consider the function $f(x, y) = \dfrac{1}{1 + (1 - x)^2 + y^2}$.
   (a) Compute the gradient of $f$ at the origin. **(1 p)**
   (b) Which information about the shape of the graph of $f$ in a neighbourhood of the origin is given by the direction and the length of the gradient of $f$ at the origin, respectively? **(2 p)**
   (c) Does the function $f$ have a global minimum over the $xy$-plane? **(1 p)**

2. (a) Formulate Green's Theorem in the plane including all conditions. **(2 p)**
   (b) Use Green's Theorem in order to compute the line integral
   $$\int_T (x - 2x^2 y)\,dx + (2xy^2 - y)\,dy$$
   where $T$ is the boundary to the parallel trapezoid with vertices in the points $(0, 0)$, $(2, 0)$, $(2, 4)$ and $(0, 5)$ traversed counter-clockwise. **(2 p)**

3. The plane curve $C$ given by the equation $27y^2 = x(x - 9)^2$ can be parametrized by $\mathbf{r}(t) = (3t^2, 3t - t^3)$ where $t$ goes through the real line.
   (a) Check that the parameter curve is part of the curve $C$, i.e., that the points satisfy the equation for $C$. **(1 p)**
   (b) Compute the velocity $\mathbf{r}'(t)$ for the parameter curve. **(1 p)**
   (c) Write down the integral in the parameter $t$ that computes the arc length of the loop given by the interval $-\sqrt{3} \le t \le \sqrt{3}$. Simplify as much as possible. **(2 p)**

## DEL B

4. Compute the double integral
$$\iint_D xy\,dx\,dy$$
where $D$ is the domain which in polar coordinates is given by the inequalities
$$\begin{cases} 0 \le r \le \sin 2\theta, \\ 0 \le \theta \le \pi/2. \end{cases}$$
**(4 p)**

5. (a) Let $f(x, y)$ be a function of two variables. Explain what it means that a point $(x_0, y_0)$ is a critical point, local maximum and local minimum, respectively. **(2 p)**
   (b) The function $f(x, y) = e^{x-x^3/3-y^2}$ has critical points in $(1, 0)$ and $(-1, 0)$. Determine whether these are local maxima, local minima or neither. **(2 p)**

6. The curve $C$ is a connected part of the hyperbola $xy = 1$ from $(1, 1)$ to the point $P$. Determine $P$ when
$$\int_C (2x + y)\,dx + (x - 8y)\,dy = 3.$$
**(4 p)**

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## DEL C

7. Consider the equation $F(x,y)=0$ where $F(x,y)=xe^y+ye^x$.
   (a) Show that there is a function $g$ with $g(0)=0$ such that $F(x,g(x))=0$ for $x$ close to $0$. **(1 p)**
   (b) Compute the Taylor polynomial of degree two for $g$ at $x=0$. **(3 p)**

8. The function $f$ is given by
$$f(t)=\iint_D \exp\left(\frac{tx}{y^2}\right)\,dxdy$$
where $t>0$ and the domain $D$ is defined by $t\le x\le 2t$ and $t\le y\le 2t$. Show that
$$f(t)=Ct^2$$
for some constant $C$. **(4 p)**

9. Let $g(r)$ be a twice continuously differentiable function with $g'(4)=1$. Compute the integral
$$\iint_D \left(f''_{xx}+f''_{yy}\right)\,dxdy,$$
where $f(x,y)=g\left(x^2+y^2\right)$ and $D$ is a circular disk with radius 2 centered at the origin. **(4 p)**