KTH, Matematik, Maurice Duits

SF2795 Fourier Analysis
Homework Assignment for Lecture 16

1. Check that for any function f on G, the determinant of the $\#G \times \#G$ matrix $[f(x_1 g_2^{-1})]$ can be expressed as

$$\det[f(x_1 g_2^{-1})] = \prod_{\phi \in \hat{G}} \#G \hat{f}(\phi)$$

and us this to prove a primitive variant of Exercise 4.2.4

2. Check the following variant of the Poisson summation formula:

$$\sum_{G/H} \left| \sum f(gh) \right|^2 = \#G\#H \sum_{\phi \in (G/H)^\wedge} |\hat{f}(\phi)|^2.$$

Hint: apply the Plancherel identity to $f^0(g) = \sum_{h \in H} f(gh)$.

3. The group $G = \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$ (n-fold and with addition) is placed in 1:1 correspondence with the set $Q = 0, 1, \ldots, 2^n - 1$ by mapping

$$g = (k_0, \ldots, k_{n-1}) \to \sum_{j=0}^{n-1} k_j 2^j,$$

in which $k_j = 0$ or 1 for $0 \leq j \leq n$. This permits you to think of Q as a group isomorphic to G. Prove that $j : G \to Q$ is an isomorphism iff

$$j(g) = \sum_{i=0}^{n} [1 - e_i(g)] 2^{i-1}$$

in which $e_i : 0 \leq i < n$ is a basis of the dual group $\hat{G}$, that is to say, every character e can be expressed in precisely on way as a product $e = e_0^{k_0} \cdots e_{n-1}^{k_{n-1}}$ with $0 \leq k_i < 2$.

(For the interested student we refer to exercise 6 for an interesting continuation)