## PROBLEM SET A

1. The stationary heat equation for a metal rod with one end held at fixed temperature g, a constant outflux of heat at the other end, and a given heat source function $f(x)$ is given by

$$\begin{aligned} -u''(x) &= f(x), \quad x \in (0,1) \\ u(0) &= g \\ u'(1) &= 1 \end{aligned}$$

   a) Derive the weak formulation of the problem.
   b) Introduce a uniform grid. Discretize the weak form of the differential equation using continuous piecewise linear functions, derive the finite element method, and give the elements of the matrix and the right hand side in the resulting linear system of equations.

2. Consider the Poisson equation:

$$\begin{aligned} -\Delta u(x) &= 1 \quad x \in \Omega \\ u(x) &= 0 \quad x \in \Gamma \end{aligned}$$

   with $x = (x_1, x_2)$, and $\Omega \subset \mathbb{R}^2$ the rectangular domain with corners in the points $(0,0)$, $(2,0)$, $(0,1)$ and $(2,1)$, and with boundary $\Gamma$.

   a) Define a finite element mesh $\mathcal{T}_h$ over the domain $\Omega$ with at least 3 internal (non-boundary) nodes.
   b) Define a discrete finite element approximation space $V_h$ over the mesh $\mathcal{T}_h$.
   c) Formulate a Galerkin finite element method (FEM) using the space $V_h$.
   d) Compute the corresponding stiffness matrix $A$ and load vector $b$. You do not have to solve the resulting system of equations.