Solutions, Tutorial 1, 2016

1.

$$\rho = \frac{mv_\perp}{qB} \qquad f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\frac{qB}{m}$$

Since nothing is said about how large a part of the velocity is perpendicular to **B**, we assume that the particle kinetic energy perpendicular to **B** is $W_\perp = \frac{2}{3}W$ and get

$$\frac{2}{3}W = \frac{mv_\perp^2}{2} \quad \Rightarrow \quad v_\perp = \sqrt{\frac{\frac{4}{3}W}{m}}$$

$$\rho = \frac{m\sqrt{\frac{\frac{4}{3}W}{m}}}{qB} = \frac{\sqrt{\frac{4}{3}mW}}{qB}$$

a)

With W = 5 keV = 5·1000·1.6·10^{-19} , B = 50·10^{-6} T, m_e = 0.91·10^{-30} kg we get

$$\rho = \frac{\sqrt{\frac{4}{3}\cdot 0.91\cdot 10^{-30}\cdot 5\cdot 1.6\cdot 10^{-16}}}{1.6\cdot 10^{-19}\cdot 50\cdot 10^{-6}} = 3.9\,\text{m}$$

$$f_c = \frac{1}{2\pi}\frac{eB}{m_e} = \frac{1.6\cdot 10^{-19}\cdot 50\cdot 10^{-6}}{2\pi\cdot 0.91\cdot 10^{-30}} = 1.4\cdot 10^{6}\ \text{Hz}$$

The rest of the problems are solved in a similar way. Note that in c), the charge of the alpha particle is $q = 2e$, and its mass is approximately $4m_p$. You can use B = 0.3 T (compare exercise 3).

Answers for the remaining subproblems:

	ρ_L	f_c
a)	3.9 m	1.4 MHz
b)	167 m	762 Hz
c)	0.025 m	2.3 MHz
d)	75 km	0,08 Hz
e)	0.017 m	1.4 MHz

In **d)** we have used a typical solar wind magnetic field of 5 nT (See Fälthammar p. 123).

2.

TheParker spiral angle is given by

$$\Psi = \arctan\left(\frac{\omega_{sun} r}{u_{SW}}\right)$$

where

$$\omega_{sun} = \frac{2\pi}{T_{sun}} = \frac{2\pi}{27 \cdot 24 \cdot 2600} = 2.7 \cdot 10^{-6} \mathrm{s}^{-1}$$

and we can take u_{SW} to be its average value of 320 kms^{-1}.

a) At the closest approach to the sun (perihelion) r = 0.31 AU which gives

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 0.31 \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^{3}}\right) = 21^{\circ}$$

At aphelion r = 0.47 AU which gives

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 0.47 \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^{3}}\right) = 30^{\circ}$$

b) r = 1.0 AU $\Rightarrow$

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^{3}}\right) = 52^{\circ}$$

c) r = 4.95 AU (perihelion) $\Rightarrow$

$$\Psi = \arctan\left(\frac{2.7\cdot 10^{-6}\cdot 4.95\cdot 1.5\cdot 10^{11}}{320\cdot 10^{3}}\right) = 81^{\circ}$$

3.

The pressure inside and outside a sunspot is, respectively

$$p_{in} = 2n_{in}k_B T_{in} + \frac{B_{in}^2}{2\mu_0}$$

$$p_{out} = 2n_{out}k_B T_{out} + \frac{B_{out}^2}{2\mu_0}$$

The factor 2 comes from the fact that electrons and ions contribute nk_BT each.

$$p_{in} = p_{out} \quad \Rightarrow$$

$$n_{in} = \frac{2n_{out}k_B T_{out} + \frac{B_{out}^2}{2\mu_0} - \frac{B_{in}^2}{2\mu_0}}{2k_B T_{in}}$$

With

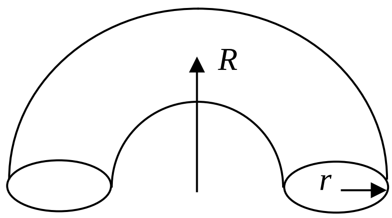
B_{in} = 0.3 T
B_{out} = 0 T
T_{in} = 4000 K
T_{out} = 6000 K
$n_{out} = 2.1\cdot 10^{19}$ cm^{-3}

we get

$$\frac{n_{in}}{n_{out}} = 1.48$$

4.

We know the size of flares is comparable to sunpots. Model the flare by a half torus with minor axis r = 1000 km, and major axis R = 10 000 km.



Let this half-torus be filled with a magnetic field of strength B ~ 0.3 T (similar to that in sunspots). If the volume of the half-torus is V and the magnetic energy density is p_B, the total energy is

$$W = V\,p_B = \pi R \pi r^2 \frac{B^2}{2\mu_0}$$

With the above numbers we get $W \sim 10^{24}$ J, which is similar to the energy released in a solar flare.

5.

a) Wien's displacement law gives

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{T} = \frac{2.9 \cdot 10^{-3}}{310} = 9.4 \cdot 10^{-6}\text{m} = 9400 \text{ nm} = 9.4 \text{ µm}.$$

This is infra-red radiation.

b)

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{4200} = 6.9 \cdot 10^{-7}\text{m} = 690 \text{ nm}$$

Dark red.

c)

$$P_{sun} = \sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2$$

$$r_{sun} = \frac{1.39 \cdot 10^9}{2} \text{ m} = 6.95 \cdot 10^8 \text{ m}$$

$$P_{with\ spot} = \sigma_{SB} T_{sun}^4 \cdot \left(4\pi r_{sun}^2 - \pi r_{spot}^2\right) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2$$

Then

$$\frac{P_{with\ spot}}{P_{sun}} = \frac{\sigma_{SB} T_{sun}^4 \cdot \left(4\pi r_{sun}^2 - \pi r_{spot}^2\right) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2}{\sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2}$$
$$= \frac{T_{sun}^4 \cdot \left(4 r_{sun}^2 - r_{spot}^2\right) + T_{spot}^4 r_{spot}^2}{T_{sun}^4 \cdot 4 r_{sun}^2}$$
$$= \frac{6000^4 \cdot (4 \cdot (6.95 \cdot 10^8)^2 - (10^8)^2) + 4200^4 \cdot (10^8)^2}{6000^4 \cdot 4 \cdot (6.95 \cdot 10^8)^2} = 0.99607$$

or

$$\frac{P_{sun} - P_{with\ spot}}{P_{sun}} = 1 - \frac{P_{with\ spot}}{P_{sun}} = 1 - 0.99607 = 0.4\ \%$$