Solutions, Tutorial 3

1.

$$\mathbf{E} = E_x \hat{x}$$

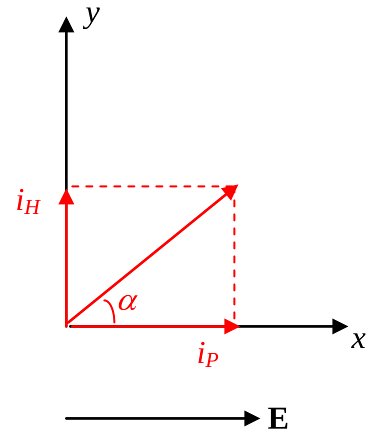
$$\mathbf{i}_{\parallel} = \sigma_{//} E_{//} = \sigma_{//} E_z = \sigma_{//} \cdot 0 = 0$$

$$\mathbf{B} = B_z \hat{z}$$

$$\mathbf{i}_{\perp} = \sigma_P \mathbf{E}_{\perp} + \sigma_H \frac{\mathbf{B} \times \mathbf{E}_{\perp}}{B} = \mathbf{i}_P + \mathbf{i}_H$$

$$\mathbf{i}_P = \sigma_P E_x \hat{x} = 0.8 \cdot 0.1\, \hat{x} = 0.08\, \hat{x}$$

$$\mathbf{i}_H = \sigma_H \frac{(B_z \hat{z}) \times (E_x \hat{x})}{B_z} = 1.2 \cdot 0.1\, \hat{y} = 0.12 \hat{y}$$

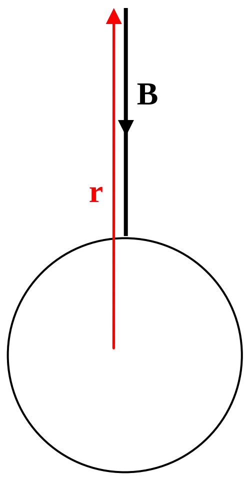


$$\tan\alpha = \frac{i_H}{i_P} = \frac{0.12}{0.08} \quad \Rightarrow$$

$$\alpha = 56^\circ$$

Note that we never needed $\sigma_{//}$, since $E_{//}$ was zero.

2.



r = R_E + 1000 km

The geomagnetic field strength is:

$$B = \sqrt{B_r^2 + B_\theta^2} = \sqrt{(B_p \frac{R_E^3}{r^3}\cos\theta)^2 + (\frac{B_p}{2}\frac{R_E^3}{r^3}\sin\theta)^2}$$

With $\theta = 0°$ this reduces to

$$B = B_p \frac{R_E^3}{r^3} = B_p \frac{R_E^3}{(R_E + 1000\,\text{km})^3} = 0.646 B_p = 40.1\,\mu\text{T}\ ,$$

since B_p = 62 μT (Fälthammar p. 85)

The induced electric field is given by

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

Since all the angles are 90° , and we are only interested in the absolute value of E, we get

$$E = vB = vB_p \frac{R_E^3}{(R_E + 1000\,\text{km})^3}$$

Thus

E = 0.28 V/m

3. a)

$$\mathbf{v} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

$$F = mg$$

$$\theta = \frac{\pi}{2},\ r = R_E + 1000\,\text{km},\ a = 8.0 \cdot 10^{22}\,\text{Am}^2 \quad \Rightarrow$$

$$B = \frac{\mu_0 a}{4\pi r^3} = 2.0 \cdot 10^{-5}\,\text{T}$$

Right angles, so

$$v = \frac{mg}{qB} = \frac{m \cdot 7.3}{1.6 \cdot 10^{-19} \cdot 2.0 \cdot 10^{-5}\,\mathrm{T}} = 2.3 \cdot 10^{24}\, m$$

$\because$

$$v_e = 2.3 \cdot 10^{24} \cdot 0.91 \cdot 10^{-30} = 2 \cdot 10^{-6}\,\mathrm{ms}^{-1}$$

$$v_O = 2.3 \cdot 10^{24} \cdot 16 \cdot 1.67 \cdot 10^{-27} = 0.06\,\mathrm{ms}^{-1}$$

b)

$$v_{E \times B} = \frac{E}{B} = \frac{10 \cdot 10^{-3}}{2.0 \cdot 10^{-5}\,\mathrm{T}} = 500\,\mathrm{ms}^{-1}$$

$$\frac{v_{E \times B}}{v_g} = \frac{500}{0.06} = 8140$$

4.

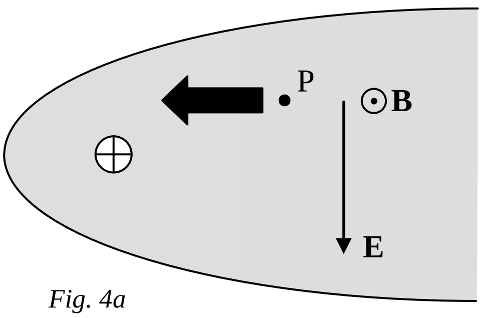


Fig. 4a

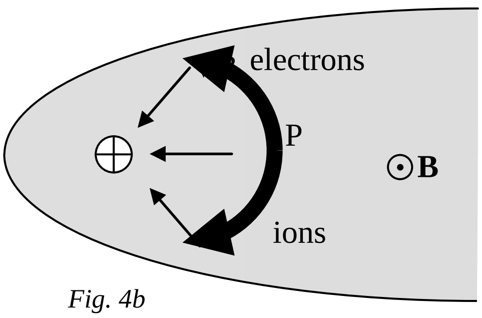


Fig. 4b

$$|\mathbf{v}| = \left| \frac{\mathbf{E} \times \mathbf{B}}{B^2} \right| = \frac{E}{B} = \frac{2.5 \cdot 10^{-3}}{120 \cdot 10^{-9}} = 20.8\, km\, s^{-1}$$

5.

I choose $n_e = 1$ cm^{-3}, and assume that the magnetosphere contains 50% protons and 50% oxygen ions. Then

$\rho = (0.5 \cdot m_p + 0.5 \cdot m_O) n_e = (0.5 + 0.5 \cdot 16) m_p n_e = 0.5 \cdot 17 \cdot\ m_p n_e = 1.4 \cdot 10^{-20}$ kg/m^3.

I approximate the magnetospheric volume by the volume of a cylinder with length 100 R_E and radius 10 R_E:

$V = \pi \cdot (10\ R_E)^2 \cdot 100\ R_E \approx 8 \cdot 10^{24}$ m^3.

The mass of the magnetosphere then becomes

$\rho V \approx 110$ tonnes

6.

Pressure balance between kinetic and magnetic pressure gives

$$\rho_{SW} v_{SW}^2 = \frac{B^2}{2\mu_0}$$

For a dipole field:

$$B^2 = B_r^2 + B_\theta^2 = (\frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos\theta)^2 + (\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \sin\theta)^2$$

In the equatorial plane $\theta = 90°$, and we get

$$B^2 = (\frac{\mu_0 a}{4\pi} \frac{1}{r^3})^2$$

If we assume that the solar wind contains only protons

$\rho = n_e m_p$

and the pressure balance becomes

$$n_e m_p v^2 = \frac{\mu_0^2 a^2}{16\pi^2} \frac{1}{r^6} \frac{1}{2\mu_0}$$

Solving for v, we get

$$v = \left(\frac{\mu_0 a^2}{32\pi^2 n_e m_p r^6} \right)^{\frac{1}{2}}$$

With the given numbers, we get

v = 504 km/s, which is not a totally unusual solar wind speed.