Solutions, Tutorial 6, 2016

Solutions, Examination EF2240, 2010-10-21

1.

a)

I take the following values for day 131:

V_R = 250 km s^{-1} = 2.5·10^5 m s^{-1}
N_P = 10^{-1} cm^{-3} = 10^5 m-3

The standoff distance is given by

$$N_p m_p V_R^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 a}{4\pi r^3}\right)^2 \quad \Rightarrow$$

$$r = \left(\frac{\mu_0 a^2}{32\pi^2 N_p m_p V_R^2}\right)^{1/6}$$

<u>For day 131:</u>

$$N_p m_p V_R^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 a}{4\pi r^3}\right)^2 \quad \Rightarrow$$

$$r = \left(\frac{4\pi \cdot 10^{-7}\left(8 \cdot 10^{22}\right)^2}{32\pi^2 \cdot 10^5 \cdot 1.67 \cdot 10^{-27} \cdot \left(2.5 \cdot 10^5\right)^2}\right)^{1/6} = 1.16 \cdot 10^8 \text{ m} = 18.2 \text{ R}_\text{E}$$

<u>For day 130:</u>

V_R = 4.0·10^5 m s^{-1}
N_P = 4·10^6 m-3

$$r = \left(\frac{4\pi \cdot 10^{-7}\left(8 \cdot 10^{22}\right)^2}{32\pi^2 \cdot 4 \cdot 10^5 \cdot 1.67 \cdot 10^{-27} \cdot \left(4 \cdot 10^5\right)^2}\right)^{1/6} = 5.4 \cdot 10^7 \text{ m} = 8.4 \text{ R}_\text{E}$$

The ratio between the stand-off distance is 2.2

b)

$$\Psi = \arctan\left(\frac{\omega_{sun} r}{V_R}\right)$$

$$\omega_{sun} = \frac{2\pi}{25 \cdot 24 \cdot 3600} = 2.91 \cdot 10^{-6}$$

r = 1 AU

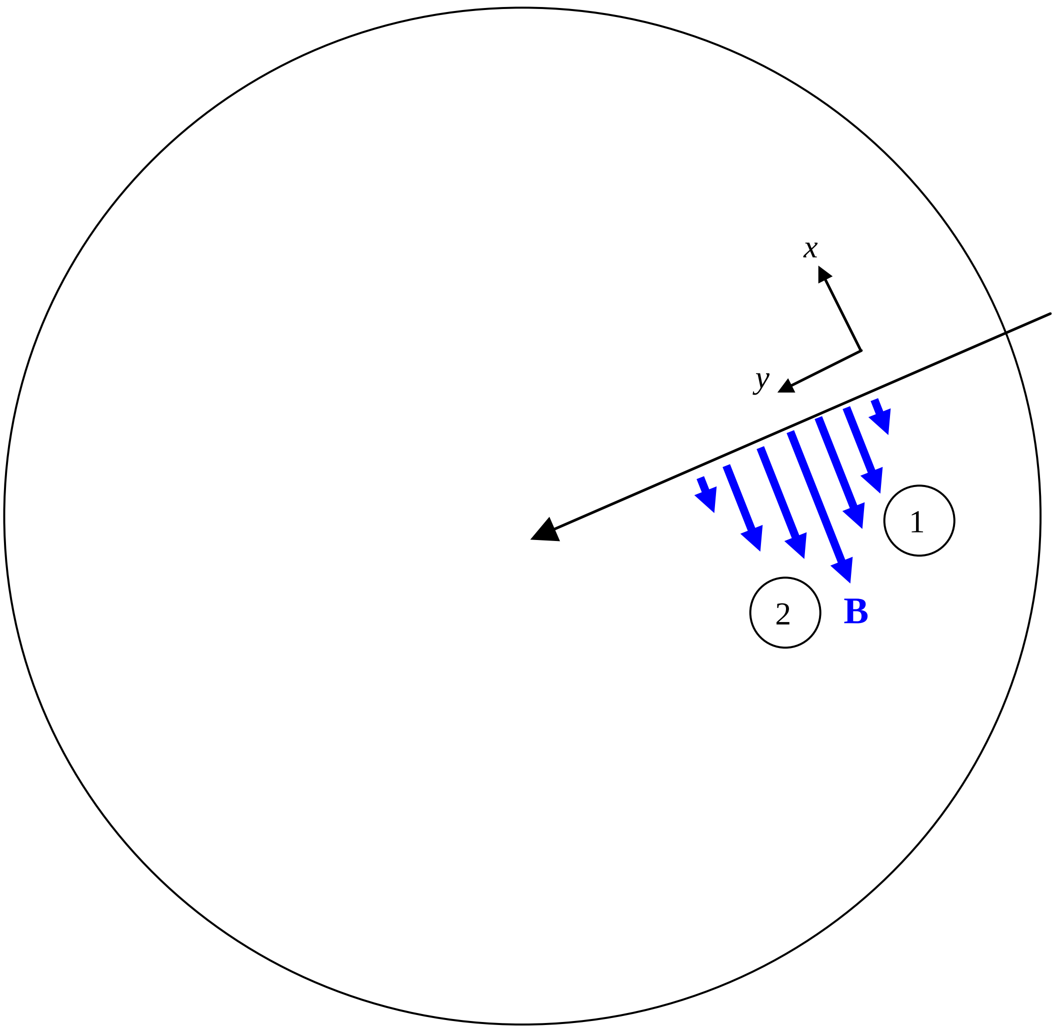
Then

$$\Psi = \arctan\left(\frac{4.36 \cdot 10^5}{V_R}\right)$$

$$\Psi_{\min} = \arctan\left(\frac{4.36 \cdot 10^5}{5 \cdot 10^5}\right) = 42.6^\circ$$

$$\Psi_{\max} = \arctan\left(\frac{4.36 \cdot 10^5}{2.5 \cdot 10^5}\right) = 61.5^\circ$$

2.



$$j_z = -\frac{1}{\mu_0}\frac{\partial B_y}{\partial x}$$

<u>Current sheet 1:</u>

$\frac{\partial B_y}{\partial x} < 0 \quad \Rightarrow \quad j_z > 0$ which means it is an upward current, which is consistent with the statistical result.

$$\Delta B_y \approx \frac{15\,\mathrm{mm}}{22\,\mathrm{mm}} \cdot 1000 \cdot 10^{-9} = 6.8 \cdot 10^{-7}\,\mathrm{T}$$

$$\Delta x \approx \frac{10\,\mathrm{mm}}{10\,\mathrm{mm}} \cdot \frac{2^\circ}{360^\circ} 2\pi \left(R_E + 800\,\mathrm{km}\right) = 250 \cdot 10^3\,\mathrm{m}$$

Then

$$j_z \approx -\frac{1}{\mu_0}\frac{\Delta B_y}{\Delta x} = 2.2 \cdot 10^{-6}\,\mathrm{Am^{-2}}$$

<u>Current sheet 2</u>

$\frac{\partial B_y}{\partial x} > 0 \quad \Rightarrow \quad j_z < 0$ which means it is an downward current, which is consistent with the statistical result.

$$\Delta B_y \approx \frac{18\,\mathrm{mm}}{22\,\mathrm{mm}} \cdot 1000 \cdot 10^{-9} = 8.2 \cdot 10^{-7}\,\mathrm{T}$$

$$\Delta x \approx \frac{10\,\mathrm{mm}}{10\,\mathrm{mm}} \cdot \frac{2^\circ}{360^\circ} 2\pi \left(R_E + 800\,\mathrm{km}\right) = 250 \cdot 10^3\,\mathrm{m}$$

Then

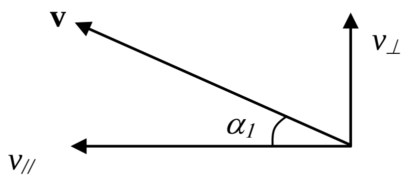
$$j_z \approx -\frac{1}{\mu_0}\frac{\Delta B_y}{\Delta x} = -2.6 \cdot 10^{-6}\,\mathrm{Am^{-2}}$$

3.

a)

$$\alpha_{lc} = \arcsin\sqrt{\frac{B_1}{B_2}} = \arcsin\sqrt{\frac{4000}{50000}} = 16.4^\circ$$

b)



W = $10^3 \cdot 1.6 \cdot 10^{-19}$

$$v = \sqrt{\frac{2W}{m_e}} = \sqrt{\frac{2\cdot 10^3 \cdot 1.6\cdot 10^{-19}}{0.91\cdot 10^{-30}}} = 1.88\cdot 10^7\,\text{ms}^{-1}$$
$$v_\perp = v\sin\alpha_1 = 6.43\cdot 10^6\,\text{ms}^{-1}$$
$$v_{//} = v\sin\alpha_1 = 1.77\cdot 10^7\,\text{ms}^{-1}$$

For the particle to be in the loss cone, we need to increase $v_{//}$ so that

$$\tan\alpha_{lc} = \tan 16.4^\circ = \frac{v_\perp}{v_{//,new}} \quad \Rightarrow$$

$$v_{//,new} = \frac{v_\perp}{\tan 16.4^\circ} = 2.18\cdot 10^7\,\text{ms}^{-1}$$

Then

$v_{//,new} - v_{//} = 0.41\cdot 10^7$ ms^{-1}.

c)

Thus the extra parallel energy needed is

$$\frac{m_e v_{//,new}^2}{2} - \frac{m_e v_{//}^2}{2} = \frac{0.91\cdot 10^{-30}}{2}\left(\left(2.18\cdot 10^7\right)^2 - \left(1.77\cdot 10^7\right)^2\right) = 7.36\cdot 10^{-17}\,\text{J} = 460\text{ eV}$$

which is the energy gained by an electron accelerated by 460 V potential drop, ($W = qV$), which is at the lower end of typical auroral acceleration potentials (typically 0.5-10 kV).

4.

a) Wien's displacement law gives

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{T} = \frac{2.9 \cdot 10^{-3}}{310} = 9.4 \cdot 10^{-6}\text{m} = 9400\ \text{nm} = 9.4\ \mu\text{m}.$$

This is infra-red radiation.

b)

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{4200} = 6.9 \cdot 10^{-7}\text{m} = 690\ \text{nm}$$

Dark red.

c)

$$P_{sun} = \sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2$$

$$r_{sun} = \frac{1.39 \cdot 10^9}{2}\ \text{m} = 6.95 \cdot 10^8\ \text{m}$$

$$P_{with\ spot} = \sigma_{SB} T_{sun}^4 \cdot \left(4\pi r_{sun}^2 - \pi r_{spot}^2\right) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2$$

Then

$$\frac{P_{with\ spot}}{P_{sun}} = \frac{\sigma_{SB} T_{sun}^4 \cdot \left(4\pi r_{sun}^2 - \pi r_{spot}^2\right) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2}{\sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2} = \frac{T_{sun}^4 \cdot \left(4 r_{sun}^2 - r_{spot}^2\right) + T_{spot}^4 r_{spot}^2}{T_{sun}^4 \cdot 4 r_{sun}^2} = \frac{6000^4 \cdot (4 \cdot (6.95 \cdot 10^8)^2 - (10^8)^2) + 4200^4 \cdot (10^8)^2}{6000^4 \cdot 4 \cdot (6.95 \cdot 10^8)^2} = 0.99607$$

or

$$\frac{P_{sun} - P_{with\ spot}}{P_{sun}} = 1 - \frac{P_{with\ spot}}{P_{sun}} = 1 - 0.99607 = 0.4\ \%$$

5.

a)

$$f_{day} = \frac{0.3}{0.9} + 6 = 6.3\,\mathrm{MHz}$$

$$f_{night} = \frac{0.1}{0.9} + 4 = 4.1\,\mathrm{MHz}$$

$$\left(2\pi f_{pe}\right)^2 = \frac{n_e e^2}{\varepsilon_0 m_e} \quad \Rightarrow$$

$$n_e = \varepsilon_0 m_e \left(\frac{2\pi f_{pe}}{e}\right)^2 = 0.0124\, f_{pe}^2$$

$$n_{e,day} = 4.9 \cdot 10^{11}\,\mathrm{m}^{-3}$$

$$n_{e,night} = 2.1 \cdot 10^{11}\,\mathrm{m}^{-3}$$

b)

$$\Delta t = 2\,\mathrm{h} = 7200\ \mathrm{s}$$

<u>Chapman layer: (See Tutorial 2, Problem 4)</u>

$$\frac{dn_e}{dt} = q - \alpha n_e^2$$

$$q = 0 \quad \Rightarrow$$

$$\frac{dn_e}{dt} = -\alpha n_e^2 \qquad \Rightarrow$$

$$\int \frac{dn_e}{n_e^2} = -\alpha \int dt \qquad \Rightarrow$$

$$-\frac{1}{n_e} = -\alpha t + C \qquad \Rightarrow$$

$$\alpha t = \frac{1}{n_e} + C$$

Determine C:

$$n_e(t=0) \equiv n_{e0} \quad \Rightarrow$$

$$C = -\frac{1}{n_{e0}} \quad \Rightarrow$$

$$\alpha t = \frac{1}{n_e} - \frac{1}{n_{e0}} \quad \Rightarrow$$

$$n_e = \frac{1}{\frac{1}{n_{e0}} + \alpha t} = \frac{n_{e0}}{1 + n_{e0}\alpha t} = \frac{4.9 \cdot 10^{11}}{1 + 3 \cdot 10^{-14} \cdot 4.9 \cdot 10^{11} \cdot 7200} = 4.6 \cdot 10^{9}\,\mathrm{m}^{-3}$$

<u>Bradbury layer:</u>

With $q = 0$, we get

$$\frac{dn_e(t)}{dt} = -\beta n_e(t) \quad \Rightarrow$$

$$\frac{dn_e}{n_e} = -\beta dt \quad \Rightarrow$$

$$\ln(n_e) + C = -\beta t$$

Let us rename the constant C to $-\ln(n_{e0})$. Then

$$\ln(n_e) - \ln(n_{e0}) = -\beta t \quad \Rightarrow$$

$$\ln\left(\frac{n_e}{n_{e0}}\right) = -\beta t \quad \Rightarrow$$

$$n_e = n_{e0} e^{-\beta t} = 4.9 \cdot 10^{11} \cdot e^{-\left(10^{-4} \cdot 7200\right)} = 2.4 \cdot 10^{11} \mathrm{m}^{-3}$$

<u>Conclusion:</u>

The Bradbury layer is the more realistic model, which reflects that atomic oxygen dominates over molecular oxygen at the altitude of the F2 region.

Solutions, Examination EF2240, 2011-10-21

1. a)

v = 6/11·500 m/s = 273 m/s

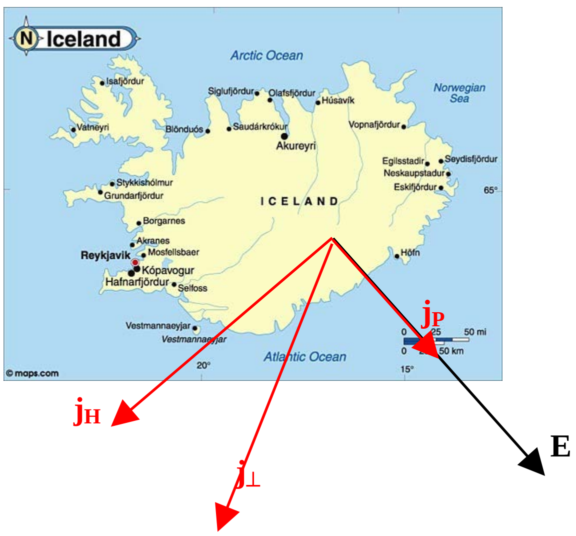
$$B = \sqrt{B_r^2 + B_\theta^2} = \sqrt{B_P^2 \left(\frac{R_E}{r}\right)^6 \cos^2\theta + \left(\frac{B_P}{2}\right)^2 \left(\frac{R_E}{r}\right)^6 \sin^2\theta} =$$

$$B_P \left(\frac{R_E}{r}\right)^3 \sqrt{\cos^2\theta + \frac{\sin^2\theta}{4}} = B_P \left(\frac{R_E}{R_E + 300\ \text{km}}\right)^3 \sqrt{\cos^2 25^\circ + \frac{\sin^2 25^\circ}{4}} =$$

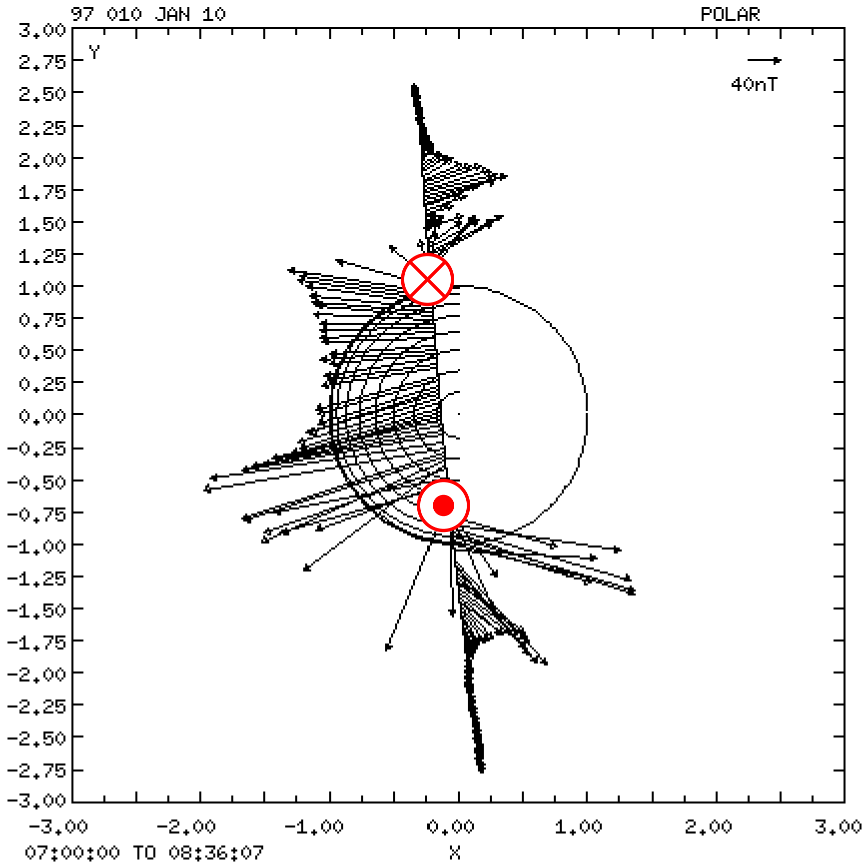
= 50 266 nT.
Then
$E = vB$ = 13.7 mV/m.

b)

Using solar maximum values at 100 km altitude, I get (night side values)
$\sigma_P = 8\cdot10^{-7}$ S/m
$\sigma_H = 7\cdot10^{-6}$ S/m
Then
$j_P = \sigma_P E = 7.3\cdot10^{-9}$ A/m^2
$j_H = \sigma_H E = 6.4\cdot10^{-8}$ A/m^2



2. a)



b)

For the upward current, e.g., we have

$$\left|j_{\square}\right| = \frac{1}{\mu_0}\frac{\partial B_x}{\partial y} \approx \frac{1}{\mu_0}\frac{(200+200)\cdot 10^{-9}}{0.3\cdot 6378\cdot 10^3} = 0.17\cdot 10^{-6}\ \mathrm{Am^{-2}}$$

3. a)

Using the scale of the sun, I estimate the radius of the CME to be

$$r_{CME}\,\frac{23}{3}r_{sun} = 5.3\cdot 10^9\ \mathrm{m}$$

From the plasma frequency, we get the number density:

$$n_e = \varepsilon_0 m_e \left(\frac{2\pi f_{pe}}{e}\right)^2$$

Assuming that the CME contains only of hydrogen ions, we get the mass density

$$\rho = n_e m_p$$

The total kinetic energy of the CME is then

$$\frac{mv^2}{2} = \rho\frac{4\pi r_{CME}^3}{3}\cdot\frac{v_{CME}^2}{2} = n_e m_p \frac{2\pi r_{CME}^3 v_{CME}^2}{3} = \varepsilon_0 m_e \left(\frac{2\pi f_{pe}}{e}\right)^2 m_p \frac{2\pi r_{CME}^3 v_{CME}^2}{3} = 1.3\cdot 10^{23}\,\mathrm{J}$$

b)

Evaluate the magnetic Reynolds number:

$$R_m = \mu_0 \sigma l_c v_c$$

We can use r_{CME} as the typical length scale, and v_{CME} as the typical velocity. Using a temperature of $2 \cdot 10^6$ K, we can evaluate the conductivity, remembering that the temperature should be given in eV. We get the conversion from

$$W = \frac{3}{2} k_B T$$

which gives the result that 1 eV corresponds to a temperature of 7729 K. We then get T = 259 eV, and

$\sigma = 7.9 \cdot 10^6$ S/m

Putting in the numbers I get

$R_m = 6.3 \cdot 10^{16} >> 1$

c)

Then the kinetic energy density is

$$\frac{1.3 \cdot 10^{23}}{4\pi r_{CME}^3 / 3} = 2.1 \cdot 10^{-7}\,\mathrm{Jm^{-3}}$$

From the gyro frequency, we get the magnetic field strength:

$$B = \frac{2\pi f_{ce} m_e}{e} = 6.1 \cdot 10^{-8}\,\mathrm{T}$$

The magnetic energy density is then

$$\frac{B^2}{2\mu_0} = 1.5 \cdot 10^{-9}\,\mathrm{T}$$

The ratio between the kinetic and magnetic energy densities is approximately 140, thus the plasma motion determines the magnetic field configuration, and not the other way around.

4. a)

Pressure balance between kinetic and magnetic pressure gives

$$\rho_{SW} v_{SW}^2 = \frac{B^2}{2\mu_0}$$

For a dipole field:

$$B^2 = B_r^2 + B_\theta^2 = (\frac{\mu_0 a}{2\pi}\frac{1}{r^3}\cos\theta)^2 + (\frac{\mu_0 a}{4\pi}\frac{1}{r^3}\sin\theta)^2$$

In the equatorial plane θ = 90°, and we get

$$B^2 = (\frac{\mu_0 a}{4\pi}\frac{1}{r^3})^2$$

If we assume that the solar wind contains only protons

$\rho = n_e m_p$

and the pressure balance becomes

$$n_e m_p v^2 = \frac{\mu_0^2 a^2}{16\pi^2}\frac{1}{r^6}\frac{1}{2\mu_0}$$

Letting the standoff distance be the Mercury radius r_M, we can solve for the velocity

$$v = \sqrt{\frac{\mu_0 a^2}{n_e m_p 32\pi^2 r_M^6}} = 504 \text{ km/s.}$$

b)

For Earth, the magnetic dipole moment is $8\cdot 10^{22}$ Am^2(Fälthammar p 85), and we can use a typical solar wind electron density of 8 cm^{-3}.

$$v = \sqrt{\frac{\mu_0 a_{Earth}^2}{n_e m_p 32\pi^2 r_{Earth}^6}} = 1.7\cdot 10^8 \text{ m/s} = 17\,000 \text{ km/s}$$

which is totally unrealistic.

c)

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d)

The standoff distance is

$$r = \left(\frac{\mu_0 a^2}{32\pi^2 n_{e,SW} m_p v_{SW}^2}\right)^{1/6}$$

For a solar wind velocity of 300 km/s we get a standoff distance of 2901 km.
Here the standoff distance is so small compared to the Mercury radius that we should really equate the gyro radius of the proton with $r - r_M$ = 461 km $=\Delta r$

$$r = \frac{m_p v}{eB}$$

We estimate the magnetic field to be constant with value B_0, from **c)**. Then

$$v = \frac{eBr}{m_p} = \frac{ev_{SW}\sqrt{2\mu_0 m_p n_{e,SW}}\,r}{m_p} = ev_{SW}r\sqrt{\frac{2\mu_0 n_{e,SW}}{m_p}} = 5.4\cdot 10^6 \text{ ms}^{-1}$$

which gives a kinetic energy of

$$\frac{m_p v^2}{2} = 2.5\cdot 10^{-14} \text{ J} = 0.15 \text{ MeV}.$$

5.

With FUV the flux of photons per unit area, the Strömgren radius is

$$r_S = \left(\frac{3N_{UV}}{4\pi\alpha_H n_H^2}\right)^{\frac{1}{3}} = \left(\frac{3\cdot 4\pi r_{star}^2 F_{UV}}{4\pi\cdot\alpha_H n_H^2}\right)^{\frac{1}{3}} \quad \Rightarrow$$

$$r_{star} = \left(\frac{r_s^3 \alpha_H n_H^2}{3F_{UV}}\right)^{\frac{1}{2}}$$

The temperature of 8000 K gives a recombination coefficient of

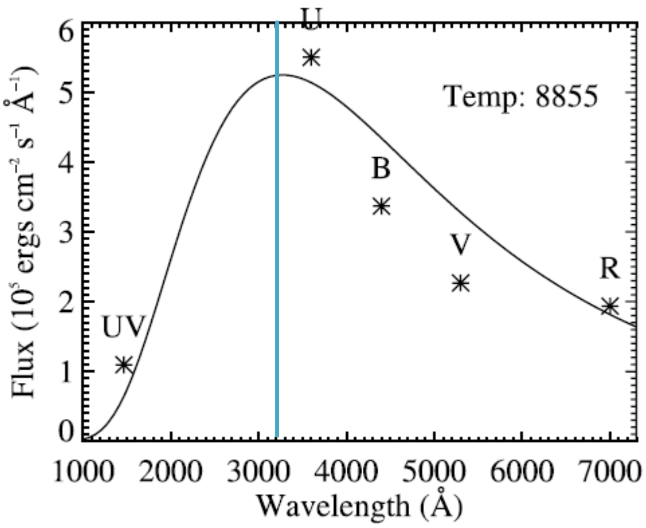
$\alpha_H = 2.4\cdot 10^{-19}$ m^{3}s^{-1}. Then

$r_{star} = 1.3\cdot 10^8$ m

Solutions, Exam 2013-10-30

1.

a)

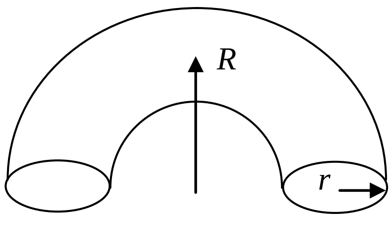


The maximum flux corresponds to a wavelength of 3200 Å = 320 nm. Wine's displacement law the gives

$$T = \frac{2.9 \cdot 10^{-3}}{\lambda_{\max} 2.9 \cdot 10^{-3}} = \frac{2.9 \cdot 10^{-3}}{320 \cdot 10^{-9}} = 9060 \text{ K.}$$

b)

Model the coronal loop by a half torus with minor axis r = 2 R_E, and major axis R = 8 R_E.



$$f_{ge} = \frac{\omega_{ge}}{2\pi} = \frac{eB}{2\pi m_e} \Rightarrow$$

$$B = \frac{2\pi m_e f_{ge}}{e} = \frac{2\pi \cdot 0.91 \cdot 10^{-30} \cdot 4 \cdot 10^{9}}{1.6 \cdot 10^{-19}} = 0.14 \text{ T}$$

Then let this half-torus be filled with a magnetic field of strength B = 0.14 T. If the volume of the half-torus is V and the magnetic energy density is p_B, the total energy magnetic is

$$W_B = Vp_B = \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi 8 R_E \pi (2R_E)^2 \frac{0.14^2}{2\mu_0} = 6.4 \cdot 10^{26} \text{ J}$$

To get the total thermal energy we need the number density, which we get from the plasma frequency:

$$n_e = \frac{(2\pi f_{pe})^2 \varepsilon_0 m_e}{e^2} = \frac{(2\pi \cdot 900 \cdot 10^9)^2 \cdot 8.854 \cdot 10^{-12} \cdot 0.91 \cdot 10^{-30}}{(1.6 \cdot 10^{-19})^2} = 1.0 \cdot 10^{22} \text{ m}^{-3}.$$

Then (assuming the ions and the electrons have the same temperature) the total thermal energy is

$$W_{Th} = Vp_{Th} = \pi R \pi r^2 (n_e + n_i) k_B T = \pi R \pi r^2 2 n_e k_B T =$$
$$= \pi \cdot 8 R_E \pi (2R_E)^2 2 \cdot 1.0 \cdot 10^{22} \cdot 1.38 \cdot 10^{-23} \cdot 9060 =$$
$= 5.1 \cdot 10^{25}$ J.

2.

The stand-off distance is given by

$$r = \left(\frac{\mu_0 a}{4\pi}\right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2\right)^{-1/6}$$

where the Earth’s magnetic dipole moment is $a = 8 \cdot 10^{22}$ Am2

Taking original values of

v_{sw} = 500 km/s
$n_{e,sw} = 5 \cdot 10^6$ m^{-3}

and values during the CME as

v_{CME} = 780 km/s
$n_{CME} = 15 \cdot 10^6$ m^{-3}

we get

r_{sw} = 8.4 R_E

and

r_{CME} = 6.1 R_E

which gives a difference of 2.3 R_E.

3.

Using the infinite current sheet approximation, we have (neglecting the sign)

$$j = \frac{1}{\mu_0}\frac{\partial B_x}{\partial y} \sim \frac{1}{\mu_0}\frac{\Delta B_x}{\Delta y}$$

From the figure, we get ΔB_x = 33 nT, and $\Delta y = \frac{8^\circ}{360^\circ} \cdot 4 \cdot 60260 \cdot 10^3 \cdot 2\pi$ = 3.8·10^7 m

Then

$$j =\sim \frac{1}{\mu_0}\frac{\Delta B_x}{\Delta y} = \frac{1}{\mu_0}\frac{33 \cdot 10^{-9}}{3.8 \cdot 10^7} = 6.9 \cdot 10^{-10} \text{ Am}^{-2}.$$

4.

a)

Pressure balance between the solar wind kinectic pressure and the magnetic pressure from the Mercury dipole magnetic field gives

$$\rho_{SW} v_{SW}^2 = \frac{B^2}{2\mu_0} \quad \Rightarrow$$

$$B = \sqrt{2\mu_0 \rho_{SW}}\, v_{SW} = \sqrt{2\mu_0 m_p n_{e,SW}}\, v_{SW} = \sqrt{2\mu_0 \cdot 1.67 \cdot 10^{-27} \cdot 27 \cdot 10^6} \cdot 250 \cdot 10^3$$
= 84 nT

where we have made the ordinary assumption that the solar wind consists only of protons and electrons, and we neglect the mass of the electrons as compared to the protons.

Knowing the magnetic field, we can calculate the gyro frequency and gyro radius

$$\omega_{ce} = \frac{eB}{m_e} = \frac{1.6\cdot 10^{-19}\cdot 84\cdot 10^{-9}}{0.91\cdot 10^{-30}} = 14.8 \text{ kHz}$$

$$r_L = \frac{m_e v_\perp}{eB} = \frac{m_e\sqrt{\dfrac{2\cdot\frac{2}{3}W}{m_e}}}{eB} = \frac{\sqrt{m_e\frac{4}{3}W}}{eB} = \frac{\sqrt{0.91\cdot 10^{-30}\cdot\frac{4}{3}\cdot 0.5\cdot 10^{3}\cdot 1.6\cdot 10^{-19}}}{1.6\cdot 10^{-19}\cdot 84\cdot 10^{-9}}$$

= 730 m

b)

Pressure balance in the equatorial plane in terms of the dipole field gives

$$\rho_{SW} v_{SW}^2 = \frac{B^2}{2\mu_0} \quad \Rightarrow$$

$$\rho_{SW} v_{SW}^2 = \frac{\left[\dfrac{\mu_0 a}{4\pi r^3}\right]^2}{2\mu_0} \quad \Rightarrow$$

$$a = \sqrt{2\mu_0 \rho_{SW} v_{SW}^2}\,4\pi r^3\,\frac{1}{\mu_0} = \sqrt{\frac{2 n_{SW} m_p v_{SW}^2}{\mu_0}}\,4\pi r^3 =$$

$$= \sqrt{\frac{2\cdot 27\cdot 10^{6}\cdot 1.67\cdot 10^{-27}\cdot\left(250\cdot 10^{3}\right)^2}{\mu_0}}\cdot 4\pi\left(1.4\cdot 2440\cdot 10^{3}\right)^3$$

= $3.4\cdot 10^{19}$ Am^{-2}.

c)

For the particle to reach the planet, its gyro radius has to be comparable to or greater than the stand-off distance r_s. Using the expression for the gyro radius,

$$r_L = \frac{m v_\perp}{eB}$$

we have (dropping the subscript on the velocity, since motion in the equatorial plane implies motion parallel to the magnetic field)

$$r_L = \frac{mv}{eB} > r_s \quad \Rightarrow$$

$$mv > eBr_s \quad \Rightarrow$$

$$v^2 > \left(\frac{eBr_s}{m}\right)^2 \quad \Rightarrow$$

$$\frac{mv^2}{2} > \frac{(eBr_s)^2}{2m} =$$

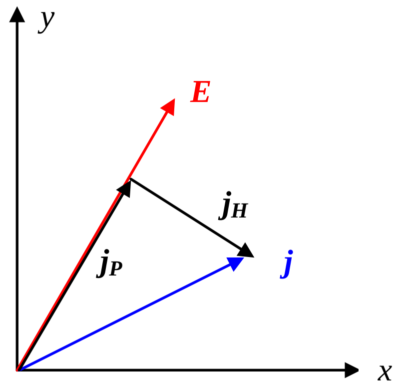
$$= \frac{\left(e \cdot 84 \cdot 10^{-9} \cdot 1.4 \cdot 2440 \cdot 10^3\right)^2}{2 \cdot 1.67 \cdot 10^{-27}}$$

= 7.9 MeV

5.

a)

Let us establish a coordinate system, in which x points in the Eastward direction, and y towards North.



In this coordinate system we can write the perpendicular electric field vector as

E = (10,20) mV/m

end the perpendicular current density as

j = (7,4) μAm^{-2}.

The Pedersen current is the projection of the current **j** on the electric field vector:

$$j_P = \frac{\mathbf{j} \cdot \mathbf{E}}{|\mathbf{E}|} = \frac{7 \cdot 10 + 4 \cdot 20}{\sqrt{10^2 + 20^2}} = 6.7\ \mu\text{Am}^{-2}.$$

Then

$$\sigma_P = \frac{j_P}{E} = \frac{6.7 \cdot 10^{-6}}{\sqrt{10^2 + 20^2} \cdot 10^{-3}} = 3 \cdot 10^{-4} \text{ Sm}^{-1}.$$

In vector form the Pedersen current can be written as

$$\boldsymbol{j_P} = j_P \frac{\boldsymbol{E}}{|\boldsymbol{E}|} = 6.7 \cdot (0.45,0.9) = (3.0,6.0) \ \mu\text{Am}^{-2}.$$

and the Hall current is

$$\boldsymbol{j_H} = \boldsymbol{j} - \boldsymbol{j_P} = (7,4) - (3.0,6.0) = (4.0,-2.0) \ \mu\text{Am}^{-2}$$

Then

$$j_H = \sqrt{4^2 + 2^2} = 4.5 \ \mu\text{Am}^{-2}$$

and

$$\sigma_H = \frac{j_H}{E} = \frac{4.5 \cdot 10^{-6}}{\sqrt{10^2 + 20^2} \cdot 10^{-3}} = 2.0 \cdot 10^{-4} \text{ Sm}^{-1}.$$

Solutions 2015-10-28

(1) $\Psi = \arctan \dfrac{\omega_\odot r}{u}$

$$\omega_\odot = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{27 \cdot 24 \cdot 3600} = 2.7 \cdot 10^{-6}\ s^{-1}$$

$$r = 1\ A.U. = 1.496 \cdot 10^{11}\ m$$

<u>$u = 320$ km/s:</u>

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 1.496 \cdot 10^{11}}{320 \cdot 10^{3}}\right) = 52^\circ$$

<u>$u = 1000$ km/s:</u>

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 1.496 \cdot 10^{11}}{1000 \cdot 10^{3}}\right) = 22^\circ$$

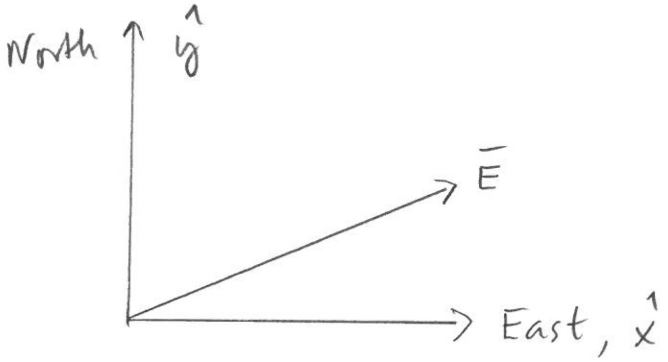
$$\Delta\Psi = 30^\circ$$

② i)

a) At 150 km:

$\sigma_P = 7 \cdot 10^{-5}$ S/m

$\sigma_H = 10^{-5}$ S/m



$\bar{E}_\perp = E_x \hat{x} + E_y \hat{y}$, $E_x = 10\,\text{mV/m}$, $E_y = 5\ \text{mV/m}$

$\bar{j} = \sigma_P \bar{E}_\perp + \sigma_H \dfrac{\bar{B} \times \bar{E}_\perp}{B}$, $\bar{B} = B\hat{z}$ $\Rightarrow$

$$j = \sigma_p (E_x \hat{x} + E_y \hat{y}) + \sigma_H \hat{z} \times (E_x \hat{x} + E_y \hat{y}) =$$

$$= \sigma_P E_x \hat{x} + \sigma_P E_y \hat{y} + \sigma_H E_x \hat{y} - \sigma_H E_y \hat{x} =$$

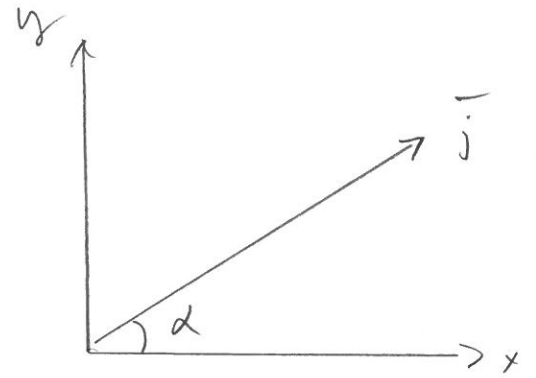
$$= (\sigma_P E_x - \sigma_H E_y)\hat{x} + (\sigma_P E_y + \sigma_H E_x)\hat{y} =$$

$$= (7 \cdot 10^{-5} \cdot 10 \cdot 10^{-3} - 10^{-5} \cdot 5 \cdot 10^{-3})\hat{x} +$$

$$+ (7 \cdot 10^{-5} \cdot 5 \cdot 10^{-3} + 10^{-5} \cdot 10 \cdot 10^{-3})\hat{y} =$$

$$= 6.5 \cdot 10^{-7}\,\hat{x} + 4.5 \cdot 10^{-7}\,\hat{y} \qquad (A/m^2)$$

ii)



$$\alpha = \arctan\left(\frac{j_y}{j_x}\right) = \arctan\left(\frac{4.5}{6.5}\right) = 35^\circ$$

$\bar{j}$ will have an angle of 35° to the Eastward direction.

$$|\bar{j}| = \sqrt{6.5^2 + 4.5^2} \cdot 10^{-7}\ A/m^2 =$$

$$= 7.9 \cdot 10^{-7}\ A/m^2 = 0.79\ \mu A/m^2$$

b) $\bar{E} = E_x \hat{x} \Rightarrow$

$\bar{j} = \sigma_p E_x \hat{x} + \sigma_H E_x \hat{y}$.

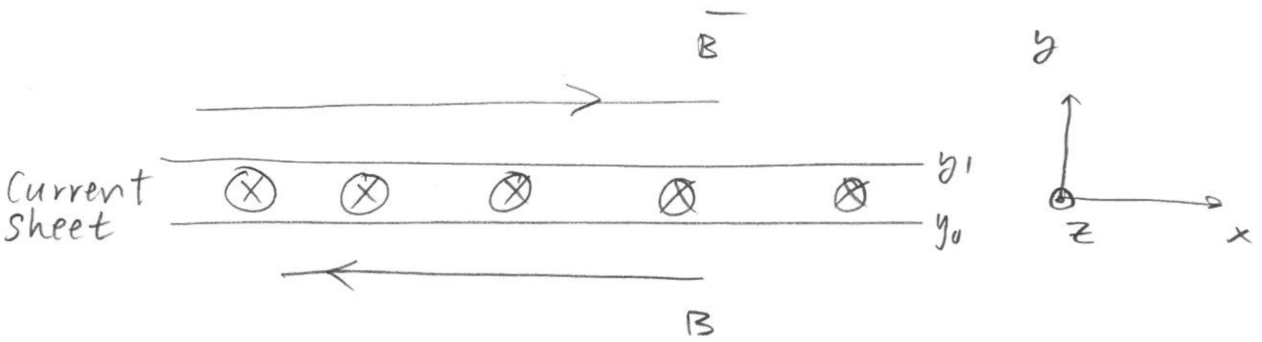
The angle is $45°$ when

$\sigma_p = \sigma_H$.

From the diagram this happens at an altitude of around

$\boxed{120 \text{ km}}$.

③ a) i)



$$j_z = -\frac{1}{\mu_0}\frac{\partial B_x}{\partial y} \Rightarrow$$

$$I = \int_{y_0}^{y_1} j_z \cdot dy \cdot L \quad , \quad L = 1\ R_J \Rightarrow$$

$$I = \frac{-L}{\mu_0}\int_{y_0}^{y_1} \frac{\partial B_x}{\partial y}\, dy = -\frac{L}{\mu_0}\left[B_x(y)\right]_{y_0}^{y_1} =$$

$$= \frac{-L}{\mu_0}\left(B_x(y_1) - B_x(y_0)\right) = \frac{-L}{\mu_0}\Delta B \Rightarrow$$

$$|I| = \frac{L}{\mu_0}|\Delta B|$$

<u>For $r \approx 45\ R_J$</u> :

$$\left.\begin{aligned} B_r(y_0) &= -18\ nT \\ B_r(y_1) &= 18\ nT \end{aligned}\right\} \Rightarrow |\Delta B| = 36\,nT \Rightarrow$$

$$|I| = \frac{R_J}{\mu_0}\cdot 36\cdot 10^{-9} = 2.0\cdot 10^{6}\ A$$

ii)

For $r \approx 30\ R_J$:

$$\left.\begin{aligned} B_r(y_0) &= -40\ nT \\ B_r(y_1) &= 30\ nT \end{aligned}\right\} \Rightarrow |\Delta B| = 70\ nT \Rightarrow$$

$$|I| = \frac{R_J}{\mu_0} \cdot 70\ nT = 3.9 \cdot 10^6\ A$$

Solutions, Examination EF2240, 2015-10-28

4.

a)

From the graph we get the wavelength for the maximum irradiation $\lambda_{max} \approx 0.45 \cdot 10^{-6}$ m. From Wien's displacement law

$$T = \frac{2.9 \cdot 10^{-3}}{\lambda_{\max}} = \frac{2.9 \cdot 10^{-3}}{0.45 \cdot 10^{-6}} = 6444\,\mathrm{K}$$

b)

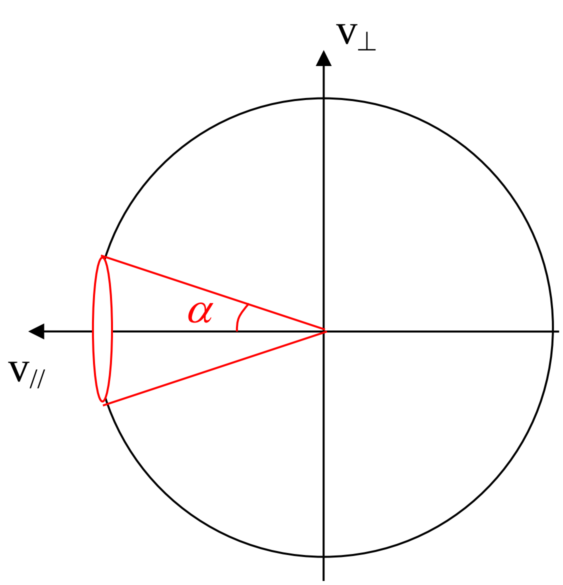
The magnetic fields at the top and the bottom of the loop are, respectively

$B_{top} = 0.1$ T
$B_{bot} = 0.4$ T

That means that the loss cone angle is

$$\alpha_{lc} = \arcsin\sqrt{\frac{B_{top}}{B_{bot}}} = \arcsin\sqrt{\frac{1}{4}} = 30^\circ$$

The isotropic particles cover a sphere in $v_{//}$ - $v_{\perp}$ space:



The solid angle of a cone is

$$\Omega = 2\pi(1 - \cos\alpha) = 0.84$$

Thus the ratio of particles in the loss cone is

$$\frac{0.84}{4\pi} = 7\%$$

5 Determin the standoff distance r_s first: i)

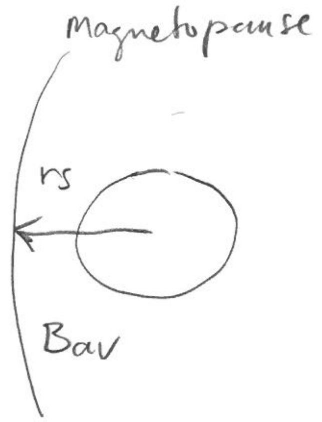
$P_{kin} = P_B \Rightarrow$

$$n_e m_p V_{sw}^2 = \frac{B^2}{2\mu_0} \Rightarrow$$

$$n_e m_p V_{sw}^2 = \left[\theta = \frac{\pi}{2}\right] = \frac{1}{2\mu_0}\left(\frac{\mu_0 a}{4\pi}\right)^2 \frac{1}{r^6} \Rightarrow$$

$$r = \left(\frac{\mu_0 a^2}{32\pi^2 n_e m_p V_{sw}}\right)^{1/6} =$$

$$= \left(\frac{4\pi\cdot 10^{-7}\cdot(3.0\cdot 10^{19})^2}{32\pi^2\cdot 35\cdot 10^6\cdot 1.67\cdot 10^{-27}\cdot(300\cdot 10^3)^2}\right)^{1/6} = 2966 \text{ km}$$

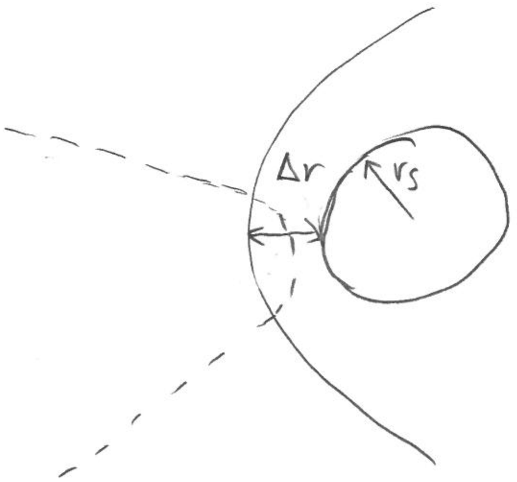


Assume Bav is equal to the average of B at the magnetopause and B at the planetary surface:

$$B_{av} = \left(\frac{\mu_0 a}{4\pi R_m^3} + \frac{\mu_0 a}{4\pi r_s^3}\right) / 2 =$$

$$= \frac{\mu_0 a}{8\pi}\left(\frac{1}{R_m^3} + \frac{1}{r_s^3}\right) = \frac{4\pi\cdot 10^{-7}\cdot 3\cdot 10^{19}}{8\pi}\left(\frac{1}{2440^3} + \frac{1}{2966^3}\right)\cdot\frac{1}{1000^3} =$$

$= 161 \cdot 10^{-9}$ T



The particle turns if its gyro radius r_L is smaller than Δr:

$$r_L = \frac{p_\perp}{q B_{av}} < \Delta r \implies$$

$$p_\perp < \Delta r \, q \, B_{av}$$

Relate momentum to energy, assume the particle is a proton, and drop the '$\perp$' sign:

$$E^2 = p^2 c^2 + m^2 c^4 \implies$$

$$p^2 = \frac{E^2 - m^2 c^4}{c^2} < (\Delta r \, q \, B_{av})^2 \implies$$

$$E^2 < (c \Delta r \, q \, B_{av})^2 + m^2 c^4 \implies$$

$$E^2 < \left(3\cdot10^8 \cdot 526\cdot10^3 \cdot 1.6\cdot10^{-19} \cdot 161\cdot10^{-9}\right)^2 + \left(1.67\cdot10^{-27}\right)^2 \left(3\cdot10^8\right)^4$$

$$= \underbrace{1.65\cdot10^{-23}}_{E_{kin}^2} + \underbrace{2.26\cdot10^{-20}}_{E_{rest}^2} = 2.260659\cdot10^{-20}\ \text{J}$$

$$E_{kin} = E - E_{rest} = \sqrt{E^2} - mc^2 = 5.5\cdot10^{-14}\ \text{J} =$$

$$= 3.4\cdot10^5\ \text{eV} = \underline{\underline{340\ \text{keV}}}$$

Compare if we calculate non-relativistically:

$$r_L = \frac{m v_\perp}{q B_{av}} < \Delta r \Rightarrow$$

$$v_\perp < \frac{q B_{av} \Delta r}{m} = \frac{1.6\cdot10^{-19}\ 161\cdot10^{-9} \cdot 526\cdot10^3}{1.67\cdot10^{-27}} =$$

$$= 8.1\cdot10^6\ \text{m/s} \Rightarrow$$

$$E_{kin} = \frac{mv^2}{2} = \frac{1.67\cdot10^{-27}\cdot\left(8.1\cdot10^6\right)^2}{2} = 5.5\cdot10^{-14}$$

$$= 3.4\cdot10^5\ \text{eV} = \underline{\underline{340\ \text{keV}}}$$

The velocity was small enough that we could have calculated non-relativistically. (But we couldn't know that, a priori.)