KTH, Matematik, Maurice Duits

SF2795 Fourier Analysis Homework Assignment for Lecture 15

1. (3.9.1) Deduce form the Szász Müntz Theorem that the power $x^{n_k}: k \geq 1$, augmented by $x^0 = 1$, span C([0,1]) iff $\sum n_k^{-1} = \infty$; "span" is understood to mean that

$$\inf \left\| f - c_0 - \sum_k C_j x^{n_k} \right\|_{\infty} = 0$$

for every $f \in C([0,1])$. Hint $||f||_{\infty} \le ||f'||_2$ for smooth functions vanishing at x=0.